ASTR 680 Problem Set 3: Due Thursday, October 20, 2022

1. **8 points** Assume you have a black hole of initial mass M_0 and some specified initial spin, and that it accretes matter at the innermost stable circular orbit. Write a computer program to calculate the dimensionless spin parameter $j = a/M = J/M^2$ of the hole as a function of its current mass M. For this I'll need an e-mail copy of your code, which must be able to compile and run on the astro machines (I'm not going to install anything!). I'll also need hardcopy of graphs showing j versus M for the two cases below.

(a) If it starts with $j = 0$ and always accretes matter in the prograde direction, then to within 0.1% how much mass does it accrete to get to j=0.5, 0.9, and the extremal value 1.0?

(b) Suppose you start with a near-extremal Kerr hole ($j = 0.999$) and accrete matter in *retrograde* orbits at the innermost stable circular orbit. To within 0.1%, how much mass does it accrete to get to j=0.99, 0.9, 0.5, and 0?

For this problem you need the following formulae. The specific angular momentum (i.e., per unit rest mass) of a particle in a circular geodesic at radius r around a black hole of mass M and spin parameter $a = jM$ is

$$u_\phi = \pm \frac{\sqrt{Mr}\left(r^2 \mp 2a\sqrt{Mr} + a^2\right)}{r\left(r^2 - 3Mr \pm 2a\sqrt{Mr}\right)^{1/2}} \ . \tag{1}$$

Here the upper sign is for prograde orbits and the lower sign is for retrograde orbits. The specific energy of a particle in that same circular orbit is

$$-u_t = \frac{r^2 - 2Mr \pm a\sqrt{Mr}}{r(r^2 - 3Mr \pm 2a\sqrt{Mr})^{1/2}} \tag{2}$$

where again the upper sign is for prograde orbits and the lower sign is for retrograde orbits. The radius of the innermost stable circular orbit is

$$r_{\rm ISCO} = M\left\{3 + Z_2 \mp \left[(3 - Z_1)(3 + Z_1 + 2Z_2)\right]^{1/2}\right\} \ , \tag{3}$$

where once again the upper sign is for prograde and the lower is for retrograde. Here we use

$$Z_1 = 1 + \left(1 - j^2\right)^{1/3}\left[(1 + j)^{1/3} + (1 - j)^{1/3}\right] \tag{4}$$

and

$$Z_2 = \left(3j^2 + Z_1^2\right)^{1/2} \ . \tag{5}$$

2. **8 points** Dr. Sane has a new model for active galactic nuclei. He asserts that galaxies at $z \sim 2 - 5$ (the peak of AGN activity) had proto-molecular clouds at their center, with typical

temperatures of 100 K and typical densities of $n = 10^2$ cm^{-3}. In his model white dwarfs, with initial masses $M_{\rm init} = 0.6\,M_\odot$ and velocities relative to the medium of 20 km s^{-1}, accrete from the clouds until their masses exceed the Chandrasekhar mass $M_{\rm Ch} = 1.4\,M_\odot$ and they collapse in free fall into neutron stars with radii 10 km from an initial radius of 1000 km. The gravitational energy released powers the AGN; in addition, angular momentum conservation means that the energy is often released in jets along the rotation axis. State at least three reasons why this model cannot explain AGN. Recall that typical observed energy outputs are $\sim 10^{44}$ erg s^{-1}, that the active phase might last for $< 10^9$ yr, that variability timescales are ∼minutes to ∼years, and that many of these sources have persistent and large-scale jets. You must make quantitative calculations supporting at least two of the problems you bring up. The other can be qualitative, i.e., something that Dr. Sane's model probably can't account for but is more difficult to quantify.