ASTR 680 Problem Set 5: Due Thursday, November 17, 2022

1. Fermi energy.

Suppose that you have a star that is supported by nonrelativistic degeneracy (of electrons or neutrons, it doesn't matter). Derive the dependence of the radius R of the star in equilibrium on its mass M (for example, if radius dependend on mass to the seventh power, which it doesn't, you would write $R \propto M^7$).

To do this, first make the simplifying assumption that the density is constant, so that if the mass is carried by protons and neutrons and the degenerate particles form a fixed fraction of the total particles; therefore, the number density of degenerate particles is $n \propto M/R^3$. The Fermi momentum is $p_F \propto \hbar/\Delta x \sim \hbar n^{1/3}$, and for nonrelativistic degeneracy the Fermi energy *per particle* is $E_F = p_F^2/2m$. To get the total energy per particle, add this to the (negative) gravitational energy per particle. Then, for fixed mass M, minimize the total energy per particle with respect to the radius R and determine the dependence of R on M.

2. What about magnetic fields?

Dr. Sane has once again stunned the astrophysical community with his unmatched breadth and vision. Ordinary researchers have noted the "ankle" in the cosmic ray spectrum above $\sim 10^{18}$ eV without realizing its significance. Dr. Sane has rushed to fill this well-needed gap in the literature.

More specifically, he notes that a neutron star with a magnetic field B, radius R, and rotation frequency Ω (in radians per second) can produce a potential drop of $B\Omega R^2/c$ in statvolts (recall that one statvolt equals 300 volts). He asserts that neutron stars can therefore accelerate protons to $> 10^{18}$ eV, and in his model it is necessary that a given star be able to do this for a thousand years to explain the ankle.

The cosmic ray community is contemplating awarding Dr. Sane the Yodh Prize for career contributions to cosmic ray research. They have, however, wisely consulted you first. To make your evaluation, assume $R = 10^6$ cm. However, you should be open-minded about B (allowing it to be between 10^8 G and 10^{16} G) and Ω (allowing it to be anywhere between 0 and 8000 rad s^{-1}). With these parameters, what can you say about Dr. Sane's latest idea?

3. Spinup of neutron star. Suppose you have a neutron star that is initially nonrotating. It has a dipolar magnetic field of polar strength B, and matter accretes on the star at a rate $\dot{M}$. Assuming that the matter couples to the field exactly at the Alfvén radius, derive the characteristic time for the star to reach spin equilibrium. By "characteristic time" we mean the equilibrium spin frequency divided by the *initial* rate at which the spin frequency changes (in reality the spin frequency would asymptote to equilibrium, but we want just the characteristic time). What is the characteristic

time for a star with $B = 10^{12}$ G and $\dot{M} = 10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$, which might be typical for a young neutron star in a high-mass X-ray binary?

4. Helium core flash.

Let's get some insight into thermonuclear flashes in a slightly different context: the evolution of a massive star. Given the equations below (which indicate the energy generation rate per time in triple-alpha fusion of helium, plus how the temperature adjusts to that energy), write a code to follow the temperature with time.

I'll need from you (a) a plot of the log temperature as a function of time (in units of days), and (b) an e-mail copy of your code, which has to be able to compile and run on the astro machines. Note that the temperature rises very suddenly during the flash, so you need to resolve the rise; you don't have to have a tiny time step the whole way, though.

The details:

Fix the density at $\rho = 2 \times 10^5$ g cm^{-3} and assume the composition is always pure helium. The initial temperature is $T = 1.5 \times 10^8$ K. The energy generation rate for the triple-alpha reaction is

$$\epsilon_{3\alpha} = \frac{5.1 \times 10^8 \rho^2 Y^3}{T_9^3} e^{-4.4027/T_9} \quad \mathrm{erg\,g^{-1}\,s^{-1}}\ . \tag{1}$$

where $T_9 \equiv T/10^9$ K and $Y = 1$ is the helium mass fraction. For a given time step, in which some number of ergs per gram is produced, the change in temperature is determined by the sum of the specific heats of the helium and the electrons. That is:

$$\frac{dT}{dt} = \frac{1}{c_V(\mathrm{He}) + c_V(\mathrm{e})} \epsilon_{3\alpha}\ . \tag{2}$$

Here $c_V(\mathrm{He}) = 3.1 \times 10^7$ erg g^{-1} K^{-1} is the specific heat for helium and

$$c_V(e) = 0.346(T/1\ \mathrm{K})\ \mathrm{erg\ g^{-1}\ K^{-1}} \tag{3}$$

is the specific heat for the electrons (these are not general formulae; the coefficients are specific to this problem).