Inflationary Theory

The hot Big Bang theory (basically that as one goes backward in time the universe was hotter and denser) is extremely successful in accounting for the cosmic microwave background and Big Bang nucleosynthesis, among other observations. However, like any good theory, there are additional questions that it has stimulated. Here we will describe several of these questions, and how the idea of inflation tries to answer them.

The Flatness Problem

We have noted throughout this class that the densities of different components of the universe evolve with different powers of the scale factor a. Relativistic particles such as photons evolve as $\rho_r \propto a^{-4}$; nonrelativistic particles such as baryons (at the present epoch) evolve as $\rho_{\rm nr} \propto a^{-3}$; curvature evolves as $\rho_k \propto a^{-2}$; and a cosmological constant would evolve as $\rho_\Lambda \propto a^0$. The current curvature of the universe is $|\Omega_k| < 0.04$, from CMB observations. That means that $\Omega_r + \Omega_{\rm nr} + \Omega_\Lambda \approx 1$. If we go back in time, however, we see that the matter+radiation density had to be stunningly close to 1. For example, consider the epoch of the microwave background. This happened at $a/a_0 \approx 10^{-3}$, and nonrelativistic matter dominated. This means that at this epoch we had $\Omega_\Lambda \approx 7 \times 10^{-4}$ and $\Omega_{\rm nr} \approx 0.9993$. That's pretty close to 1! Of course, the agreement with unity gets even better if you go farther back in time. At even smaller scale factors the cosmological constant contribution is utterly negligible, and so is the contribution from curvature. For example, at the epoch of nucleosynthesis (1 second after the Big Bang), $|\Omega_k| < 10^{-18}$, and at the time of electroweak symmetry breaking (10^{-12} seconds after the Big Bang), $|\Omega_k| < 10^{-30}$.

That's pretty amazing. Put another way, if the total mass-energy density had been 1 part in 10^{29} larger at the era of electroweak symmetry breaking, then the universe would have recollapsed before any galaxies could form! Sure seems impressive that things were fine-tuned to that extent. Did we just get spectacularly lucky, or is there some natural way that the universe would be close to spatially flat?

The Horizon Problem

We discussed how the CMB is very close to isotropic: just a part in 10^5 variation across the sky, once our motion and foregrounds are taken into account. Perhaps at first glance this seems natural, but a deeper examination reveals a real puzzle.

The crux of this puzzle has to do with how far light can travel in a given time. Outside of that range, things can't communicate with each other and therefore there is no reason for regions beyond that distance to have similar properties. Now recall how the scale factor changes with time:

$$\begin{aligned} a &\propto t^{1/2}\ , \qquad \text{Radiation dominated} \\ a &\propto t^{2/3}\ , \qquad \text{Nonrelativistic matter dominated}\ . \end{aligned} \tag{1}$$

For comparison, the distance light can travel in time t is obviously ct.

Suppose, therefore, that we consider the universe at roughly the time of the CMB, when $a = 10^{-3}a_0$. This means that the age of the universe was a factor of $(10^{-3})^{3/2}$ times less than it is now, or about 30,000 times less. Light, therefore, had been able to travel only $1/(3\times10^4)$ of the current size of the universe. Since what we see of the universe was $1/1000$ of the current size, though, we have to conclude that almost all of what we see now could not have been in causal contact at that stage. As with the flatness problem, this only gets much worse as we go to higher and higher redshifts. This means that all that matter had almost exactly the same temperature despite not having had a chance to interact!

This situation is akin to the following. Suppose that you arrest two suspects for a crime. You nab them separately, take them to the station separately, interrogate them separately, and generally make sure they cannot talk to each other at all. Nonetheless, the stories they tell are word for word identical. How can this be?

The Problem with Heavy Relics

Yet another difficulty has to do with the generic expectation in grand unified models of particle physics that there are a lot of stable superheavy particles that should have existed in the very early universe. By "superheavy" we mean particles with rest mass-energies of 10^{16} GeV, where for comparison protons and neutrons have mass-energies of roughly 1 GeV. The motivating example circa 1980 was magnetic monopoles, but other particles would also fit the bill. The problem comes back to how energy densities scale: for relativistic particles $\rho \propto a^{-4}$ but for nonrelativistic, $\rho \propto a^{-3}$. Therefore, if superheavy particles were abundant in the very early universe, they should completely dominate by now and they obviously don't. What could prevent this?

Inflation

In the late 1970s and early 1980s (which, perhaps coincidentally, was a period of relatively high inflation for the US economy!), a number of people including Demos Kazanas, Alan Guth, and Andrei Linde came up with a set of ideas to solve the preceding problems. The general point is that if the early universe experienced a period of exponential expansion, and then resumed "ordinary" expansion, all would be well.

How does this work? Recall how things would operate if the universe were dominated by a cosmological constant:

$$\begin{aligned} (\dot{a}/a)^2 &= \Lambda/3 \\ a &\propto \exp(\sqrt{\Lambda/3})\ . \end{aligned} \tag{2}$$

If something like that happened early on, the universe could increase its size by many tens of orders of magnitude within 10^{-34} seconds or so(!!).

Before discussing how this would solve our three problems, let's return to a standard conceptual block that we mentioned many lectures ago. When people are told about this exponential expansion, a natural reaction is that this must violate special relativity, because it is inevitable that points that had previously been in causal contact will be driven to move apart much faster than the speed of light.

This, however, plays on the common but incorrect thought that moving spacetime is like any other movement. Note, for example, that two galaxies that both move with the Hubble flow and thus both have equal claim to be at "rest" can nonetheless have a relative redshift that is the equivalent of a 100,000 km s^{-1} speed. In the case of inflation, imagine an infinite grid. Suppose that in a given amount of time the distance between any pair of grid points is doubled. For close points, this doesn't have to indicate any particularly high speed, but for more distant points the relative speed can be as high as you like. All this means, really, is that spacetime points that separate faster than the speed of light are outside each other's horizons and thus can't influence each other. This is the kind of topic for which it is good to meditate on the consequences in order to reach understanding.

In any case, how does this help us? First consider the flatness problem. Using as an analogy a normal sphere (which therefore has a two-dimensional surface and is thus called a 2-sphere), the curvature can be defined in terms of the reciprocal of the radius. If we blow up the sphere by a huge factor, then it remains a sphere but the curvature gets really close to zero. An alternate picture of this comes from general relativity and the equivalence principle: in a small enough region, spacetime looks flat. Here, though, rather than us looking at a very small region, spacetime has expanded so that our normal view encompasses only a tiny portion of what we'd seen before. It amounts to the same thing. Therefore, whatever the natural curvature was before the huge expansion, it goes to almost zero after the expansion. See Figure 1 for a visual representation.

Now the horizon problem. Consider a patch of the universe that was in causal contact before inflation. It can therefore thermalize and get into equilibrium with itself. When inflation commences, most regions of the patch go out of causal contact, meaning that they evolve independently. However, their initial conditions are the same so the temperature, density, etc. will stay about the same (note that the cosmological principle of uniformity works here). After inflation ceases, you have a huge region with similar properties but not

Fig. 1.— A visual demonstration that inflation will naturally lead to a nearly flat geometry. From http://ircamera.as.arizona.edu/NatSci102/images/inflation1.jpg

in causal contact. Now, think about the subsequent expansion in the radiation-dominated era (where $a \propto t^{1/2}$) and later in the matter-dominated era (where $a \propto t^{2/3}$). Light travel distances are $d = ct \propto t$, so the amount of matter within causal contact increases with time. Therefore, more stuff comes into the horizon. This explains the uniformity of the CMB: yes, what we see now wasn't in causal contact at $z \approx 1000$, but it was in causal contact at the time of inflation and thus was able to maintain an approximately uniform temperature.

This actually brings up another puzzle that inflation resolves. We know that the CMB isn't *entirely* uniform; it has fluctuations. Without inflation, there would be no way to generate the variability, because those regions didn't know about each other. However, quantum fluctuations prior to inflation would have imposed a spectrum of variation at all scales. Inflation, in turn, would amplify these and allow them to serve as the seeds of structure.

Now to our final problem: relic particles. The idea here is that even if pre-inflation the density of superheavy particles was high, inflation will dilute the densities of all particles to practically nothing. Of course, you may then wonder how it is that we have any particles at all; wouldn't the argument work for them as well as for the hypothesized superheavies? Yes, indeed it would. However, once inflation ends the universe is still extremely hot. As a result, particles are produced by various reactions (e.g., if you have very energetic photons then there will be pair creation to produce electrons and positrons, as well as lots of other stuff). If the temperature after inflation was low enough (i.e., below $kT \sim 10^{16}$ GeV) that the superheavies couldn't be produced, everything works.

Particle Physics and the Amount of Inflation

The *minimum* amount of inflation we need can be estimated simply. Using the same arguments as earlier in the lecture, we find that at 10^{-34} s we had $|\Omega_k| < 10^{-53}$. Since $\Omega_k \propto a^{-2}$, this means that a mere 27 orders of magnitude of inflation will do the job. Note that exponential expansion can do this just fine: if $H^{-1} = 10^{-36}$ s, then after 10^{-34} s the expansion factor is $e^{100} \approx 10^{43}$. The actual factor is unknown, and could be millions (yes, millions!) of orders of magnitude more.

But what actually drives the expansion? Aye, there's the rub. There isn't a particularly compelling and corroborated model. The idea that has attracted the most attention is that when a phase transition occurs, the dynamics can be governed by something called a scalar field (think of this as a field in all space that has a scalar value, rather than a vector value as would be more familiar from electric and magnetic fields). As the universe expands and cools, at some point it undergoes such a transition. One can then think of the universe "rolling down" a hill in the value of the scalar field, to some minimum (at which point the inflation stops). Generic models of this type, called "slow-roll" inflation because the rolling down the hill takes much longer than the characteristic expansion time, do very well in predicting the spectrum of fluctuations in the universe (this is a genuine prediction, in addition to the pre-existing puzzles that it solves). However, the functional form of the scalar field is utterly ad hoc, so there is a long way to go in this theory. Nonetheless, inflation is decently corroborated at this point and can be considered to be a part of the standard cosmology.

Intuition Builder

Suppose someone comes up with a 10^{12} GeV particle. What is the earliest time after the Big Bang that inflation could stop if we want to avoid getting overwhelmed by those particles?