ASTR 422
Problem Set 4
Due Friday, November 2, 2007

1. A certain globular cluster at a distance of 15 kpc has as its brightest main sequence stars ones that have an apparent visual magnitude of $\mathrm{m_V} = 22.0$. There is no extinction towards the cluster. To within 20%, calculate the age of the cluster. For this problem I assume you will do Web research to look up quantities such as the absolute magnitude of the Sun, the relation of luminosity to mass in main sequence stars, and main sequence lifetimes, so please give me any URLs that you used.

2. Dr. Sane has decided to try his hand at data analysis, by looking at the catalog of data from the MACHO microlensing project. He has announced that he has discovered a population of $10^6\, M_\odot$ black holes in our galaxy, typically at 4 kpc from us, that lens stars in our Galactic bulge (typically 8 kpc from us). The signatures he sees are full lensing events, in which the entire Einstein radius of the BH at that distance crosses our line of sight to the star. Typical speeds at which the BHs move across our line of sight are 100 km s^{-1}, and the MACHO project has been going on for roughly 15 years. If Dr. Sane is right then he has solved the mystery of dark matter, so the Nobel Academy has asked you to comment on his work. Do a quantitative calculation to see if he could have done what he claims.

3. In a rotating disk, the Jeans criterion for formation of stars or planets is modified. Assume you have a disk of pure hydrogen in Keplerian rotation at a distance r from a star of mass M (we assume that the mass of the disk is small compared to M). The disk has a uniform surface density of Σ kg m^{-2} and a uniform temperature T.

In a way analogous to how we derived the Jeans mass, derive a criterion for instability to collapse in this rotating disk (in the form of an equation), for a circular section of radius $b \ll r$ cut out of the disk (you may assume that the gravitational potential energy of this section is $\sim -Gm^2/b$, where m is the mass of the section). Remember to include the effects of velocity shear as well as of temperature. That is, if the relative speed $v_{\rm rel}$ between the inner and outer edge of the section is such that $\frac{1}{2}mv_{\rm rel}^2 > Gm^2/b$, then the section cannot collapse. Similarly, if $(m/m_p)(3/2)kT > Gm^2/b$, the normal Jeans criterion prevents collapse. An answer to within a factor of 3 is fine as long as you get all the dependences right.

4. The first stars are thought to have formed at redshifts $z \sim 20$ (or somewhat larger). Here we want to calculate the masses of the dark matter halos that house them.

In order to retain baryons, the escape speed of halos must be greater than the thermal speed of the baryons. We will assume that the temperature of the baryons is 10^4 K (this is what can be achieved with atomic hydrogen cooling), and that the baryons are pure hydrogen. The escape speed from a halo of mass M is

$$V_{\rm esc} \approx 220\ \mathrm{km\ s^{-1}} [g(c)]^{1/2} (M/10^{10}\ M_\odot)^{0.27} \tag{1}$$

where c, the so-called concentration parameter, is given by

$$c = 24[M/10^{10}\ M_\odot]^{-0.13}(1+z)^{-1} \tag{2}$$

at redshift z and

$$g(c) = [\ln(1+c) - c/(1+c)]^{-1}\ . \tag{3}$$

With this information, calculate to within a factor of 5 the minimum halo mass needed at redshift $z = 20$ to retain pure hydrogen at $T = 10^4$ K.