ASTR 498
Problem Set 6
Due Thursday, May 8

1. Gamma-ray burst flux distribution. Let us suppose that all gamma-ray bursts are standard candles in the sense that they all have the same luminosity L and emit isotropically. Looking out into the sky, the number observed that have a flux F or greater depends on the flux like $N(>F) \propto F^{-\alpha}$. In each of the following two cases, please derive the power law α.

(a) [**2 points**] The sources are distributed homogeneously throughout all space, with constant number density.

(b) [**2 points**] We are at the center of a distribution of sources that are being continually produced, and all the sources are drifting away from that center at a uniform speed. There is an equal probability of a burst occurring anywhere along each individual trajectory, but note that this means the number density *decreases* with increasing distance from us (and part of the problem is to calculate this decrease). This would be one consequence of a model in which the sources are high-velocity neutron stars ejected from our galaxy.

2. One of the arguments in favor of the magnetar model of soft gamma-ray repeaters involves the maximum luminosity they can sustain while still holding on to their plasma. The normal Eddington luminosity would be about $L_E = 2 \times 10^{38}$ erg s^{-1}, but these sources can sustain 10^{42} erg s^{-1} or more without apparently ejecting large amounts of mass.

A suggested reason for this is that very strong magnetic fields decrease the electron scattering cross section by a factor $\sigma/\sigma_T = (\hbar\omega/\hbar\omega_{\rm cyc})$. Here the photon energy is $\hbar\omega$, and $\omega_{\rm cyc}$ is the electron cyclotron energy.

For [**4 points**], derive the minimum magnetic field in Gauss such that the critical luminosity is at least 10^{42} erg s^{-1}, assuming photon energies of $\hbar\omega = 20$ keV. Do your derivation with dimensional analysis, obtaining $\hbar\omega_{\rm cyc}$ using only $\hbar$, c, the electron charge e, the electron mass m_e, and the magnetic field B (no factors of 2, π, or the like). **Hint:** the square of the magnetic field, B^2, has units of energy density, and $e^2/(\hbar c)$ is dimensionless. As a further hint, there is only one factor of e in the numerator.

3. [**4 points**] This problem shows the limits of order of magnitude calculations in some cases. Let's say you'd like to estimate the recoil speed of a merged black hole remnant, due to linear momentum carried away by gravitational radiation. To simplify things, suppose we have two nonrotating black holes of masses M_1 and M_2 that collide head-on, so there is no spin at any point. A theorem from black hole thermodynamics says that the square of the irreducible mass of the final black hole cannot be less than the sum of the squares of the irreducible masses of the

initial black holes. For nonrotating black holes, this becomes

$$M_{\rm final}^2 \geq M_1^2 + M_2^2 \; . \tag{1}$$

Like the increase in entropy, this is an *inequality*, but for our order of magnitude estimate we will assume $M_{\rm final}^2 = M_1^2 + M_2^2$.

With that assumption, compute the final speed of the remnant (as a fraction of the speed of light, and as a function of M_1 and M_2) assuming that all the radiated energy is carried away in a single direction. For comparison, the maximum kick speed in practice, for any masses or angular momenta, is somewhat under 4,000 km s^{-1}.

4. [**4 points**] Dr. I. M. N. Sane has come to you with a brilliant idea. He has realized that LISA will be the ideal instrument to detect moons around extrasolar planets. In particular, he envisions a $m = 6 \times 10^{26}$ g moon (about 10% of Earth's mass, bigger than any moon in the Solar System) in a circular orbit of frequency of $f_{\rm orb} = 5 \times 10^{-5}$ Hz around a planet with mass $M = 2 \times 10^{31}$ g, about ten times Jupiter's mass. At gravitational wave frequencies $f_{\rm GW} < 10^{-3}$ Hz, LISA's spectral density sensitivity at signal to noise of 1 is $10^{-19}(10^{-3}\ {\rm Hz}/f_{\rm GW})^2\ {\rm Hz}^{-1/2}$. Assuming an observing time of 10^8 seconds, evaluate the detection prospects if the system is at a distance of 10 parsecs from us (about 3×10^{19} cm).