The Thermal History of the Universe

We will now work our way backward in time to see what the universe was like in the really old days. An especially important phase involved the formation of the light elements (hydrogen, deuterium, helium, and small amounts of lithium and beryllium), aka "Big Bang Nucleosynthesis". This provides one of the earliest tests of the hot Big Bang model, and as such we will devote two future lectures to it.

Here we explore the temperature and density of the universe as a function of scale factor and time, to get a general overview. In the next lecture we will focus on neutrinos.

Recent Temperature Evolution

The temperature evolution of the photons is easy. Remember that the wavelength of an individual photon simply scales with the overall scale factor a, as long as the photon does not interact with anything. This, you recall, is the way we were able to assert that the cosmic microwave background as we see it now should have the form of a blackbody, and how we are able to predict its current temperature. In fact, if nothing else happened, we could confidently predict that the photon temperature would remain proportional to $1/a$ even arbitrarily far back in time, because its self-interactions maintain the same blackbody form. As we'll see later, though, other things do happen, so it's a little more complicated.

What about nonrelativistic particles? Here the situation is slightly trickier. To get a sense of how the temperature evolves, we need to use a Newtonian analogy and, in fact, to get a basic sense of what temperature is.

What is temperature? You might be tempted to define it in terms of the typical velocity of individual particles, and that's not too bad a definition. Note, though, that the velocity itself has no absolute meaning (you can go to a different reference frame and the velocity will change). It is therefore best to talk about the velocity dispersion in a small region; relative to the center of mass velocity of the region, what is the standard deviation of the speeds? Even more precisely, the region needs to be in thermodynamic equilibrium for us to really talk about temperature, but we won't worry too much about that. What this means is that we could imagine a group of particles moving past us at 99% of the speed of light that is actually quite cold because they all move together.

With this in mind, consider a point explosion out in the middle of space (i.e., no gravity) that sends out a huge number of fragments that have different nonrelativistic velocities (where here we mean different speeds as well as different directions). If we pick an imaginary box of size Δr centered on a distance r from the center of the explosion, we can estimate the velocity spread in, say, the radial direction by noting that the particles had to have speeds within a fractional range $(\Delta r)/r$ of each other to be in the box. Therefore, $\Delta v \propto 1/r$ for a fixed (and unimportant) Δr. Since the temperature scales as $T \propto (\Delta v)^2$, that in turn means that $T \propto 1/r^2$.

Going back to cosmology, this means that without interactions relativistic and nonrelativistic temperatures evolve differently with the scale factor:

$$\begin{aligned} T_{\rm rel} &\propto 1/a \\ T_{\rm nonrel} &\propto 1/a^2\ . \end{aligned} \tag{1}$$

As a result, once the nonrelativistic matter no longer interacts significantly with the relativistic matter (mostly photons), it cools below the photon temperature until something else happens. Dark matter presumably is in this state for a long time, but baryons and electrons actually do couple decently with photons. This leads to two obvious questions. First, **Ask class:** when do they think the the baryons will no longer interact sufficiently with photons to maintain about the same temperature? A reasonable answer would be that this happens at the same time as the photons decouple from the baryons and we see the CMB; after all, isn't that practically the definition of decoupling? However, it's not quite like that. Remember that there are billions of photons per baryon. Therefore, even though after decoupling the typical photon will never again interact with a baryon, one cannot say that the typical baryon will never again interact with a photon! In fact, the photons are thermally dominant down to $z \sim 100$ or so. One important aspect of this is that after decoupling there is still some ionization left, so electrons get hit with photons. This is an indication that the universe is not quite in thermodynamic equilibrium; continued expansion means that not every electron is able to find a safe home in an atom.

The other question is **Ask class:** what happens from that point on? The baryon temperature does decrease relative to the photon temperature for a while. However, when stars and galaxies form, their light ionizes the universe and heats it up, leading to the present-day situation that intergalactic gas is pretty hot, $\sim 10^6$ K or so. This reionization phase is of great interest to cosmologists. The really cool thing, though, would be that if 21 cm absorption lines could be seen at the $z < 100$ pre-ionization phase (due to the cool matter against the CMB background), this would in principle give us what has been called 3-D tomography of the early universe and its density fluctuations. That would give us spectacular amounts of information about the fluctuations and their growth, potentially dwarfing the CMB acoustic peaks in terms of characterization of the linear universe. However, it is highly unlikely that any instrument built in the next several decades at least will have the requisite sensitivity.

Matter-Radiation Equality

Now let's go backwards in time, starting at the epoch of the CMB. To see what happens, note that the current energy density in the CMB is just given by its temperature: $U_{\rm CMB} = aT^4$, where here $a = 7.565 \times 10^{-16}$ J m^{-3} K^{-4} is the Stefan-Boltzmann constant. Putting in $T = 2.726$ K gives $U_{\rm CMB} = 4.2 \times 10^{-14}$ J m^{-3}. In comparison, recall that $\Omega = 0.27$, and that $\rho_{\rm crit} = 10^{-26}$ kg m^{-3}, meaning that the mass-energy density in nonrelativistic matter is $U_{\rm nonrel} = 0.27 \times 10^{-26} \times c^2 = 2.4 \times 10^{-10}$ J m^{-3}.

Now remember that whereas $\rho_{\rm nonrel} \propto a^{-3}$, $\rho_{\rm rel} \propto a^{-4}$. That implies that

$$\frac{U_{\rm rel}}{U_{\rm nonrel}} = 1.8 \times 10^{-4}(a/a_{\rm now})^{-1} \ . \tag{2}$$

Thus nonrelativistic matter dominates now, but at redshifts more than $\sim 10^4$ and thus temperatures $T >$ few $\times 10^4$ K, the relativistic matter dictated the evolution of the universe. As you recall, this implies, for example, that $a \propto t^{1/2}$ rather than $a \propto t^{2/3}$ if the matter density is close to critical (and also note that this is really close to correct for $z \gg 1$, which is the situation of interest).

By the way, one might have guessed intuitively that since at sufficiently high temperature the baryons will become relativistic, maybe that's when relativistic matter becomes dominant. However, it's not true. The comparison to make is the ratio $x \equiv kT/mc^2$. For electrons, the lightest of the normal matter components, $x > 1$ only for $T > 6 \times 10^9$ K, which is far beyond the actual point of matter-radiation equality. In reality, therefore, there are just an enormous number of photons around.

Hotter and Hotter

Now suppose that we continue to higher and higher redshift. The photons, baryons, and electrons are all tightly coupled and thus have the same temperature. Dark matter presumably goes about its own business. Neutrinos will stream freely after the temperature decreases below around 10^{10} K (more about this in the next lecture). What else is going on?

To get a sense for this, we can consider the typical energies that correspond to a given temperature, and then compare those energies with various other characteristic energies. Note that the useful unit of energy here is the electron volt (eV), which is 1.6×10^{-19} J. The translation is then $T = E/k \Rightarrow T = 1.16 \times 10^4$ K$(E/1$ eV$)$.

Typical atomic energies are a few eV, so we would expect that when the temperature is a few tens of thousands of Kelvin atoms would be fully ionized. As we saw two lectures ago, in fact the threshold is smaller, but this is of the right general magnitude.

Typical nuclear energies are a few MeV, so we expect that when $T \sim 10^{10}$ K nuclei were broken apart into their separate nucleons. Running time forward, this suggests that this is the epoch when compound nuclei such as helium could form. This is also the approximate mass-energy of electrons, meaning that around this time one expects that positrons as well as electrons could exist.

At above $T \sim 10^{12}$ K, laboratory evidence and theory suggest that individual nucleons (i.e., protons and neutrons) give way to a quark-gluon plasma. At this point, data are tough to come by, and the situation gets rather fuzzy.

Nonetheless, particle physics experiments indicate that at energies $> 10^{11}$ eV, or $\sim 10^{15}$ K, one expects that the electromagnetic and weak forces will unify, in the sense that if you were around at the time, you would only measure aspects of three different forces instead of four.

At much greater energies (perhaps around 10^{25} eV), it is thought based on extrapolations of coupling constants that the strong force unites with the electroweak force. This, however, is sufficiently beyond laboratory capabilities (to put it mildy!) that this is a realm of wild speculation.

Finally, at the Planck energy (which is what you get when you combine G, c, and $\hbar$), or around 10^{28} eV, gravity is thought and hoped to be unified with the other forces.

The general picture one gets is that the initial state of the universe was one of great symmetry and equality. As it cooled, however, the symmetry was spontaneously broken. A good analogy to keep in mind is of water as it cools. As steam, water has great symmetry: the molecules are every which way, and oriented randomly. As liquid water, the polar nature of the molecule causes correlations that didn't exist before. As ice, there is a crystalline structure that is produced (actually, there are more than a dozen different forms of ice, depending on pressure and temperature; this is more by a lot than any other known solid). There, directions are clearly different and the symmetry is thus substantially reduced.

There are quite a few puzzles about why things happened the way that they did in this phase. For example, the universe has an excess of matter compared to antimatter. Why? What caused this particular asymmetry? In general, how were baryons produced? When the universe transitioned from a quark-gluon phase to the current hadron phase, did it do so via a first-order or second-order phase transition? The distinction, involving whether fundamental quantities such as density are discontinuous (first-order) or just their derivatives are discontinuous (second-order) might make a difference to the detectability of gravitational waves during this phase. Are primordial black holes formed during any of these early phases? See Figure 1 for a characterization of the history of the universe.

Pretty heady stuff! Now, though, let's go back to some relatively well-understood portions of the early universe.

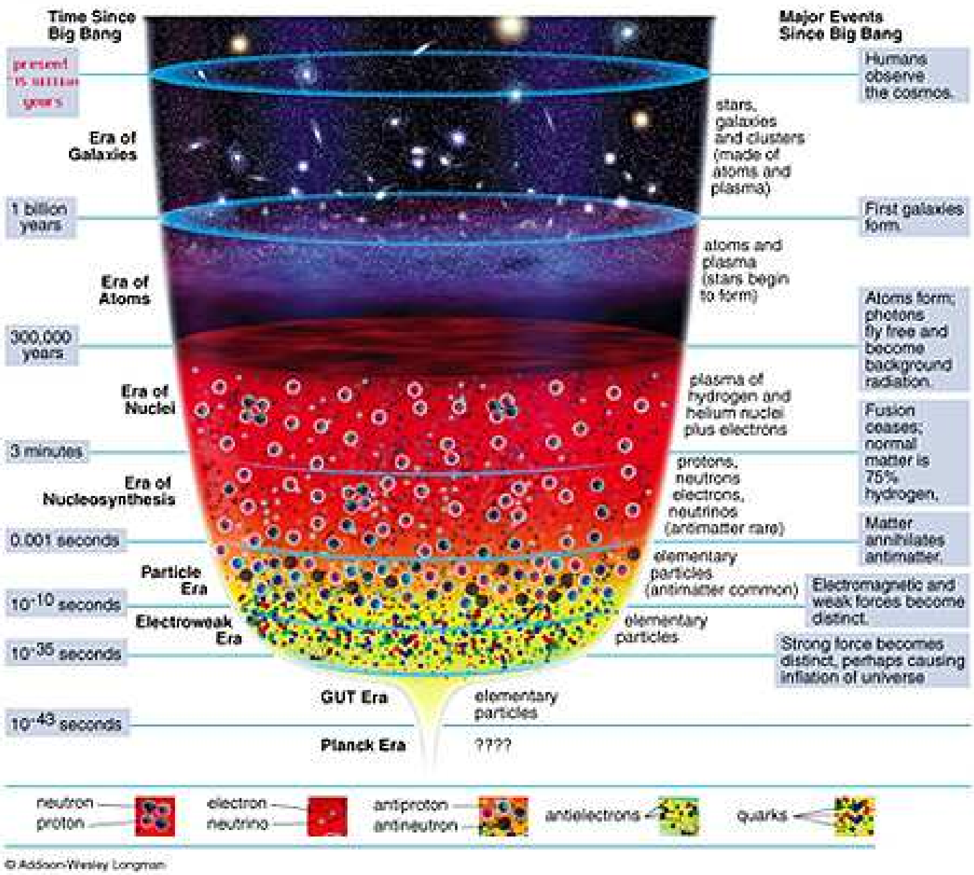

Fig. 1.— The history of the universe, with temperatures and events. From http://rst.gsfc.nasa.gov/Sect20/eras_of_universe.jpg

Conversion of Matter Into Radiation

Earlier we said that "if nothing else happened", we could say that the photon temperature would scale as $1/a$ as far back as we liked. However, other stuff does happen. In particular, when the temperature is high enough that the photons have energies comparable to the rest-mass energy of electrons, then electron-positron pair creation is possible:

$$\gamma\gamma \rightarrow e^- + e^+ \; . \tag{3}$$

Yes, the reverse process (annihilation) also happens, but at a high enough temperature the two processes are in equilibrium, meaning that the energy that at lower temperatures is all in photons, is actually shared between photons, electrons, and positrons. It may be easier to conceive of this going forward in time: when the temperature drops sufficiently, all the positrons annihilate with the electrons to produce extra photons, so the photon temperature goes up.

Our first guess about the magnitude of the temperature jump might be that because prior to annihilation there are three relativistic species (electrons, positrons, and photons) and after there is just one (photons), the number density of photons increases by a factor of 3. Since the number density in equilibrium scales as T^3, that would suggest that the temperature jump is a factor of $3^{1/3}$.

This, however, is not quite right. Photons are bosons, but electrons are fermions, and this affects the number densities. In fact, the energy density in relativistic fermions is proportional to

$$U_{\text{fermion}} \propto I_+ \equiv \int_0^\infty \frac{x^3 dx}{e^x + 1} \tag{4}$$

where $x \equiv pc/kT$, whereas the energy density in relativistic bosons is proportional to

$$U_{\text{boson}} \propto I_- \equiv \int_0^\infty \frac{x^3 dx}{e^x - 1} \; . \tag{5}$$

We therefore expect there to be more bosons than fermions (this is reasonable; multiple fermions can't occupy the same state, whereas multiple bosons can). We actually only need the *relative* values of I_+ and I_- rather than their absolute values, and we can get the relative values by a neat algebraic trick. Note that

$$\frac{1}{e^x - 1} - \frac{1}{e^x + 1} = \frac{2}{e^{2x} - 1} \; . \tag{6}$$

Integrating, and changing variables on the right hand side to $y = 2x$ gives

$$I_- - I_+ = \frac{1}{8} I_- \tag{7}$$

or $I_+ = \frac{7}{8} I_-$. Therefore, prior to annihilation into photons, the total number density that was sharing the energy was $1+2(7/8) = 11/4$ times the total number density afterwards, and thus $T_{\rm after} = (11/4)^{1/3} T_{\rm before}$. Of course, as we go yet farther back in time, more and more particles participate in this effect (e.g., protons and antiprotons, neutrons and antineutrons, and eventually the whole Standard Model zoo). Therefore, if we receive relic information from some of these early phases (e.g., neutrinos) they are expected to have a smaller temperature than the CMB.

Intuition Builder

When we've discussed the photon temperature in the current universe, we have talked only about the CMB. How does the current energy density in the CMB compare with the current energy density from all the starlight ever produced? How about the energy produced by accretion onto black holes?