

ASTR 680 Problem Set 6: Due Thursday, December 1

1. **8 points** One model for short hard gamma-ray bursts is the tidal disruption of a neutron star by a black hole. If this tidal disruption happens inside the ISCO, the whole neutron star falls in at once and it is doubtful whether any emission can last for more than a few milliseconds. However, for tidal disruption outside the ISCO there is at least a possibility that a longer-lived disk could form, as would be required to explain the observed durations of tenths of a second.

In this problem, you should write a computer program to calculate the maximum mass of a black hole that would disrupt a neutron star outside the ISCO, as a function of the hole's dimensionless spin parameter $j = a/M = cJ/GM^2$. Assume that the neutron star has a mass of $1.4 M_{\odot}$ and a radius of 10 km. Also assume that the spacetime is Kerr with the hole's spin parameter, so that you can compute the ISCO radius using the formulae given in problem set 3. In determining the tidal radius, you may use Newtonian formulae to calculate when the difference in gravitational acceleration across the neutron star from the black hole is greater or less than the acceleration from self-gravity of the star. As usual I'll need an e-mail copy of your code and a hardcopy of the graph showing M_{\max} (in solar masses) versus j .

2. **8 points** Dr. Sane has a remarkable new idea for the production of short hard gamma-ray bursts, which has not been considered by the establishment community. In a galactic nucleus or a globular cluster, two neutron stars that were unbound relative to each other simply collide at high speed, producing the burst. This is distinct from the standard idea of two stars *orbiting* each other and eventually coalescing. Giving Dr. Sane the benefit of the doubt, we will say that either a nucleus or a cluster has a typical number density of neutron stars of $n = 1000 \text{ pc}^{-3}$, that the typical relative speeds are 10 km s^{-1} in a globular and 100 km s^{-1} in a nucleus, that there are a total of 10^5 neutron stars in a globular system around one galaxy and 10^4 neutron stars in a single galactic nucleus, and that the number density of galaxies is 0.003 Mpc^{-3} . Also note that the cross section for a collision at a closest approach distance of r_p is $\pi r_p (2GM_{\text{tot}}/v^2)$, where M_{tot} is the total mass of the two stars and v is the relative speed at a large distance (the factor in parentheses is due to gravitational focusing).

As a result of this idea, the High Energy Astrophysics Division of the AAS is considering awarding Dr. Sane its prestigious Rossi Prize. They have brought you in as an external expert. Based on the notes, please do a quantitative evaluation of whether Dr. Sane's model can work. **Hint:** we see about one gamma-ray burst per day, and obviously miss some GRBs. Maybe 20–30% of observed GRBs are short GRBs.