ASTR 601 Problem Set 4, Due Thursday, November 2

1. **8 pts** This is your numerical problem, and we'll use this opportunity to give you practical experience with statistics. I did a whole course on Bayesian statistics (note that in the following, the "∼" will need to be typed by hand in the URL; it doesn't copy from the pdf):
`https://www.astro.umd.edu/∼miller/teaching/astrostat/`,
which I think would be useful for you to read when you have time, but for this problem I will only require you to read Lectures 6 and 7 from that course:
`https://www.astro.umd.edu/∼miller/teaching/astrostat/lecture06.pdf` and
`https://www.astro.umd.edu/∼miller/teaching/astrostat/lecture07.pdf`.
Then, perform the analysis describe in detail in Lecture 7 (where there are uncertainties in *both* the baryonic mass and the rotational speed) with the whole data set (which is on our website, next to the pdf for this problem set). Your output needs to be a plot of the $\Delta\chi^2 < 2.3$ and $\Delta\chi^2 < 6.18$ regions in $\theta - b$ space (see the lectures for definitions), a plot of the marginalized one-parameter posteriors $P(\theta)$ and $P(b)$, the values of θ and b at the minimum χ^2 in the full $\theta - b$ space, the values of θ and b at the peaks in their respective one-dimensional marginalized distributions, and the minimum χ^2 and the number of degrees of freedom. Note that Lectures 6 and 7 give details about how to perform these operations, and the results for small subsets of the data, so you can check your results. As usual, I need you to send me your code (in any language but in a form that can be compiled [please send instructions!] and run on my departmental desktop) before the class starts.

2. **4 pts** Polarization of the cosmic microwave background has been observed with WMAP, Planck, and other experiments. Let's focus on observations at 1 GHz. If the universe had a strong enough magnetic field with a direction that varied on angular scales smaller than the beam of the detector (note that the "beam" of the detector is its angular resolution limit), Faraday depolarization would have destroyed the polarization signal. Your task will be to determine what this means for the possible strength of the universe's magnetic field.

(a) First, let's think about the current universe. The polarization was observed at approximately the expected strength. Assuming that the universe is fully ionized hydrogen at a number density of 2×10^{-8} cm^{-3}, place an upper limit (to within two orders of magnitude) on the net parallel magnetic field strength on scales smaller than the beam. The size of the universe at low redshift is around 10^{28} cm.

(b) Now what about at earlier times? Consider the universe in the redshift range $z_0/2$ to z_0. Assume that the number density scales as $(1+z)^3$, and that the path length from z to $z+dz$ is roughly $10^{28}\,\mathrm{cm}(1+z)^{-2}dz$. Also remember that if we observe a frequency ω, the frequency in the rest frame was $\omega(1+z)$. Within two orders of magnitude, what is the limit on the magnetic field that existed at $z_0 = 1000$? We're still assuming fully ionized hydrogen.

(c) A primordial magnetic field that expands passively with the universe has a strength that scales as $n^{2/3}$, where n is the number density. Let us suppose that reionization happened at $z = 6$; that is, when we do our physics properly we realize that the universe was actually mainly neutral from the CMB epoch $z \approx 1100$ until stars and the first quasars reionized the universe at $z = 6$. Thus take your magnetic field limit from part (b), evaluated at $z = 6$, and passively evolve that primordial field to $z = 0$. What does that imply about the upper limit to the *current* strength of the primordial component of the magnetic field?

3. **4 pts** Dr. Sane has become fascinated by the problem of missing mass. He thinks he has the perfect candidate: "Sane nebulae". Sane nebulae are clouds of fully ionized pure hydrogen, that typically have temperatures of 10^5 K and number densities of 0.1 cm^{-3}. They have no internal energy sources. Dr. Sane thinks that Sane nebulae have existed with these properties more or less unchanged for the last 10 billion years. Evaluate his claim, assuming that Sane nebulae have negligible interaction with anything external to them. **Hint:** Dr. Sane's nebulae are small enough that any radiation generated in them escapes without interacting further.