Frontiers: Sunyaev-Zeldovich effect

An effect predicted more than three decades ago, the S-Z effect is coming into its own now as a probe of cosmological conditions, due to instrumental advances and a certain amount of cleverness! Here we'll talk about the basic effect itself, with a little discussion of some of the subtleties. We'll then discuss a number of the implications that observations of the S-Z effect has for estimation of cosmological parameters, such as the Hubble constant. Finally, we'll see the current state of the observations themselves.

Basics of the S-Z effect

We'll take much of the following from the ARA&A articles by Yoel Rephaeli (1995, 33, 541) and Carlstrom et al. (2002, 40, 643).

The idea is that clusters are in the foreground of the cosmic microwave background, so the CMB is affected by clusters. In its essence, the S-Z effect is simply Compton scattering. Photons in the cosmic microwave background scatter off of the hot gas ($T \sim 10^8$ K) in clusters. **Ask class:** what is the typical direction of change of photon energy as a result (i.e., does the photon become more or less energetic)? More energetic, because the electrons have a high temperature. Therefore, photons that had been at low energies become higher energy. As a result, if the initial CMB spectrum was a perfect blackbody, after scattering the spectrum is distorted. Scattering does not create or destroy photons, so the net result is transfer of photons from the low-energy (Rayleigh-Jeans) portion of the spectrum to the high-energy (Wien) portion of the spectrum. The CMB is in radio wavelengths, so radio observations are best at detecting both the decrement and the increment. Since the fractional frequency shift in a single Compton scattering is $\sim kT/m_e c^2$, the signal along a given line of sight is proportional to the integrated pressure, $\int nT\,dx$, where x is the coordinate of location in the cluster. Note that the magnitude of this effect is entirely independent of the redshift; it depends *only* on the cluster properties. This makes is practically unique as a cosmological probe. Virtually anything else you could imagine observing has serious surface brightness effects, usually like $(1+z)^{-4}$ or at best $(1+z)^{-2}$ for the flux of point objects, which make observations at high redshift very difficult. The S-Z effect therefore has substantial future promise.

This effect was first considered by Sunyaev and Zeldovich in the context of a hypothesized hot gas that pervades the universe, but it is now most often considered in relation to clusters. To estimate the significance of this effect, let's first compute the optical depth to scattering through the cluster. **Ask class:** for photons from a 2.7 K thermal bath scattering off of 10^8 K electrons, what is the appropriate limit of scattering? Thomson, because in the rest frame of the electrons the photon energy is much less than $m_e c^2$. So, we can calculate the typical optical depth through a cluster. If the number density of electrons is 10^{-3} cm^{-3}, and the size of the cluster is 1 Mpc, then the optical depth is $\tau = n\sigma d \approx 10^{-3} \times 6 \times 10^{-25} \times 3 \times 10^{24} \approx 2 \times 10^{-3}$. This is an extremely small optical depth, so the majority of photons from the CMB never scatter at all in a given cluster. This also means that one cannot treat the Comptonization process in the diffusion limit, as was done initially. Moreover, the speed of electrons at cluster temperatures is $v \approx (kT/511\ \text{keV})^{1/2}c \approx 0.1c$, so the motion is mildly relativistic and corrections have to be made. Another point is that in addition to the thermal Comptonization featured above, the directed movement of clusters relative to the Hubble flow can also produce a "kinematic S-Z effect". The kinematic S-Z effect is usually a factor of 10 or more smaller than the thermal S-Z effect. An exception comes with measurements near where the modified spectrum intersects the original spectrum (it has to, to go from a decrement to an increment). To first order, this "null" in the change occurs at $h\nu = 3.83kT$, or about 217 GHz for $T = 2.726$ K.

Other subtleties that people have considered include (1) the incident spectrum might not be quite a blackbody, e.g., if the hot intercluster gas has already changed the spectrum, (2) as a variant, cluster positions are correlated with each other, so Comptonization in a supercluster is a possibility, and (3) if the cluster is dynamically collapsing, then the gravitational potential of the cluster might change significantly during the radiation crossing time. All of these tend to be small effects compared to the dominant S-Z effect.

S-Z effect as probe of clusters and cosmology

Cluster properties.—The thermal S-Z effect depends on the integrated product of number density n and temperature T, which is the integrated pressure along the line of sight. The kinematic S-Z effect depends on the product of the peculiar velocity of the cluster and the integrated column depth. Therefore, when combined with X-ray measures of the cluster (the temperature directly, from the spectrum; and the luminosity, which depends on $n^2T^{1/2}$), the variety of dependences on number density and temperature allow redundant checks of many different cluster properties. This gives a great deal of information about the gas clustering properties.

Determination of H_0.—A long-anticipated (and now realized) benefit of the S-Z effect is that one can use it to estimate the Hubble constant in a way that is completely independent of all other estimates of H_0. Consider for simplicity a cluster that has a characteristic radius R, a characteristic number density n, and a characteristic temperature T. The magnitude of the thermal S-Z effect depends on nTR. The temperature comes directly from the X-ray spectrum, and the bremsstrahlung surface brightness depends on $n^2T^{1/2}R$. Measurement of the S-Z effect, the spectrum, and the luminosity therefore give independent determination of n, T, and R. Now, suppose that the redshift and apparent angular size of the cluster have been measured (in the best case, from an X-ray image, but it could also be optically). If you make the further assumption that the cluster is spherical, then from R and the angular size of the cluster you know the angular diameter distance.

At low redshift, the angular diameter distance depends only on H_0. One can, therefore, use this combination of observations to determine the Hubble constant. At higher redshift, the angular diameter distance also depends on Ω_m and Ω_Λ, so one could in principle use it to estimate Ω_m and Ω_Λ as well!

Ask class: what are some uncertainties or problems that could crop up in this determination? That is, which of the assumptions above could be wrong, and what would be the direction of the bias produced? One problem is that clusters might not be spherical. Suppose a cluster is prolate, and oriented with the long axis along our line of sight. **Ask class:** would the estimate for H_0 be high or low? Low. R is determined along the line of sight, and for a prolate cluster is larger than the orthogonal radius. Therefore, when the angular diameter is measured, it appears that the cluster is more distant than it really is (that is, the distance in Mpc is too large). But the redshift is fixed, so H_0, which is redshift/distance, is too small. Similarly, if the cluster is oblate and along our line of sight, H_0 is too large. Now suppose that the cluster is spherical and isothermal, but lumpy, so that there are a number of clumps with higher number density than average. **Ask class:** what is the effect on the derived Hubble constant? The derived H_0 is too high. Too see this, look again at the quantities that are derived: T, nTR, and $n^2T^{1/2}R$. If T is known, nR and n^2R are measured. But if the matter is clumpy, the average density is unaffected but the average squared density $\langle n^2\rangle$ is increased, so nR is the same but n^2R is larger than it would be. Taking the ratio of the two, n appears larger than it is, so because nR is known, R appears smaller than it is. Therefore, the distance appears smaller than it is, so the derived H_0 is too high.

The actual measurements of H_0 have enormous uncertainties. Early measurements tended to be a little too low (in the 40-60 km s^{-1} Mpc^{-1} range, compared to 70 km s^{-1} Mpc^{-1} from Cepheid results). One must be careful about the derivation of H_0, for many reasons in addition to those above. The assumption of an isothermal, constant density cluster gas is not correct, so instead one usually uses a beta model for the gas distribution. The recent high angular resolution measurements with Chandra suggest that the gas is not very clumped, and in any case that would tend to *increase* H_0, so that is probably not a factor. For redshifts $z \not\ll 1$, it is also important to include the possible presence of a cosmological constant. The main qualitative effect of a cosmological constant is to make the universe "bigger" at intermediate to high redshifts. That means that the angular diameter distance for fixed z and H_0 is larger when $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ than when $\Omega_m = 1$. So, a few years ago, measurements of H_0 for clusters with, say, $z = 0.5$ gave smaller values than they do now! Yet another cautionary point when interpreting data. The most recent results, taking overall cosmology into account and using the most updated X-ray data, are now in fact consistent with the Cepheid values for H_0 (e.g., Bonamente et al. 2006, ApJ, 647,25).

Measurements of the S-Z effect

The first attempts at measuring the S-Z effect were with single dish receivers, pointed at relatively low-redshift clusters ($z < 0.2$). The state of the art is that such observations have indeed been made, but they are difficult due to the difficulty of subtracting out point sources and eliminating background contamination. The big advance in the past few years has been the use of interferometers, particularly by John Carlstrom and his collaborators. The high stability and 2-D mapping capability of interferometry has made it the method of choice. But it isn't trivial. The initial problem was that interferometry as normally used has too great an angular resolution to detect the S-Z effect easily. Even at substantial redshifts, clusters have angular sizes on the order of tens of arcseconds to an arcminute, so obviously if the angular scale of an interferometer is much less than this, the signal will get averaged out. Carlstrom et al. dealt with this in a clever way: they used cm-wave receivers mounted on the BIMA and OVRO mm-wave interferometric arrays. The result was that the angular size probed better matched the size of the clusters, and the signal was much clearer. However, it was also necessary to move the individual telescopes very close to each other!

I have a personal identification with this effect. In 1997, when comet Hale-Bopp came by, it was bright enough to be seen with the naked eye even in Chicago. Looking at it normally, however, it didn't seem all that spectacular, because its light was spread out compared to the stars. With my glasses off, on the other hand, the light from the stars was spread out by a comparable amount, and Hale-Bopp really stood out!

The future promise of the S-Z effect

The near-independence of the S-Z effect from redshift effects, plus its independence from other types of cosmological estimates of quantities such as H_0, Ω_m, and Ω_Λ, means that it has a unique role in the ongoing data-rich era in cosmology. In particular, it will be possible to do an unbiased survey of cluster masses and temperatures and their evolution. From the last lecture, you recall that the evolution of clusters is a major clue to the value of Ω_m in particular, which therefore provides a complementary measure (along with CMB power spectra and supernova distance measurements) in the $\Omega_m - \Omega_\Lambda$ plane. The S-Z effect in the cores of clusters is especially important, because it depends on the product of n and T, both of which are expected to evolve very differently for different values of the mass parameter.

In addition, as cosmic microwave background experiments are able to probe higher and higher wavenumbers ℓ (i.e., smaller and smaller angular scales), then the S-Z effect will go from being small at low ℓ to dominant at higher ℓ, say $\ell = 2000$. This will provide an excellent complement to the pointed observations from dishes on the ground. It's an exciting future, and CARMA will be in the center of all of it!