Measuring Ages

Now that we have a framework with which to calculate the time since the Big Bang in different cosmological models, how does that age stack up against measured ages? Here we will discuss a few methods by which ages can be estimated. In a way similar to distance estimates, we note that any age of cosmological interest cannot be measured directly, since we haven't been around that long! We therefore require ways to extrapolate, anchored at small ages with direct measurements. We will start with radiometric dating, then follow it with discussions of fitting main sequence turnoffs and white dwarf cooling curves. Some of this information came from Ned Wright's excellent cosmology tutorial: http://www.astro.ucla.edu/∼wright/age.html.

Radiometric Dating

Some atomic nuclei are completely stable in isolation as far as we know. If you let a carbon-12 nucleus alone, it will apparently sit there unaltered until the end of time. In contrast, other nuclei change spontaneously with time. For example, carbon-14 (with six protons and eight neutrons) reaches a lower energy state via the decay

$$ {}^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + e^- + \bar{\nu}_e \ . \tag{1}$$

That is, carbon-14 decays into nitrogen-14, an electron, and an electron antineutrino. Laboratory experiments demonstrate that it is not possible to predict when any given nucleus will decay. In addition, the probability of decay in the next time interval dt is *completely independent* of how long the nucleus has lasted up to that point, be it an attosecond or a billion years. What this means is that the probability that a nucleus will decay at a time between t and $t + dt$ after its formation is

$$P(t)dt = e^{-t/t_0}dt \tag{2}$$

where t_0 depends on the particular nucleus. This is often phrased in terms of the *half-life* $t_{1/2}$, which is the time by which half the nuclei would have decayed: $t_{1/2} = \ln(2)t_0 \approx 0.7t_0$. Because nuclei are very compact compared to the distance between atoms, external effects have negligible impact on the spontaneous decay rates.

There are two points worth stressing here. The first is that the nature of this process allows half-lives to be measured even if they are extremely long compared to a laboratory timescale. For example, suppose that you have 10^{20} atoms of uranium-238. After one hour, you determine that there have been approximately 1.8 million decays. From the equation $P(t) = e^{-t/t_0}$ you know that in a time $t \ll t_0$ a fraction $\approx t/t_0$ of the nuclei will decay. Therefore, in this case $t_0 \approx 6.43 \times 10^9$ yr and $t_{1/2} \approx 4.46 \times 10^9$ yr.

The second point is the somewhat amusing one that regardless of how large t_0 is, the one-second period with the most decays will be the first one-second period after formation. The reason is that decays are independent of what has gone before (there is no “memory”), and after that first one-second period there are fewer undecayed nuclei!

This is all very well, but how does it help us determine ages? The simplest case would be one in which we know the initial amount of the radioactive substance. For later purposes, we will call the initial amount P, which will stand for “parent nucleus.” Similarly, D will be the initial amount of “daughter nucleus.” For example, ${}^{14}_{6}$C is a parent nucleus, and ${}^{14}_{7}$N is a daughter nucleus. Suppose that P_t is the amount of the parent nucleus at time t. The time is then simply

$$t = t_0 \ln(P/P_t) \ . \tag{3}$$

Fine, except that we *don't* know the initial quantity of the parent nucleus. Luckily there is a circumstance in which we can know something almost as valuable: the *ratio* of the initial quantity of a radioactive *isotope* of a nucleus to a nonradioactive isotope of the same element. I'm talking, of course, about radiocarbon dating, which since its discovery in 1949 by Willard Libby and colleagues has been the workhorse for dates within historical times.

Carbon, with six protons, has a very common stable isotope (carbon-12), an uncommon stable isotope (carbon-13), and a moderately common unstable isotope (carbon-14). Neutrons in cosmic rays entering the Earth's atmosphere can interact with nitrogen to form this isotope:

$$n +^{14}_{7} \mathrm{N} \rightarrow^{14}_{6} \mathrm{C} + p \ . \tag{4}$$

The cosmic rays do most of their work at high altitudes, 9–15 km, but the carbon gets taken up in carbon dioxide and spreads around all altitudes and latitudes. Plants acquire it during photosynthesis, and animals acquire it by eating plants. The net result is that for living things there is an approximately constant ratio

$$n_{{}^{14}_{6}\mathrm{C}}/n_{{}^{12}_{6}\mathrm{C}} \approx 10^{-12} \ . \tag{5}$$

After the animal or plant dies, however, there is no additional intake of ${}^{14}_{6}$C, so the ratio decreases steadily on the carbon-14 half-life $t_{1/2} = 5730$ years. As a result, measurement of the isotopic ratio (most sensitively using mass spectrometry) tells us the age of a given sample. In practice, ages beyond about 10 half-lives are inaccurate because there are so few carbon-14 nuclei left. However, within historical times this method is outstanding.

Naturally it isn't quite as simple as that. Various things can alter the steady-state fraction of ${}^{14}_{6}$C in the atmosphere, including fossil fuel burning (adding only ${}^{12}_{6}$C to the atmosphere), volcanic eruptions, and atomic bomb tests. People therefore use calibrated ages. These are the equivalent of direct distance measurements that are used to calibrate other methods. The idea is to have independent and exact knowledge of the age of something, then use that to calibrate radiocarbon estimates. One particularly cool way to determine ages independently uses tree rings, and is called dendrochronology. This goes back farther in time than you might at first think, because one can look at patterns of thick and thin rings and map back longer than any individual tree lived, to more than 10,000 years ago in some locations.

Isochron Dating

For astronomical purposes, though, radiocarbon dating is worthless because we want billions of years, not thousands! Luckily there are lots of radioactive isotopes, and some have half-lives of billions of years. Unluckily, there are no others for which we have independent measurements of the initial ratio of radioactive to non-radioactive isotopes of an element. What can be done?

When one does not know the initial concentration of the parent or daughter nucleus, the key is a method known as isochron dating (isochron="same time"). Consider strontium, which has many isotopes. $^{87}_{38}\mathrm{Sr}$ is a *radiogenic* nucleus (i.e., it can be generated by radioactive decay), and can be produced by beta decay from $^{87}_{37}\mathrm{Rb}$ (rubidium). In contrast, $^{86}_{38}\mathrm{Sr}$ is *non-radiogenic*, so it has no precursors. Both strontium isotopes are stable. The point is to note that *chemically*, there is no distinction between isotopes (because it's the electrons that do all the reacting), and as a result the isotopes are evenly spread. Therefore, consider a mineral from which one can extract many samples. In each sample, we measure the current amounts of (1) the radiogenic daughter isotope, (2) the non-radiogenic isotope, and (3) the parent isotope. Let us define D as the *initial* (and thus unknown) concentration of the radiogenic daughter, D_i as the initial, and constant, concentration of the non-radiogenic isotope, and P as the initial unknown concentration of the parent isotope. Also, define ΔP_t as the amount of the parent that has decayed into the radiogenic daughter after the unknown time t. Algebraic manipulation therefore gives

$$\frac{D+\Delta P_t}{D_i} = \frac{\Delta P_t}{P-\Delta P_t}\left(\frac{P-\Delta P_t}{D_i}\right) + \frac{D}{D_i}\ . \tag{6}$$

The left side is the *current* ratio of the radiogenic to non-radiogenic daughter isotope. The factor in parentheses is the *current* ratio of the parent to the non-radiogenic daughter. The factor before the parentheses is just related to how much of the parent has decayed and thus is a direct indicator of age. Therefore, multiple samples from a given rock should give a straight line when radiogenic/non-radiogenic is plotted against parent/non-radiogenic. The slope tells you the age, and the intercept tells you the initial ratio of radiogenic/non-radiogenic.

This method has the advantage that such a plot will give you an idea of its accuracy. For example, one could imagine a rock that had been heated at some point in its history so that the parent flows out but the daughter doesn't. Different places will have different fractional losses, leading to scatter in the relation. Figure 1 gives an example, using neodymium-samarium dating of moon rocks. Plagioclase dating is less certain than pyroxene/olivine dating, probably because a large impact event reheated plagioclase samples.

Measurements such as these have allowed very precise and reliable dating of the oldest meteorites in the Solar System, to 4.56 Gyr. We can therefore be confident that the universe is at least this old. How much farther back can we go?

Models of nuclear evolution

Even 4.56 Gyr would have posed problems with Edwin Hubble's initial measurement of $H_0 = 550$ km s^{-1} Mpc^{-1} (there were dust extinction issues, blah, blah), which would have implied an age of $2/(3H_0)$=1.2 Gyr! With the current H_0, though, we're still at least a factor of 2 away from problems.

But we don't think that the Solar System formed at the beginning of the universe! Instead, it formed from a nebula that itself had elements gathered from previous generations of stars. In a nebula you don't have chemical differentiation the way that you do in crystals. This means that there is only one measurement of elemental abundances possible, rather than the multiple ones that allow the linear fit crucial to isochron dating. We thus have only the absolute abundances from which to estimate ages.

At first sight, it seems that we're out of luck. In particular, if the initial abundances could be anything, then we would have no information at all about ages. As a result, people have turned to models of elemental production in supernovae. This is a tricky business, and it is worth keeping in mind that first-principles simulations of the *explosion* are a long way from incorporating all of the relevant physics. However, nucleosynthetic models have a lot of data from observations of supernovae and their remnants, so all in all it isn't too bad. A particularly useful isotope pair for comparison is ^{238}U and ^{232}Th, with respective half-lives of 4.468 Gyr and 14.05 Gyr. Wright quotes a recent paper by Dauphas (2005, Nature, 435, 1203), which compares abundances in the Solar System with those in metal poor stars to get an age of 12.3–17.3 Gyr.

The lower limit to that age is *well* above the age of 9 Gyr that one would get with $\Omega = 1$, $\Omega_k = \Omega_\Lambda = 0$ and the best current value for H_0. That indicates a problem, but without independent confirmation we might happily discount this age as systematically uncertain. Do we have other info?

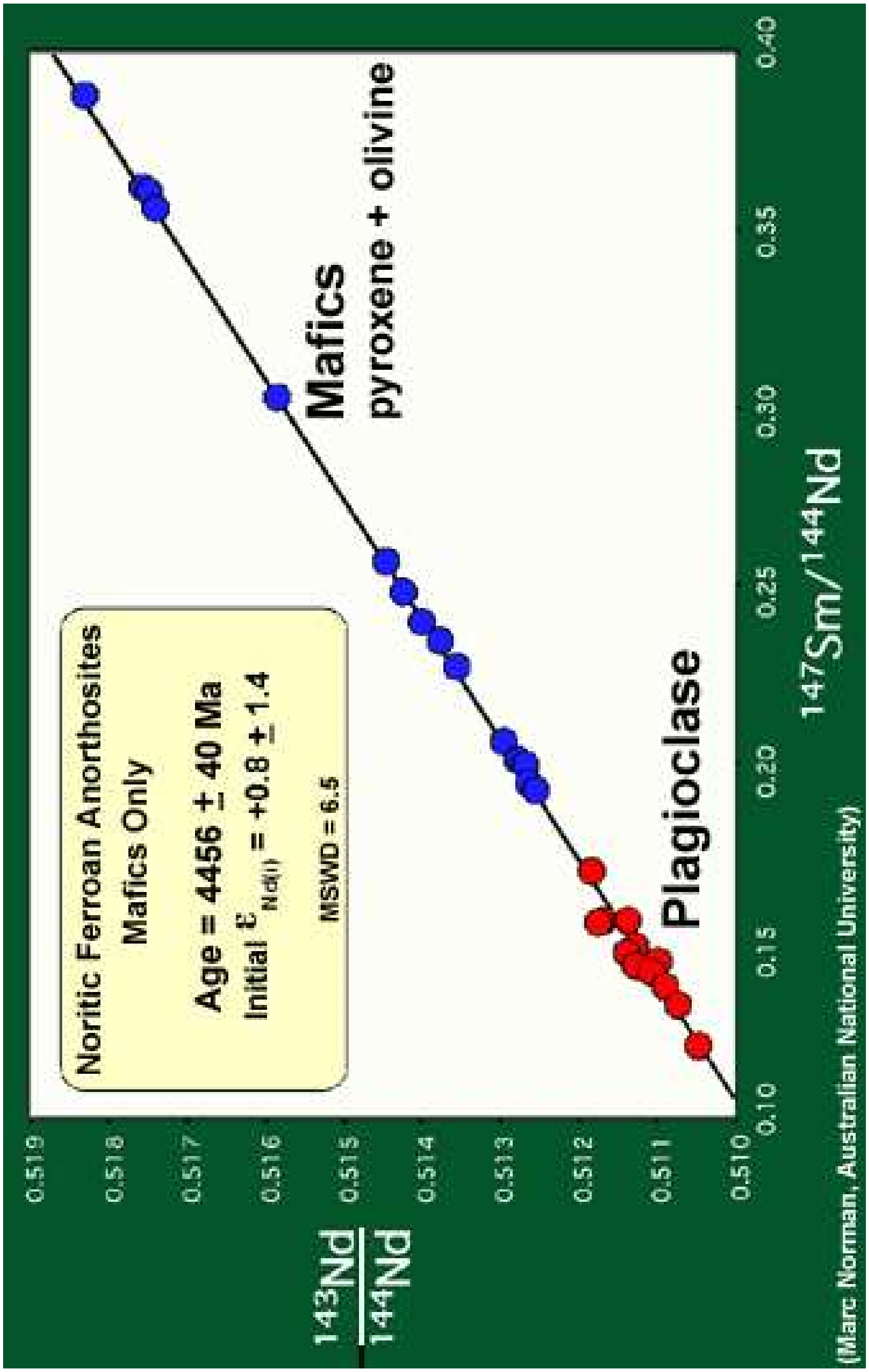

Fig. 1.— Dating of moon rocks using the isochron method. Pyroxene and olivine have a higher melting point than plagioclase minerals, leading to less scatter because they did not have differential leaking of parent and daughter nuclei. From http://www.psrd.hawaii.edu/WebImg/FAN_mafic_isochron.gif

Main Sequence Turnoff

Yes, of course we have other info or I wouldn't mention it! One approach is to look at globular clusters. These are collections of a few hundred thousand stars within a few to tens of parsecs that, from all indications, formed together in just a few million years. More importantly for our purposes, many of them appear to be extremely metal-poor and thus are excellent candidates for having formed early in the universe.

Therefore, we can use stellar modeling theory. The life of a star (especially one of low mass) depends primarily on its mass and only secondarily on other characteristics such as chemical composition. In particular, the time that it leaves the main sequence (where its energy source is the fusion of hydrogen into helium) is determined by its mass. This means that one can (1) construct a Hertzprung-Russell diagram for a cluster (plotting color versus luminosity, so this requires a distance measurement), and (2) use stellar evolutionary theory to determine the age based on the most luminous tip of the main sequence.

Note carefully that the distance to the cluster is needed for this: at a given flux, a more distant cluster has more luminous stars, which therefore live a shorter time. In fact, in the early 1990s the age problem appeared to have reached a crisis, but an overall shift of the distance scale (to larger distances for all objects) occurred because of more precise parallax measurements with the satellite Hipparcos. This (a) decreased inferred stellar ages in clusters, because the stars were more luminous than previously thought, and (b) increased the inferred age of the universe, because larger distances mean smaller H_0 and thus a larger age. The best current guess is that the oldest stars we can see currently are in the range of 11–13 Gyr.

White Dwarf Cooling

Another independent method comes from observations of white dwarfs. These are the compact remnants of the evolution of stars with initial mass $< 8\,M_\odot$, and once formed have no energy generation. They simply cool down indefinitely. In addition, they are relatively simple objects, being supported by electron degeneracy pressure, so cooling models are fairly simple although one does have to be careful about effects such as crystallization (which release energy due to the phase change). Hansen et al. (2004) estimate that in the globular cluster M4, the white dwarfs are 12.1 ± 0.9 Gyr old. The white dwarfs also had to have some time to form, so they estimate the age of the universe as 12.9 ± 1.1 Gyr from this method.

In summary, therefore, we can be pretty confident that the universe is $>$11 Gyr old. That is strongly in conflict with an $\Omega = 1$ universe and the best estimate of H_0, but in excellent agreement with the age of 13.7 ± 0.2 Gyr inferred from the cosmic microwave background, which implies $\Omega = 0.27$ and $\Omega_\Lambda = 0.73$. This is a great test of the overall cosmological model, and it is a tribute to the efforts of many scientists that we have a reasonably precise measure of an age that is a million times longer than human beings have had even the most primitive agriculture!

Intuition Builder

A form of radioactive decay we did not discuss is *electron capture*. In this process, an electron in an inner shell combines with a proton in the nucleus to form a neutron and give off a neutrino and some energy. Would you guess that such a process is more or less sensitive to environmental effects than spontaneous nuclear decay?