ASTR 680
Midterm
Tuesday, March 13

Constants and Equations

Speed of light in vacuum: $c = 2.9979 \times 10^{10}$ cm s^{-1}

Gravitational constant: $G = 6.673 \times 10^{-8}$ g^{-1} cm^3 s^{-2}

Planck's constant: $h = 6.63 \times 10^{-27}$ erg s, $\hbar = h/2\pi = 1.05 \times 10^{-27}$ erg s.

Energy conversion: 1 eV=1.6×10^{-12} erg

Thomson cross section: $\sigma_T = 6.65 \times 10^{-25}$ cm^2

Electron mass: $m_e = 9.11 \times 10^{-28}$ g=511 keV/c^2

Proton mass: $m_p = 1.6726 \times 10^{-24}$ g=938.27 MeV/c^2

Neutron mass: $m_n = 1.6749 \times 10^{-24}$ g=939.57 MeV/c^2

Solar luminosity: $L_\odot = 3.8 \times 10^{33}$ erg s^{-1}

Solar mass: $M_\odot = 2 \times 10^{33}$ g

Minkowski line element: $ds^2 = -d\tau^2 = \eta_{\hat{\alpha}\hat{\beta}} dx^{\hat{\alpha}} dx^{\hat{\beta}} = -d\hat{t}^2 + d\hat{r}^2 + d\hat{\theta}^2 + d\hat{\phi}^2$

Schwarzschild line element: $ds^2 = -d\tau^2 = g_{\alpha\beta} dx^\alpha dx^\beta$

$= -(1 - 2M/r)dt^2 + dr^2/(1 - 2M/r) + r^2(d\theta^2 + \sin^2\theta d\phi^2)$

Schwarzschild conserved quantities: u^2, u_ϕ, u_t

Transformation matrices: $\eta_{\hat{\alpha}\hat{\beta}} = e^\mu{}_{\hat{\alpha}} e^\nu{}_{\hat{\beta}} g_{\mu\nu}$, $g_{\mu\nu} = e^{\hat{\alpha}}{}_\mu e^{\hat{\beta}}{}_\nu \eta_{\hat{\alpha}\hat{\beta}}$

Schwarzschild radius: $R_s = 2GM/c^2 = 2.94(M/M_\odot)$ km

Overall hint: All of these problems are designed to be straightforward if done properly. If you think you need to write an enormous amount or go through a very long derivation, you aren't doing it correctly and should probably spend a few minutes thinking about a more efficient method.

There are four problems; see the back page for the fourth.

1. Terms and concepts. Give short (one-sentence) descriptions of the following (**1 point each**):

(a) Opacity (give cgs units as well).

(b) Eddington luminosity (I don't need the value or equation, just the physical definition).

(c) Invariant interval.

(d) Frame-dragging.

2. Particles and radiation (**4 points**).

Some blazars emit photons up to 10^{13} eV. The universe has a large infrared background at energies of $\sim 10^{-1}$ eV, meaning that in the center of momentum frame of the IR-gamma ray interaction, the product of the energies is up to 10^{12} (eV)^2. The rest mass-energy of an electron is 5.11×10^5 eV. Use these facts to argue qualitatively that there should be a sharp cutoff in the gamma-ray spectra of sufficiently distant blazars (and indicate qualitatively what criterion would define "sufficiently distant"; I don't need the actual distance, just an indication of what it means).

3. General relativity

Consider a particle with nonzero rest mass, so that $u^2 = -1$. It starts with zero speed at a large distance and is dropped radially into a Schwarzschild black hole with no energy loss. Therefore, the only nonzero space component of the velocity is the radial component. With this information, derive the following (here r and t are the usual Schwarzschild coordinates; see the equation sheet for line elements and the definition of transformation matrices):

(a) (**1 pt**) The proper radial velocity $u^r = dr/d\tau$.

(b) (**1 pt**) The radial velocity as seen at infinity, dr/dt.

(c) (**1 pt**) The radial velocity as seen by a local static observer, $d\hat{r}/d\hat{t}$.

(**1 pt**) In each of these cases, add a sentence at the end to indicate if your answer makes physical sense as $r \to 2M$. For example, "This answer makes physical sense because..." or "This answer does not make physical sense because..." if you feel your answer is wrong.

4. Black holes (**4 points**)

Dr. Sane has been banned from future Nature submissions, so he is taking his discoveries directly to the popular press. Ron Cowen, astronomy writer for Science News, has asked you to investigate Dr. Sane's latest claims. In particular:

(a) (**2 pts**) Dr. Sane believes that in some protoplanetary disks, a Jupiter-like planet (mass $M \approx 2 \times 10^{30}$ g, moment of inertia $I \approx 6 \times 10^{49}$ g cm^2, and angular velocity $\Omega \approx 2 \times 10^{-4}$ rad s^{-1}) can collapse directly into a black hole on a free-fall timescale. He thinks this is because of special interactions in the extended atmosphere of the planet. **Hint:** dimensional analysis is your friend here!

(b) (**2 pts**) Dr. Sane also thinks that the Sun is actually powered by accretion onto a central $M_{\rm BH} = 0.001\, M_\odot$ black hole (produced in the early universe), instead of by nuclear fusion. He thinks this happens by Bondi accretion (accretion rate $\dot{M} = \rho\sigma v$, where $\rho \approx 1$ g cm^{-3} is the average density of the Sun, $v = 4 \times 10^7$ cm s^{-1} is a typical speed, and the cross section is appropriate to the Bondi radius $r_{\rm Bondi} = GM_{\rm BH}/v^2$).

In both cases, give the most concise and robust possible argument showing that Dr. Sane is once again out of his gourd.