Cosmic Concordance

What does the power spectrum of the CMB tell us about the universe? For that matter, what is a power spectrum? In this lecture we will examine the current data and show that we now have remarkably tight constraints on several crucial cosmological parameters, although there is still substantial room for improvement and for self-consistency checks.

Power Spectra

Suppose that you have a one-dimensional function $f(x)$ that is defined in a finite region of dimension D. You can always Fourier decompose the function:

$$f(x) = \sum_{n=0}^{\infty} b_n \cos(2\pi nx/D + \phi_n) \ . \tag{1}$$

That is, all of the information is characterized by the set of b_n and ϕ_n. Larger n harmonics are what model smaller-scale structure in the function.

However, if we want to characterize the temperature fluctuations over the entire visible sky, we have a two-dimensional function $T(\theta, \phi)$. To decompose this we need a set of two-dimensional functions Y_{lm} called spherical harmonics, so that

$$T(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta, \phi) \ . \tag{2}$$

The azimuthal factor is simply $e^{im\phi}$, so

$$Y_{lm} = e^{im\phi} P_l^m(\cos\theta) \tag{3}$$

where the P_l^m are associated Legendre functions. What we find is that larger values of l correspond to smaller angular scales. For example, $l = 0$ is the overall constant, or the average value of T over the whole sky. $l = 1$ has an angular scale of $360/2 = 180°$, and is the angular dipole.

The theory of the temperature fluctuations is that each coefficient a_{lm} should have an average that depends only on l, not m. In addition, the distribution of the values should be Gaussian. This means that one can define an average

$$C_l \equiv \langle |a_{lm}|^2 \rangle \tag{4}$$

and thus characterize the power spectrum completely. Note that there are $2l + 1$ values of m for each l. Consider a small value of l, say $l = 3$. With only 7 independent measurements to average, the sample average could be significantly different than the "true" average that we would get for the cosmological parameters that determine the a_{lm}. That is, even with no measurement error at all, in our one and only universe we might happen to have fluctuations that give smaller or larger values of C_l than would be expected. This is called "cosmic variance". However, at larger l the number of independent measurements of a_{lm} becomes big enough that cosmic variance is no longer a limiting factor. It also becomes large enough that it is possible to do reasonably high-precision tests of the assumption of Gaussianity.

With this background in mind, we can now see what the data say.

The Temperature Power Spectrum

We say the temperature because as we will see later, there is also polarization information. For an outstanding pedagogical discussion of the physics behind this spectrum, see Wayne Hu's pages at http://background.uchicago.edu/∼whu/intermediate/intermediate.html. The only drawback is that this series of pages does not incorporate the data from the WMAP experiment, but all the physics is updated. I will follow much of his discussion.

After the initial discovery of the CMB in 1965 by Penzias and Wilson, and independently by Dicke and colleagues at the same time, many instruments were designed to characterize the CMB more and more precisely. The first of the fluctuations were discovered with the COBE instrument in the early 1990s, and through 2001 many ground-based instruments contributed, including the clear discovery of the first acoustic peak. At this moment, however (late 2007), for l values of a few hundred or less the best current data by far come from the Wilkinson Microwave Anisotropy Probe (WMAP), so we will concentrate on that.

The temperature map from the first three years of WMAP data is shown in Figure 1, and the power spectrum of this map is in Figure 2. We will concentrate on the information in the power spectrum.

The First Acoustic Peak

Starting from the left (low l, high angular scale), the first obvious feature is the first peak, at an angular scale of slightly less than 1° (l close to 200). As we discussed last time, the reason for this peak is that sound waves of the right frequency would have had just enough time to reach maximum compression when the universe became transparent. What can we learn from the measured angular scale?

Recall that the *physical* scale of the fundamental acoustic mode is well understood. This means that the angle we see on the sky depends only on the angular diameter distance to the

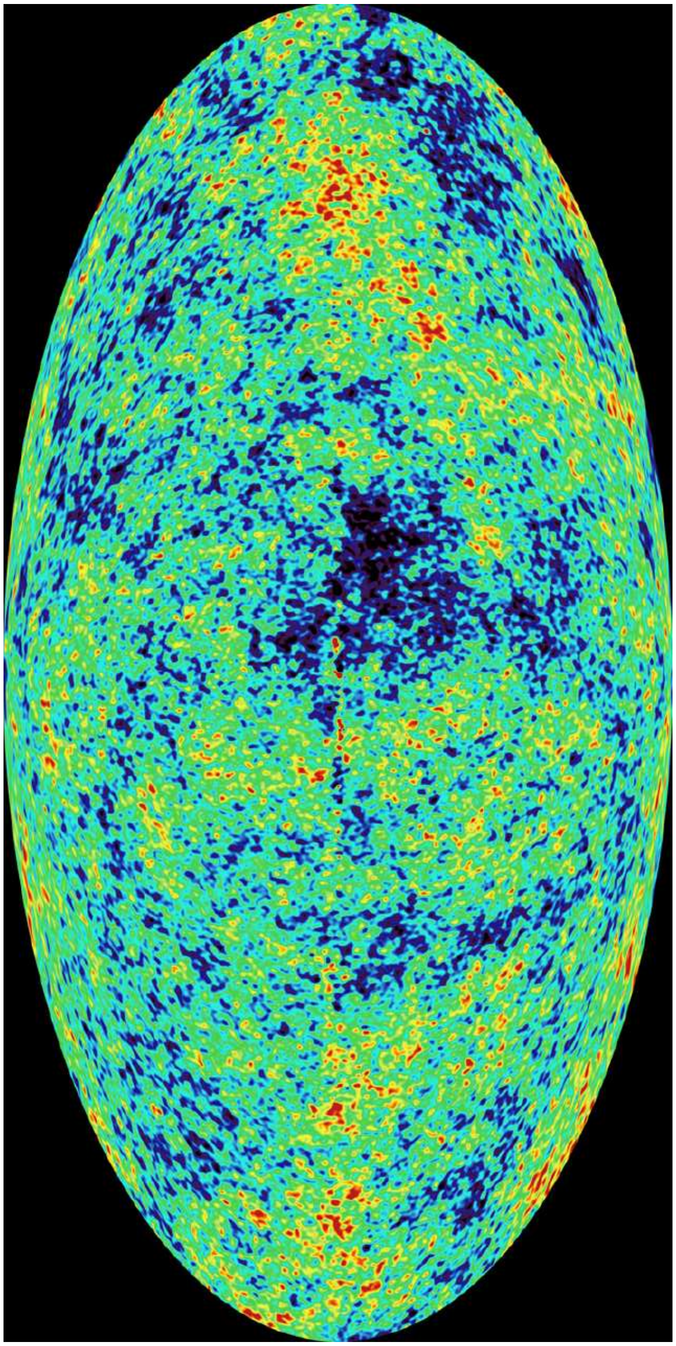

Fig. 1.— Sky map of the third-year WMAP temperature anisotropies. This map has subtracted out the average temperature, the dipolar contribution from our movement, and the Galaxy's foreground. By eye, you can see that the spots have a characteristic size.

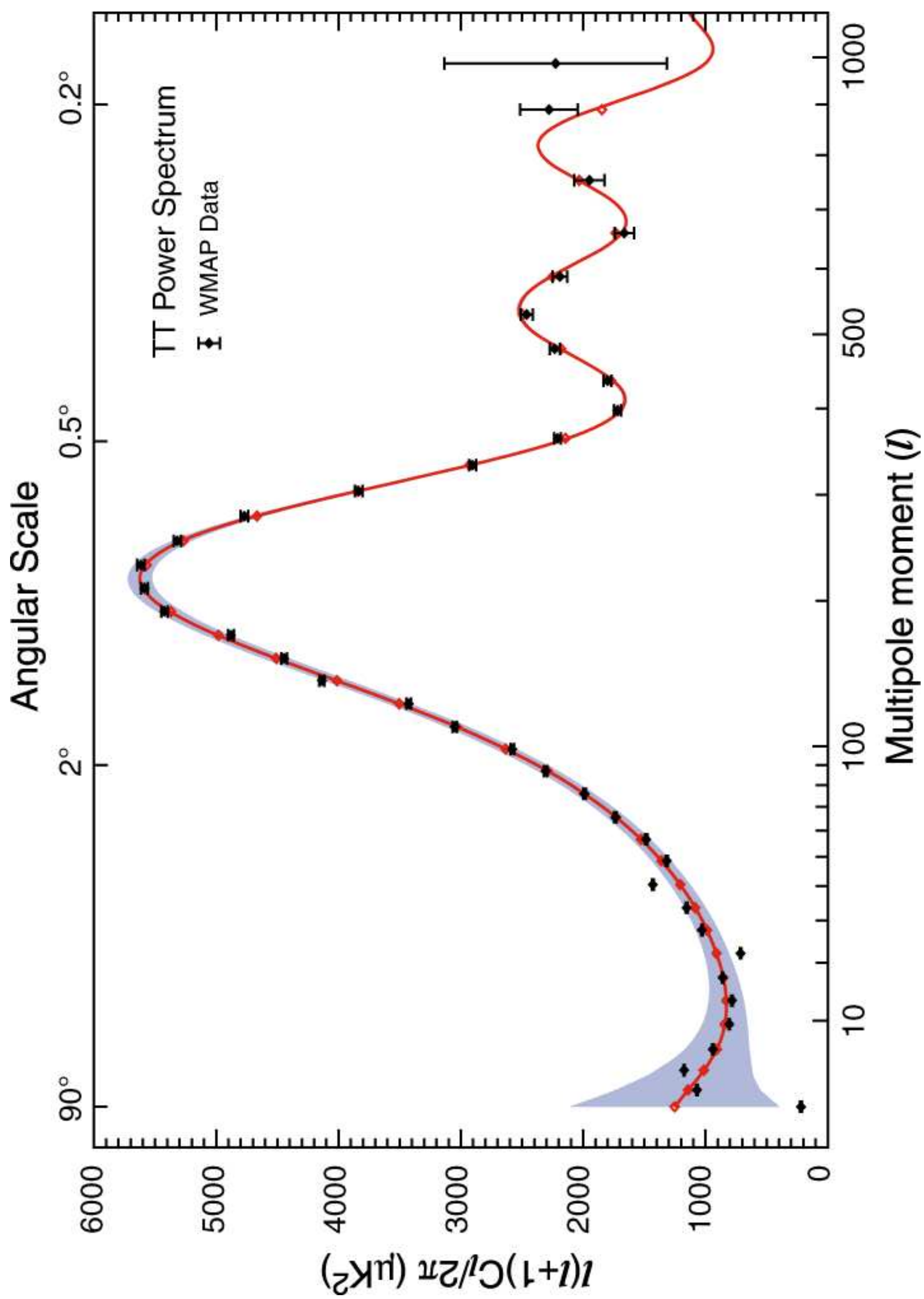

Fig. 2.— Power spectrum from WMAP.

surface of last scattering. In turn, the angular diameter distance depends on how light rays converge or diverge: in a spatially closed universe ($\Omega_k < 0$) they converge, and in a spatially open universe they diverge, relative to a flat universe. You can then see that if you have two rays that (1) both terminate at one point at redshift $z = 0$ (i.e., at our observation point), and (2) end on either side of a region of fixed size at redshift $z \sim 1100$, then compared to a flat universe a closed universe will give a larger apparent angle, and an open universe will give a smaller apparent angle.

This means that the primary information that comes from the angular location of the first acoustic peak is the geometry of the universe. The WMAP third year data (plus other data) indicate that at the 68% confidence level $|\Omega_k| < 0.04$. This is remarkably close to zero, and indicates that the universe is spatially flat to high precision.

The Second Acoustic Peak

We see that the second peak is much lower than the first. To understand this, consider a potential well with masses and springs. In this case we think of the "springs" as the pressure (here provided mainly by the radiation) and the masses as provided by the baryons. The maximum compression achieved by the masses at the bottom of the potential well depends on the pressure and the mass, and is larger for larger mass (because the masses have greater inertia). In contrast, the maximum rarefaction does not depend on the masses. This implies that if we were to fix everything but increase the baryon density, the compression peaks (first, third, fifth, etc.) would increase in height relative to the rarefaction peaks (second, fourth, sixths, etc.). As a result, the ratio of second to first peak amplitude tells us about Ω_b, which is the ratio of the baryon density to the critical closure density. The result from the WMAP data is $\Omega_b = 0.044$. This is more than has been detected, but is consistent with the independent inference from big bang nucleosynthesis (which we will discuss in a couple of weeks).

However, the baryons also have other effects. One of these is that larger amounts of baryons reduce the frequency of oscillations at all scales (think of a classical spring with a fixed spring constant: the frequency is $\omega = \sqrt{k/m}$, hence smaller for larger mass). The oscillations have higher frequencies at higher multipoles, hence the multipole of the first peak is higher when one has more baryons. Therefore, what we said earlier is not quite true: it is not *exclusively* the geometry of the universe that determines the multipole of the first peak. This is an example of *degeneracy* in cosmological parameters, and means that one actually needs additional information to get a unique separation of different effects.

The Higher Acoustic Peaks

Suppose that the universe had only radiation. When a perturbation reached maximum compression, as it expanded out the photons would continue to redshift with the universe, hence the gravitational potential would decay away. This would allow the temperature perturbation to be much greater than it would otherwise, and hence would enhance the peaks.

In contrast, nonrelativistic matter does not redshift, so the gravitational potential it produces does not redshift away. This would lead to relatively smaller fluctuations and smaller peaks.

In reality both radiation and matter contribute, but as we have seen, at smaller scale factor radiation becomes relatively more important. Therefore, higher frequencies (and thus higher multipoles) should have an enhancement in amplitude. That isn't to say that higher harmonics should have larger amplitudes in an absolute sense, just that the strength is relatively increased. The actual dependence of amplitude on multiple is therefore a measure of the total density of nonrelativistic matter in the universe, since that quantity affects the transition between radiation domination and matter domination. The multipoles of the peaks are also affected because as you recall the expansion rate of the universe is different in the relativistic regime (where $a \propto t^{1/2}$) from the nonrelativistic regime (where $a \propto t^{2/3}$). This also affects the angular size of the universe (as we see it) for a given physical size or time after the Big Bang.

The net result is that if the first three peaks can be measured with good precision, the geometry of the universe, the baryon density, and the total density of nonrelativistic matter can be determined accurately. The WMAP data do not fully characterize the third peak, but they get enough of it to estimate that the total matter density relative to critical is $\Omega_m = 0.27$, meaning that the dark matter fraction must be about $\Omega_{DM} = 0.23$.

The Damping Tail

The future space mission *Planck* (with an expected 31 July 2008 launch) will be able to measure out to much higher multipoles than WMAP, and to be cosmic variance limited to $l = 2500$. It should thus be able to measure many peaks and get far higher precision than WMAP. Ground-based observatories will also be able to measure higher multipoles, but over a smaller solid angle. A particularly important effect it should be able to see is the damping tail of the oscillations.

The surface of last scattering is not infinitesimal in thickness. This means that photons will still scatter a few times before streaming. If the typical distance they travel is larger than the wavelength of the oscillation at a given multipole, then the temperature contrasts at that multipole will be smoothed out and damped. This effect obviously becomes more pronounced at shorter wavelengths, hence higher multipoles are expected to drop dramatically in amplitude. If there are a lot of baryons around (meaning in practice a lot of electrons), then photons can't travel as far between scatterings. That implies that they random walk a shorter distance and do less smoothing of high-l oscillations. In addition, the total matter density again has an effect on the relative age of the universe at the surface of last scattering. Therefore, when the damping tail is measured there will be additional important checks on the overall theory, and on the idea that only a few parameters determine the general structure of the universe. It will also allow checks of many of the underlying assumptions, e.g., that the fluctuations are Gaussian.

Information from Other Observations

The precision of CMB observations and the simplicity of the underlying physics make it the gold standard data set for cosmology. However, we should not discount the wide variety of other observations that can be made of other things. These include:

- Supernova Type Ia observations. These suggest that the expansion of the universe is accelerating, and thus give evidence for dark energy.
- Clusters of galaxies. These are thought to be "representative" samples of the clustered part of the universe (i.e., everything but dark energy). Observations of clusters therefore allow us to estimate things such as the ratio of baryonic matter to dark matter, and also to estimate the quantity σ_8, which is the fractional root mean square fluctuation in density on a linear scale of $8h^{-1}$ Mpc, where $h \equiv H_0/100$ km s^{-1} Mpc^{-1}. That's a useful quantity because it sets an overall scale for fluctuations (e.g., inflationary theory predicts a power-law spectrum but not the normalization).
- Large scale structure. These are the superclusters, as most recently observed in good detail by, e.g., the Sloan Digital Sky Survey.
- Big Bang nucleosynthesis. We'll have a couple of full classes on this, but the upshot is that the primordial abundances of light elements such as hydrogen, deuterium, and helium allow us to estimate Ω_b.

The net result is that there is excellent agreement between all these experiments about quantities such as Ω, Ω_Λ, Ω_b, and Ω_k. This agreement has been dubbed "cosmic concordance", and it is indeed very gratifying to see things come together so well. On the other hand, it is worth keeping in mind that not everything agrees completely. For example, the value of σ_8 is either close to 0.9 or to 0.7, depending on the method used. There are also some apparent discrepancies in the observed versus predicted primordial abundance of lithium. This is a reason why, in addition to continued pursuit of CMB fluctuations with experiments such as *Planck*, it is essential to use as many other approaches as possible. In my biased opinion, X-ray observations of clusters and (eventually) gravitational radiation from merging supermassive black holes are especially promising.

Future Work and Other Information

Believe it or not, we've only examined a small fraction of what the CMB can teach us. For example, we have focused on the temperature fluctuations, but scattering also produces polarization. The polarization information is extensive: not only does it yield independent self-consistency tests, but certain modes of polarization are possible only if they are affected by gravitational radiation from the early universe (this has not been measured yet). WMAP has seen polarization fluctuations, but experiments are underway to do much better.

It will also be important to characterize the so-called secondary temperature anisotropies. Remember that although the universe is pretty transparent it isn't exactly so, meaning that the radiation scatters off of free electrons it finds on its way. This means that one can measure effects such as (1) the reionization of matter by the first stars or black holes and (2) scattering off of hot gas in clusters (the so-called Sunyaev-Zeldovich effect). There is a world of information out there, and the next several years will see huge increases in our cosmological knowledge as a result.

Intuition Builder

It is expected that, just as there is a background of photons at about 2.7 K, there should be a background of neutrinos at about 1.9 K. This would have come from the universe when it was tens of seconds old, and thus would be a record of very early stages. Why do you think there hasn't been a major effort to detect this background?