

Theory of Big Bang Nucleosynthesis

The relative abundances of the lightest elements (hydrogen, deuterium, helium-3 and helium-4, and some lithium and beryllium) provide a strong test of the hypothesis that the universe was once far hotter and denser than it is now. This is because those elements were formed when the universe was just a few seconds to a few minutes old. In comparison, the CMB comes to us from when the universe was roughly 400,000 years old. In this lecture we will draw liberally from the Annual Review of Astronomy and Astrophysics paper by Boesgaard and Steigman (1985, 23, 319–378) to discuss the theory of BBN.

Overview

So that you don't get too lost in the details, here is the basic summary of the process:

- When the universe is very hot, $T \gg 10^{10}$ K, neutrons and protons have about the same abundance.
- However, any nuclei that form at this stage (e.g., deuterium, via $p + n \rightarrow d + \gamma$) are immediately dissociated by high-energy photons (via $d + \gamma \rightarrow p + n$).
- When the universe is much older than the lifetime of a neutron in free space ($\sim 10^3$ s, at which point the temperature is around 4×10^8 K), any remaining free neutrons will have decayed: $n \rightarrow p + e^- + \bar{\nu}_e$. No nucleosynthesis can proceed beyond this point until stars form vastly later.
- However, there is a sweet spot between these phases, when there are free neutrons around that can combine with protons without immediately getting separated again. In this phase, the light elements form.
- In principle one could imagine (as George Gamow did) that one would be able to produce *all* elements this way. However, there are no stable nuclei with 5 nucleons or 8 nucleons, so this means there is a bottleneck that prevents additional nucleosynthesis (basically because hydrogen and helium-4 are by far the most stable and common things around, but H+He and He+He both make unstable things).
- The relative abundances of H, D, He, etc. depend on the relative numbers of baryons and photons in the universe. Therefore, given a measured baryon fraction the relative abundances are all predicted with no free parameters. This is a strong test of the standard model.

Early Development

In the 1940s the iconoclastic Russian physicist George Gamow and his student Ralph Alpher started to consider the thermodynamics of the early universe. It was they who originally predicted that there would be a few-Kelvin glow from when the universe became transparent (although this had been forgotten by the time of the detection). They also realized that nucleosynthesis could occur in the early universe, although as mentioned above they were aiming for production of all the elements, not just the light ones. Their paper was originally going to be Alpher and Gamow, but since Gamow had a sense of humor he added Hans Bethe to the list, because he felt it was appropriate that a paper on beginnings should be Alpher, Bethe, and Gamow! Reportedly Bethe was surprised by his inclusion and Alpher was somewhat miffed, but that's neither here nor there.

At this point it is worth motivating why we need to look at the early universe to produce anything at all. After all, we know that stars burn hydrogen to helium for most of their lives, so why not rely on this process for all the helium we see?

We can do a rough calculation of this by noting that the Galaxy has a stellar luminosity of about $2 \times 10^{10} L_{\odot}$ and a stellar mass of about $10^{11} M_{\odot}$. If the current luminosity is close to the average over the last 10 billion years, then putting in $L_{\odot} = 3.8 \times 10^{26} \text{ J s}^{-1}$ indicates that the total energy released in stars has been about $2.4 \times 10^{54} \text{ J}$. Now, suppose that originally all of the mass in stars was pure hydrogen, for a total of $2 \times 10^{41} \text{ kg}$. Fusing hydrogen to helium releases about 0.7% of the mass-energy, so fusing all the stellar hydrogen to helium would have released $1.3 \times 10^{56} \text{ J}$. To account for the inferred $2.4 \times 10^{54} \text{ J}$, therefore, we find that $2.4 \times 10^{54} / 1.3 \times 10^{56} = 2\%$ of the gas mass should be in helium. The actual fraction is closer to 25%. Indeed, the real discrepancy is much greater, because as we discussed earlier in this course, most gas is between galaxies and thus had no chance to be processed in stars.

We therefore find that the helium could *not* have just been fused in stars. Something else had to happen. Let us therefore explore the processes in the first few minutes after the Big Bang.

The Basics

This phase of the universe is especially simple. The density is overwhelmingly dominated by relativistic matter (given that nonrelativistic matter, curvature, and the cosmological constant all have weaker dependences on the scale factor a). Therefore, if we define the energy density as $\rho_R \propto T^4$, we find that the universe evolves as

$$\frac{32\pi}{3} G \rho_R t^2 = 1 \tag{1}$$

or $T \propto t^{-1/2}$. With the numbers in, $T \approx 10^{10} \text{ K}(t/1 \text{ s})^{-1/2}$. We sometimes express temperature in energy units via $E = kT$, so $10^{10} \text{ K} \approx 1 \text{ MeV}$. As a shorthand, we can write

$T_{\text{MeV}} \equiv (kT/1 \text{ MeV})$. Since we're interested in nucleosynthesis, which requires that nuclei get close to each other, a natural length scale is the *fermi*, designated f, which is 10^{-15} m. The number density of photons is then

$$n_\gamma \approx 10^{-7.5} T_{\text{MeV}}^3 \text{f}^{-3} \quad (2)$$

and thus the typical distance between photons at that temperature is around $300 T_{\text{MeV}}^{-1}$ f, far larger than nuclei. Also remember that nucleons are only a tiny fraction of the soup, so the number density is $n_N = \eta n_\gamma$, where $\eta < 10^{-9}$.

What about interaction rates? The typical distance l traveled by something before an interaction depends on the cross section σ of the interaction (with units of area) and the number density of targets. Specifically,

$$l = 1/(n\sigma) . \quad (3)$$

If this distance is much less than the horizon size $D_H = ct$ then the reactions are in equilibrium. If the distance is much more than the horizon size, the particles basically don't interact.

For example, in the very early universe, the number densities of photons, electrons, positrons, and neutrinos are all comparable to each other. The cross section for electron-photon scattering is about 10^{-28} m^2 or about 100 f^2 , so

$$\frac{l_{e\gamma}}{D_H} \approx 10^{5.5} T_{\text{MeV}}^{-3} \text{f} / 10^{23.5} T_{\text{MeV}}^{-2} \text{f} = 10^{-18} T_{\text{MeV}}^{-1} . \quad (4)$$

Here we used $D_H = ct = 3 \times 10^8 \text{ m} T_{\text{MeV}}^{-2}$. Clearly, photons, electrons, and positrons all interact very rapidly compared to the expansion of the universe.

In contrast, for neutrinos we have $\sigma \sim 10^{-48} \text{ m}^2 T_{\text{MeV}}^2 = 10^{-18} \text{ f}^2 T_{\text{MeV}}^2$, so

$$\frac{l_{e\nu}}{D_H} \approx 100 T_{\text{MeV}}^{-3} . \quad (5)$$

Therefore, neutrinos are in equilibrium for $T_{\text{MeV}} > 5$ but not afterwards.

With all of this in mind, let's look at epochs in chronological order.

$$t \sim 10^{-2} \text{ s} \quad (T \sim 10^{11} \text{ K} \sim 10 \text{ MeV})$$

At this stage, all neutrinos, photons, electrons, and positrons are in equilibrium. The number density of individual species of fermions (i.e., the neutrinos, electrons, and positrons) is 7/8 of the number density of the photons, as we discussed before (note that this statement actually relies on the multiplicity of all these particles being the same, i.e., that there are

two polarization states per photon, two spin states per electron, and two helicity states per neutrino. Only one helicity state has ever been observed for neutrinos, so this may not be right. We're not going to worry about it.)

There are also weak decays going on that convert neutrons to protons and vice versa: $n \rightarrow p + e^- + \bar{\nu}_e$, $p + e^- \rightarrow n + \nu_e$, and $n + e^+ \rightarrow p + \bar{\nu}_e$. These occur rapidly, and hence maintain the relative number densities of neutrons and protons at the equilibrium value

$$n_n/n_p = \exp(-\Delta mc^2/kT) \quad (6)$$

where Δm is the mass difference between the neutron and the proton+electron.

$$t \sim 10^{-1} \text{ s } (T \sim 3 \times 10^{10} \text{ K} \sim 3 \text{ MeV})$$

At this phase, the neutrinos decouple because they typically go across the universe without scattering at all. However, everything else is still in equilibrium.

$$t \sim 1 \text{ s } (T \sim 10^{10} \text{ K} \sim 1 \text{ MeV})$$

At this temperature, the reactions that had been keeping the neutron to proton ratio in equilibrium become too slow to do so any more. Therefore, for a while, the ratio is fixed at

$$n_n/n_p = \exp(-\Delta mc^2/1 \text{ MeV}) . \quad (7)$$

In more detail, the freeze-out temperature, and thus the freeze-out number density ratio, depends on how fast the universe expands. This in turn depends on the energy density in relativistic particles, which relates to the number of relativistic species. It was this fact that allowed early constraints on the number of neutrino flavors. If there were a huge number, this would have had a major impact on big bang nucleosynthesis.

$$t \sim 10 \text{ s } (T \sim 3 \times 10^9 \text{ K} \sim 0.3 \text{ MeV})$$

Note that the rest mass-energy of an electron or positron is 511 keV. Therefore, when the temperature is ~ 1 MeV or above, photon-photon pair production can produce plenty of pairs to replace the ones that annihilate with each other. However, below this temperature annihilations outstrip production. At this phase, therefore, we can say that in a rough sense all the positrons annihilate and pump their energy into the photons. As we discussed earlier, this leads to an increase in the temperature of the photons relative to what it had been. As we also discussed, since the neutrinos had already decoupled by this point, their relic background temperature is less than that of the photons.

What about the nucleons? The binding energy of deuterium is about 2.2 MeV, so you might think that at this point deuterium would be forming and then going on to other nuclei. However, the problem is that there are more than a billion photons per nucleon, so there are plenty out on the high-energy tail of the thermal distribution that can photodissociate deuterium. Therefore, no significant and permanent nucleosynthesis has happened yet.

$$t \sim 100 \text{ s } (T \sim 10^9 \text{ K } \sim 0.1 \text{ MeV})$$

At this point, however, it's a different story. Note that the fraction of photons above energy E if the temperature is T is around $\exp(-E/kT)$. The number of photons per nucleon with such an energy is therefore $\eta^{-1} \exp(-2.2T_{\text{MeV}}^{-1})$. When $T_{\text{MeV}} < 0.1$, this factor drops below unity and thus deuterium is safe. We have finally made the first step towards real nucleosynthesis! This also means that the temperature at which this happens, $T_{\text{D}\gamma}$, depends on η and hence the fundamental parameter η (which we currently can't predict from first principles) affects nucleosynthesis.

Now we can finally get going. From deuterium we can go to tritium, ^3He , and ^4He , of which ^4He is by far the most tightly bound per nucleon. Therefore, almost all the neutrons end up in ^4He . There are no stable nuclei with 5 or 8 nucleons, meaning that you can't just build indefinitely by adding hydrogen to helium or helium to helium.

$$t > 1000 \text{ s } (T < 3 \times 10^8 \text{ K } \sim 0.03 \text{ MeV})$$

At another order of magnitude in time, the temperature drops low enough that the nuclei can't easily overcome the Coulomb barrier that separates them. Therefore, nucleosynthesis effectively ceases at this time. In addition, the lifetime of a neutron in free space is a little over ten minutes, so any neutrons that have not been incorporated into nuclei turn into protons.

The net result of all this is that the ratio η determines all of the relative abundances of hydrogen, helium, and the (much smaller) amounts of deuterium, lithium, and beryllium. We know the photon density to high accuracy from the CMB, so this really means that big bang nucleosynthesis predicts a very tight relation between the baryon density and the relative abundances. For example, if the baryon density is increased, more neutrons can interact and become part of helium nuclei, so the helium to hydrogen ratio should go up. The relations are not quite as obvious for the other species, but the calculations are free of most uncertainties and have been done. See Figure 1 for a representation of the dependence of abundances on density.

Of course, we'd love to go ahead and determine if the predictions match the observations!

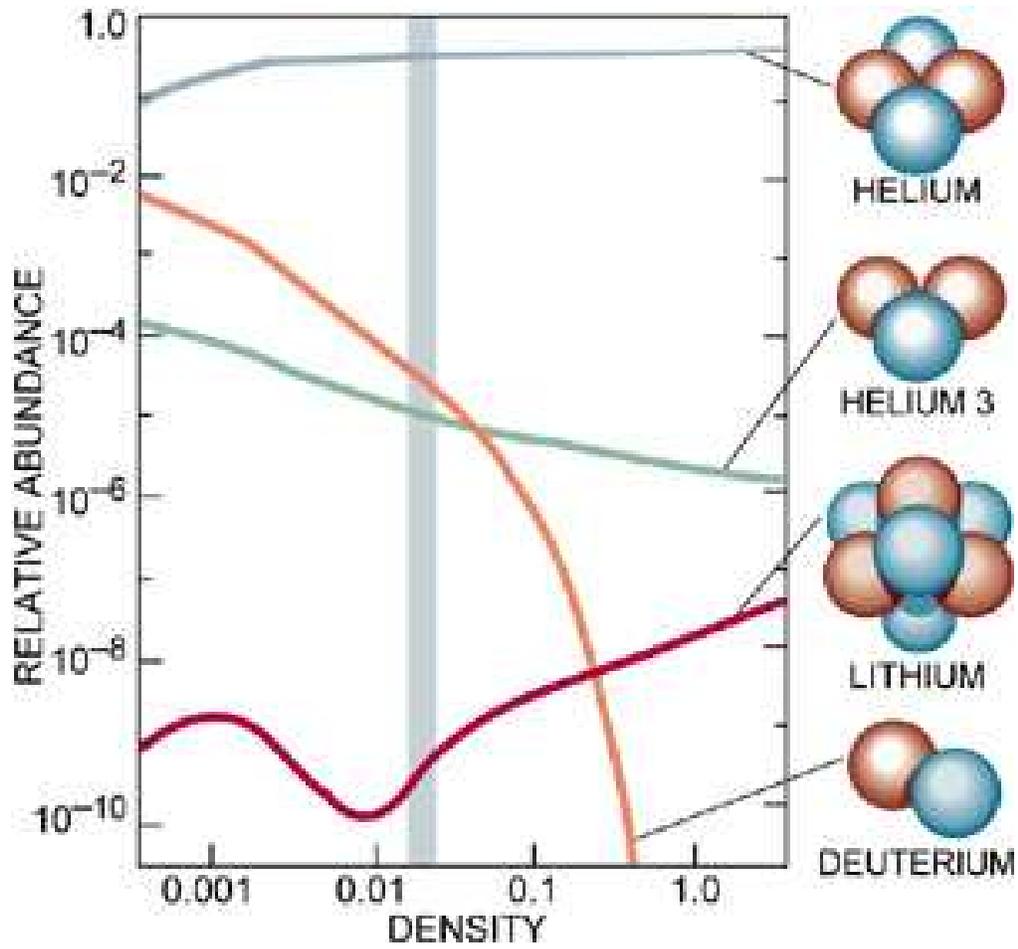


Fig. 1.— Mass abundances relative to ordinary hydrogen, as a function of baryon abundance. From <http://cosmology.berkeley.edu/Education/CosmologyEssays/images/altbbn.jpg>

This is a nontrivial task, and will be the subject of the next lecture.

Intuition Builder

The baryon density inferred in this way is about 0.04 of the critical density, versus the total matter density of 0.27. This rules out the idea that dark matter could be planets or stellar remnants. However, it does *not* rule out the possibility that black holes formed in the very early universe constitute dark matter. Why is this?