ASTR 498
Problem Set 1
Due Thursday, February 14

1. **4 points** Problem 1.8 from book: In a spaceship shuttle service from Earth to Mars, each spaceship is equipped with two identical lights, one at the front and one at the back. The spaceships usually travel at a speed v relative to Earth, such that the headlight of a spaceship approaching the Earth appears green ($\lambda = 500$ nm) and the taillight of a departing spaceship appears red ($\lambda = 600$ nm).

(a) **2 points** Show that v/c=1/11.

(b) **2 points** One spaceship accelerates to overtake the spaceship ahead of it. Show that the overtaking spaceship has to travel with a speed of $0.18c$ relative to Earth so that the taillight of the Mars-bound spaceship ahead of it looks like a headlight (i.e., green).

2. **4 points** Problem 1.9 from book: A particle as observed in a certain reference frame has a total energy of 5 GeV and a momentum of 3 GeV/c.

(a) **1 point** What is its mass in GeV/c^2?

(b) **1 point** What is its energy in a frame in which its momentum is equal to 4 GeV/c?

(c) **2 points** Use the velocity addition formulae, or the energy-momentum transformation, to find the relative speed of the two frames of reference, if the particles are moving in the same direction.

3. **4 points** Problem 3.11 from book:

The energy loss rate (i.e., energy per time) for a single, ultrarelavistic electron to synchrotron radiation is

$$\frac{dE}{dt} \approx -\frac{2}{3} r_0^2 c \gamma^2 B^2 \tag{1}$$

where r_0 is the classical radius of the electron $r_0 = e^2/(m_e c^2)$, $\gamma = 1/\sqrt{1 - v^2/c^2}$ is the Lorentz factor, and B is the strength of the magnetic field. For an ultrarelativistic electron, $E \approx \gamma m_e c^2$ and $\gamma \gg 1$.

Show that the energy changes with time as

$$E(t) = \frac{E_0}{1 + t/\tau} \tag{2}$$

where τ is the synchrotron loss time and E_0 is the energy at $t = 0$. Also derive τ; for each variable that τ depends on (examples would be the magnetic field strength B, or the charge e of the electron), discuss whether the dependence should be direct (increasing as the variable increases) or inverse (decreasing as the variable increases).

4. **4 points** The well-known theoretical gadfly Dr. I. M. N. Sane has realized that a fundamental error is being made in studies of core-collapse supernovae. When collapse to a neutron star happens, a characteristic temperature is $T = 10^{12}$ K. Ordinary theorists think that neutrinos produced in this environment have an energy of 3–5 MeV (where 1 eV=1.6×10^{-12} ergs, and the mass-energy of an electron is 511 keV), but Dr. Sane asserts that the real energy is of order kT. There are about 10^{57} electrons uniformly distributed in the inner 100 km of the star, and neutrinos scatter off of them, delivering momentum and energy. Everyone, including Dr. Sane, agrees that about 1% of neutrinos interact in this way; more than that, and supernovae would be too energetic.

The Washington Post wants to do a story on this, but has consulted you first. Do a quantitative calculation to determine if Dr. Sane's idea hangs together.