Line Broadening

Initial questions: What processes can change the profile of an atomic line? What about in the practical case, where we have a finite signal to noise?

Spectral lines are not arbitrarily sharp. There are a variety of mechanisms that give them finite width, and some of those mechanisms contain significant information. We'll consider a few of these in turn, then have a detailed discussion about how line shapes and profiles have given information about rapidly rotating accretion disks around black holes.

First, **Ask class:** why can't a spectral line be arbitrarily sharp? Ultimately, it comes from the uncertainty principle in the form $\Delta E \Delta t > \hbar/2$. If a line were arbitrarily sharp, this would imply perfect knowledge of E, which can't happen unless the atom spends an infinite amount of time before decaying into a lower state. If instead the decay time is finite, say $\tau_{\rm decay}$, then the approximate width of the line is $\Delta E \sim \hbar/\tau_{\rm decay}$. This is called *natural broadening*, and represents the limit on how sharp a line can be. If one has an atom in state n, and the spontaneous decay rate to a lower energy state n' is $A_{nn'}$, then the spontaneous decay proceeds at a rate

$$\gamma = \sum_{n'} A_{nn'} \ , \tag{1}$$

Ask class: Is this the only contribution to the decay? No, there are also induced decay processes (stimulated emission). These should be added to the spontaneous rates. The energy decays at a rate $\exp(-\gamma t)$. The energy is proportional to the square of the coefficient of the wave function, so that coefficient decays at a rate $\exp(-\gamma t/2)$. The decaying sinusoid that is obtained for the electric field then gives a line profile of the Lorentz form, as we saw in the semiclassical picture:

$$\phi(\nu) = \frac{\gamma/4\pi^2}{(\nu-\nu_0)^2 + (\gamma/4\pi)^2} \ . \tag{2}$$

Ask class: from the above discussion, can they name a state that will have zero breadth because it can persist indefinitely? The ground state is stable, so its energy can be defined with (in principle) arbitrary sharpness. If instead the level n' is itself an excited level, that energy level has breadth as well. Then, approximately, the effective width of the transition is $\gamma = \gamma_u + \gamma_l$, where γ_u and γ_l are respectively the widths of the upper and lower states.

Ask class: what's a way to broaden this line further *for a single atom*? Collisions will do it. Effectively, a collision produces an abrupt change in the phase of the wave function. Suppose that collisions occur at random times with an average frequency $\nu_{\rm col}$. Then the resulting profile still looks like a Lorentzian:

$$\phi(\nu) = \frac{\Gamma/4\pi^2}{(\nu-\nu_0)^2 + (\Gamma/4\pi)^2} \ , \tag{3}$$

where now $\Gamma = \gamma + 2\nu_{\rm col}$ includes contributions from both natural broadening and collisions.

Now suppose we have a collection of many atoms, and we are measuring the line profile from all of them combined. **Ask class:** what is another mechanism that will broaden the observed line? Doppler shifts are one way. Each atom, individually, will emit a line that has the natural width plus a collisional width, but its motion towards us or away from us will produce blueshifts or redshifts, so its line center will be displaced. Many atoms, moving in different directions with different speeds, will produce a line blend with significant width. For example, suppose the atoms are thermalized and thus have a Maxwellian distribution of velocities with some temperature T. Then if the line center frequency is ν_0, the line profile is Gaussian:

$$\phi(\nu) = \frac{1}{\Delta\nu_D\sqrt{\pi}} e^{-(\nu-\nu_0)^2/(\Delta\nu_D)^2} \tag{4}$$

where $\Delta\nu_D$, the Doppler width, is

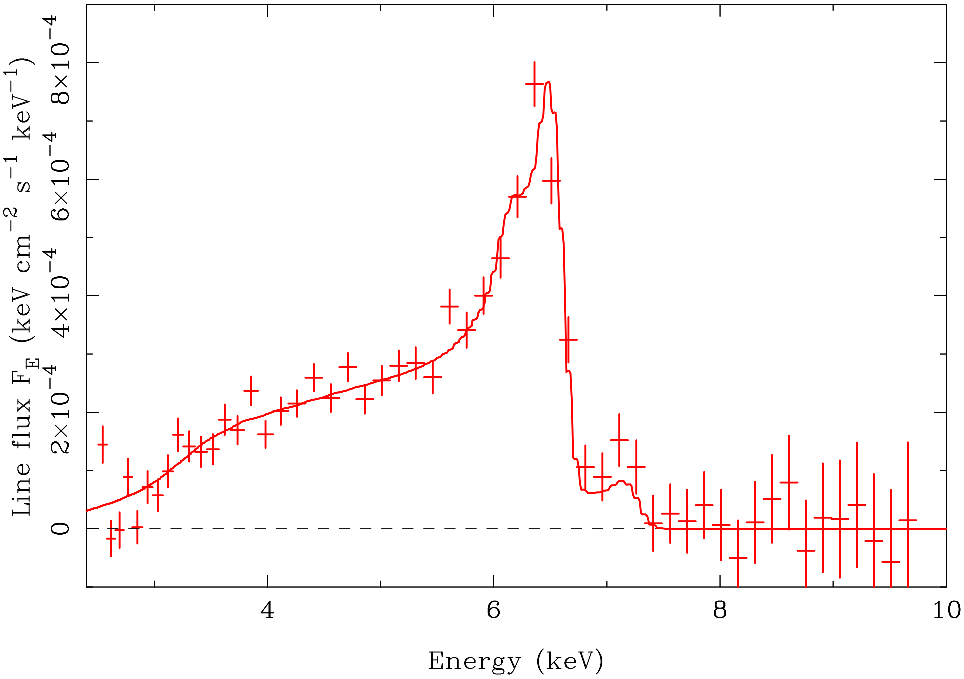
$$\Delta\nu_D = \frac{\nu_0}{c}\sqrt{\frac{2kT}{m_a}}\ . \tag{5}$$

A similar profile is obtained if one has microturbulence. However, if the Doppler shifts are from ordered motion (e.g., orbits), the profile will be different. Fundamentally, one calculates the Doppler profile by adding up the Doppler shifts from all the atoms individually. One can imagine a situation in which collisions and Doppler shifts are both important. If the Doppler shifts are due to isotropic thermal motion, the resulting line profile is called the Voigt profile, and is a convolution of a Lorentzian and a Gaussian. Note that because a Lorentzian dies off like a power law, whereas a Gaussian dies off exponentially, the line wings sufficiently far from the center will always be dominated by the Lorentzian.

Let's examine a couple of examples in which the line profile gives us physical information.

Suppose you are observing stars moving in the center of a distant galaxy. **Ask class:** If there is a supermassive black hole in the center of the galaxy then what, qualitatively, do you expect to see when you focus on a particular spectral line? It depends on whether the motion near the black hole is ordered or random. If the motion is ordered, then as one scanned across the central regions one would expect the net velocity (as measured by the redshift or blueshift of the line) to increase quickly towards the center, then abruptly change sign when the center was crossed. If the motion is random, then the line would have a width that increased towards the center. Either way, one can define a velocity or velocity dispersion that indicates the mass of the black hole. In more distant galaxies, other methods are used to estimate or constrain the mass of the black hole, because one can't observe the optical lines of stars with enough spatial resolution.

Now consider another example. The inner regions of accretion disks around black holes are hot places, and various processes mean that there are photons of energies reaching up to many keV to tens or even hundreds of keV. When a photon with an energy of 6-7 keV or more hits the accretion disk, it can photoionize the inner K shell electrons of iron, which is relatively abundant for a metal and has a high cross section for this effect. When an electron drops down into the K shell from the next shell up, it emits a line that, in the rest frame of the atom, is relatively sharp and has an energy of 6.4 keV. Motion of the atoms in an accretion disk can change this sharp rest-frame line into a broader line. Detailed interpretation of this line has given a tremendous amount of information about the properties of accretion disks and strong gravity. Let's try our hand at it. Suppose that the observed line looks like:



Ask class: What effects might account for this profile? We'll need to identify important parts and interpret them separately to put together the picture. We see that the line is (1) broad, (2) asymmetric, (3) sharply peaked. We also note that the line goes a little bit above the rest-frame energy, but a lot below. All this can be understood in terms of a relativistic disk. Emission from deep in a gravitational well can be redshifted significantly, which is why the emission can get to half(!) of the rest-frame energy. If matter is moving in a disk, then when the fast (half the speed of light or more) motion is towards us, then there is significant beaming. Put another way, since the specific intensity scales as $I_\nu \propto \nu^3$, blueshifts can increase the intensity a lot, whereas redshifts decrease it a lot. Thus, the emission is strongly peaked where there is a slight blueshift, and it is asymmetric. The sharp cutoff is also understandable, given some details of a relativistic disk model; deep in the gravitational well, orbital motion produces little if any net blueshift. These results, worked on by Chris Reynolds and Andy Young at Maryland (among others) have provided evidence for rapid rotation of supermassive black holes, and possibly of extraction of the spin energy of black holes in a couple of cases!

Recommended Rybicki and Lightman problem: 10.5