ASTR 680 Problem Set 1: Due Thursday, September 15, 2022

1. **(4 points)** Suppose that there is a population of standard candles in the universe. Regardless of their redshift, they emit a rest frame pure power law with specific intensity $I_\nu \propto \nu^{-\alpha}$, with a luminosity independent of redshift, for a rest frame time T independent of redshift. On Earth, we observe these sources with detectors that have perfect sensitivities between received frequencies $\nu_{\rm min}$ and $\nu_{\rm max}$, and no sensitivity outside this range. Please calculate:

(a) **(2 points)** The dependence of the observed flux per solid angle on the redshift of the source. Flux here is energy per time per area in the observed frequency range $\nu_{\rm min} < \nu < \nu_{\rm max}$.

(b) **(2 points)** The dependence of the observed fluence per solid angle on the redshift of the source. Fluence here is the energy per area, integrated over the observed duration of the event, in the observed frequency range $\nu_{\rm min} < \nu < \nu_{\rm max}$.

2. **(4 points)** Dr. I. M. N. Sane, an employee of your least favorite institution, has had a dazzling revelation. Just as the microwave background has given us unprecedented understanding of the universe at a few hundred thousand years, so will the thermal neutrino background tell us about the universe at a few seconds. He has proposed to the National Science Foundation that a large water tank be built to observe these neutrinos scattering off the electrons in the water. You have been asked to comment on the viability of this plan.

(a) **(3 points)** We will assume that the neutrinos are relativistic, and distributed as a blackbody. Their background has an expected temperature of $T = 1.9$ K. Assuming for simplicity that all the neutrinos are at the thermal peak $E = 2.7kT$, compute to within a factor of 5 the volume of water (at 1 g cm^{-3}) needed so that we would expect 100 scatterings in one year (so that we have good statistics and can derive interesting quantities). Assume that all electrons in the water can potentially scatter the neutrinos, and that any scattering will be detected. Find a body of water with approximately this volume.

(b) **(1 point)** Give a simple argument that, in reality, your answer is a tremendous underestimate of the volume actually needed to detect electron scattering of these neutrinos.

3. **(4 points)** There is a great deal of interest in how dark matter in our Galaxy might be detected. One possibility that has been suggested is that dark matter might decay into electrons and positrons. Indeed, anomalies in the spectra of electrons and positrons have been proposed as signatures of dark matter (but be skeptical of this...).

(a) **(2 points)** Suppose that the decay of a dark matter particle produces a 1 TeV electron. Ignoring its possible interactions with other particles, calculate to within a factor of 3 how long it will be before the electron's energy drops to half its initial value because of synchrotron radiation. Assume that the interstellar magnetic field is tangled (so that the pitch angle is random at any time) and has a uniform strength of 3 microGauss.

(b) **(2 points)** If the electron undergoes a random walk, where the step size equals the radius of curvature at any given time, then within a factor of three how large will be the net distance it travels by the time it gets down to 500 GeV? In a random walk of N steps of length d, the net distance is roughly $d_{\rm net} \approx d\sqrt{N}$. Here, though, the step size changes as the energy does, so you may need to factor that in.

4. **(4 points)** Compare the total energy ever emitted in gravitational waves with the total emitted in starlight. To do this, assume that gravitational wave energy is dominated by mergers of supermassive black holes (SMBH); that a typical current SMBH has 10^{-4} of the total mass of the stars in a galactic disk; and that in their lifetimes most SMBHs have one merger with a comparable-mass SMBH, and in doing so emit 5% of their mass-energy in gravitational waves. For the starlight, you will need to look up galaxy luminosities and multiply by the age of the universe, so please give me the luminosity and tell me the reference.