Sources of Gravitational Radiation

For our final lecture we will concentrate on one of my favorite topics: sources of gravitational radiation. As we discussed back in the fourth lecture, there are tradiationally four classes of sources to be discussed: binaries, continuous-wave sources, burst sources, and stochastic sources. Binaries are the only ones that we know for sure should be detectable by planned instruments, so we will give them the majority of the coverage, but we will also say some words about the others.

Binary Sources of Gravitational Radiation

We now turn our attention to binary systems. These obviously have a large and varying quadrupole moment, and have the additional advantage that we actually know that gravitational radiation is emitted from them in the expected quantities (based on observations of double neutron star binaries). The characteristics of the gravitational waves from binaries, and what we could learn from them, depend on the nature of the objects in those binaries. We will therefore start with some general concepts and then discuss individual types of binaries.

First, let's get an idea of the frequency range available for a given type of binary. There is obviously no practical lower frequency limit (just increase the semimajor axis as much as you want), but there is a strict upper limit. The two objects in the binary clearly won't produce a signal higher than the frequency at which they touch. If we consider an object of mass M and radius R, the orbital frequency at its surface is $\sim \sqrt{GM/R^3}$. Noting that $M/R^3 \sim \rho$, the density, we can say that the maximum frequency involving an object of density ρ is $f_{\rm max} \sim (G\rho)^{1/2}$. This is actually more general than just orbital frequencies. For example, a gravitationally bound object can't rotate faster than that, because it would fly apart. In addition, you can convince yourself that the frequency of a sound wave through the object can't be greater than $\sim (G\rho)^{1/2}$. Therefore, this is a general upper bound on dynamical frequencies.

This tells us, therefore, that binaries involving main sequence stars can't have frequencies greater than $\sim 10^{-3} - 10^{-6}$ Hz, depending on mass, that binaries involving white dwarfs can't have frequencies greater than $\sim 0.1 - 10$ Hz, also depending on mass, that for neutron stars the upper limit is $\sim 1000 - 2000$ Hz, and that for black holes the limit depends inversely on mass (and also spin and orientation of the binary). In particular, for black holes the maximum imaginable frequency is on the order of $10^4(M_\odot/M)$ Hz at the event horizon, but in reality the orbit becomes unstable at lower frequencies (more on that later).

Now suppose that the binary is well-separated, so that the components can be treated as points and we only need take the lowest order contributions to gravitational radiation.

Temporarily restricting our attention to circular binaries, how will their frequency and amplitude evolve with time?

Let the masses be m_1 and m_2, and the orbital separation be R. We argued in the previous lecture that the amplitude a distance $r \gg R$ from this source is $h \sim (\mu/r)(M/R)$, where $M \equiv m_1 + m_2$ is the total mass and $\mu \equiv m_1 m_2/M$ is the reduced mass. We can rewrite the amplitude using $f \sim (M/R^3)^{1/2}$, to read

$$\begin{aligned} h &\sim \mu M^{2/3} f^{2/3}/r \\ &\sim M_{ch}^{5/3} f^{2/3}/r \end{aligned} \tag{1}$$

where M_{ch} is the "chirp mass", defined by $M_{ch}^{5/3} = \mu M^{2/3}$. The chirp mass is named that because it is this combination of μ and M that determines how fast the binary sweeps, or chirps, through a frequency band. When the constants are put in, the dimensionless gravitational wave strain amplitude (i.e., the fractional amount by which a separation changes as a wave goes by) measured a distance r from a circular binary of masses M and m with a binary orbital frequency $f_{\rm bin}$ is (Schutz 1997)

$$h = 2(4\pi)^{1/3} \frac{G^{5/3}}{c^4} f_{\rm GW}^{2/3} M_{ch}^{5/3} \frac{1}{r} \ , \tag{2}$$

where $f_{\rm GW}$ is the gravitational wave frequency. Redshifts have not been included in this formula.

The luminosity in gravitational radiation is then

$$\begin{aligned} L &\sim 4\pi r^2 f^2 h^2 \\ &\sim M_{ch}^{10/3} f^{10/3} \\ &\sim \mu^2 M^3/R^5 \ . \end{aligned} \tag{3}$$

The total energy of a circular binary of radius R is $E_{\rm tot} = -G\mu M/(2R)$, so we have

$$\begin{aligned} dE/dt &\sim \mu^2 M^3/R^5 \\ \mu M/(2R^2)(dR/dt) &\sim \mu^2 M^3/R^5 \\ dR/dt &\sim \mu M^2/R^3 \ . \end{aligned} \tag{4}$$

This also suggests (correctly) that eccentric binary orbits will tend to circularize under the action of gravitational radiation. The reason is that the radiation is much more intense at pericenter than apocenter, so for very eccentric orbits we can treat the losses as if they only occur at pericenter. This conserves the pericenter while decreasing the semimajor axis, hence it circularizes the binary. The formulae for the evolution of a and e in the lowest-order approximation are called the Peters equations, and the change in semimajor axis has been well-confirmed by observations of binary pulsars.

When two black holes merge with each other, it is predicted that the slight asymmetry of the emission of gravitational radiation will lead to a kick of the remnant relative to the original center of mass. This kick can be tens to thousands of kilometers per second, depending on the mass ratio and the spin magnitudes and relative orientations. At the upper end this could eject the remnant black hole from any galaxy, but Bogdanović, Reynolds, and Miller (2007) argue that in mergers of gas-rich galaxies, torques from the gas will align the spins of the holes with each other and with the orbital axis, leading as it turns out to kicks less than 200 km s^{-1}. This would retain supermassive black holes, which is consistent with the observation that big galaxies that have had major mergers all appear to have black holes in their centers.

Let us now continue with other categories of binary sources. We will focus on compact objects: white dwarfs, neutron stars, and black holes.

Double white dwarf systems (WD-WD) should be extraordinarily plentiful in the Milky Way and other galaxies. In fact, they should be so common that they will provide a limiting noise for low-frequency detectors such as LISA. We can understand this at a crude level as follows. Suppose that LISA, with its frequency range of $\sim 10^{-5} - 10^{-1}$ Hz, observes for three years (about 10^8 s). Its frequency resolution is therefore 10^{-8} Hz. It can observe both polarization modes, but if more than two WD-WD sources are in the same frequency bin they can't be distinguished, and in fact they act as unresolvable noise.

If there are 10^8 WD-WD binaries in our Milky Way, this implies that, assuming a maximum frequency of ~ 1 Hz, there is on average one binary per bin. However, the strong increase of energy loss to gravitational radiation as the orbit shrinks means that binaries spend most of their time at low frequencies. The net result is that low-frequency bins are buried in unresolvable WD-WD binaries, whereas at high frequencies there is on average less than one binary per 10^{-8} Hz frequency bin, meaning that it will be possible to identify them individually and model them out of the data stream. The frequency at which one can start to resolve individual WD-WD binaries has been variously calculated to be in the 2-3 mHz range (see Farmer & Phinney 2003 for a recent discussion). In the $\sim 10^{-3} - 10^{-4}$ Hz range, it is expected that this background will be more important than the LISA instrumental background for determining sensitivities. The extragalactic WD-WD background, although smaller in amplitude, involves so many sources that it will produce an unresolvable background all the way up to $\sim$1 Hz, but at a level far below the current LISA background (see Farmer & Phinney 2003).

What about neutron stars? There are now eight known double neutron star binaries (all in or near our galaxy, of course), and five of them will merge within a Hubble time (i.e., the current age of the universe). In one of them, J0737, we observe both neutron stars as pulsars. It is interesting to note that NS-NS mergers are the only high-frequency gravitational wave source *known* to exist. Other sources are extremely likely (e.g., NS-BH or BH-BH binaries) and many suggestions have been made for sources whose strength is uncertain (e.g., continuous or burst sources). Discovery of any such source would yield important astrophysical information. However, when making the case for ground-based gravitational wave detectors, it is necessary to estimate the rate of detection of sources we can project with some confidence. There are still many uncertainties, but the best current estimates from observed sources is that the rate is about $10^{-5} - 10^{-4}$ per year per Milky Way Equivalent Galaxy (MWEG). Even greater uncertainties exist for BH-NS mergers, because no such systems are currently known.

Mergers of two black holes are special because they can happen in a wide variety of mass ranges. Mergers of stellar-mass black holes tell us about stellar dynamics. Mergers of supermassive black holes yield insight about hierarchical structure formation, and could act as good cosmological probes (although it turns out that uncertainties due to weak gravitational lensing limit their utility, as is also the case for Type Ia supernovae). Mergers of a small black hole (stellar-mass) with a much larger black hole (intermediate-mass or supermassive) would act as excellent probes of the spacetime around the larger black hole, and hence would test general relativity in the strong gravity limit.

Continuous Sources

Once we move away from binaries we enter unknown territory. All other types of sources are of unknown strength, which is another way of saying that if they are detected, we can learn a lot of astrophysics.

The first of these uncertain classes of sources that we will treat is continuous sources. A binary increases its frequency as it loses energy, meaning that searching for an unknown binary requires potentially involved data analysis. In contrast, a spinning source can in principle emit gravitational waves at a single frequency for a long time, so the signal builds up in a narrow frequency bin. As a result, particularly for high frequencies observable with ground-based detectors, continuous-wave sources are interesting because they can in principle be seen even at relatively low amplitudes.

What amplitude can we expect? From the first lecture we know that if the moment of inertia is I, then the amplitude is

$$h \sim (G/c^4)(1/r)(\partial^2 I/\partial t^2) \ . \tag{5}$$

For binaries we argued that $I \sim MR^2$, and also had a relation between $\Omega^2 \sim \partial^2/\partial t^2$ and M and R. However, for a spinning source these relations do not have to hold. For a gravitationally bound source (e.g., a neutron star and not a strange star, which is self-bound and can therefore in principle rotate faster), Ω cannot be greater than the Keplerian angular velocity, but it can certainly be less. In addition, unlike for binaries, not the entire moment of inertia is involved in gravitational wave generation (indeed, if the spinning source is axisymmetric, no gravitational radiation is emitted). Let us say that some fraction ϵ of the moment of inertia is nonaxisymmetric. Generically this could be, e.g., a lump or a wave. Therefore, $h \sim (G/c^4)(1/r)\Omega^2\epsilon I$.

The luminosity is then

$$\begin{aligned} L &\sim r^2h^2f^2 \\ &= (32/5)(G/c^5)\epsilon^2 I_3^2\Omega^6 \ , \end{aligned} \tag{6}$$

where we have put in the correct factors for rotation around the minor axis of an ellipsoid (here I_3 is the moment of inertia around that axis), and we are now defining ϵ to be the ellipticity in the equatorial plane: $\epsilon = (a-b)/(ab)^{1/2}$, where the principal axes of the ellipsoid are $a \geq b > c$.

Note the extremely strong dependence on Ω. The rotational energy is $E_{\rm rot} = \frac{1}{2}I\Omega^2$, so if the part of the star generating the gravitational waves (e.g., a lump) is coupled to the rest of the star then we have

$$\begin{aligned} I\Omega\dot{\Omega} &= -(32/5)(G/c^5)\epsilon^2 I_3^2\Omega^6 \\ \dot{\Omega} &= -(32/5)(G/c^5)\epsilon^2 I_3\Omega^5 \ . \end{aligned} \tag{7}$$

For pulsars, we can relate this to the dimensionless period derivative $\dot{P} = -2\pi\dot{\Omega}/\Omega^2$, which is between $\sim 10^{-13}$ for young pulsars and $\sim 10^{-21} - 10^{-22}$ for the most stable of the millisecond pulsars. Therefore, we have

$$\dot{P} = (64\pi/5)(G/c^5)\epsilon^2 I\Omega^3 \ . \tag{8}$$

For a typical neutron star moment of inertia $I \approx 10^{45}$ g cm^2 and a young pulsar like the Crab with $\Omega \approx 200$ rad s^{-1} and $\dot{P} \approx 10^{-13}$, this implies $\epsilon < 3 \times 10^{-4}$. The reason for the inequality is that the observed spindown can also be caused by other effects, notably magnetic braking. By the same argument, a millisecond pulsar with $\Omega \approx 2000$ rad s^{-1} and $\dot{P} \approx 10^{-21}$ has $\epsilon < 10^{-9}$.

What strain amplitudes should we expect? When the correct factors are put in, we find that the strain amplitude from a pulsar of period P seconds at a distance r is

$$h_c \approx 4 \times 10^{-24}\epsilon P^{-2}(1 \text{ kpc}/r) \ . \tag{9}$$

For the Crab pulsar, $P = 0.03$ s, $r = 2$ kpc, and $\epsilon < 3 \times 10^{-4}$, so the maximum amplitude is $h_c \approx 6 \times 10^{-25}$. For a millisecond pulsar with $P = 0.003$ s, $r = 1$ kpc, and $\epsilon < 10^{-9}$, the maximum amplitude is $h_c \approx 4 \times 10^{-28}$. These amplitudes seem extremely small, but the coherence of their signal (and the fact that the frequency is known from radio observations) means that searches can go extremely deep. For example, the LIGO sensitivity goal at 60 Hz (the frequency of the Crab signal, or twice the rotation frequency) is $\sim 10^{-22}$ Hz$^{-1/2}$. Therefore, in principle, a coherent signal at the Crab maximum could be detected in a time $[10^{-22}/6 \times 10^{-25}]^2 \approx 3 \times 10^4$ s, or less than a day. For a very stable millisecond pulsar, though, the required integration time would be more than 10^{10} s, which is prohibitively large.

Burst Sources

The next category of gravitational wave sources is burst sources. These refer to events of very limited duration that do not have to have any special periodicity. Data analysis for these will be very challenging indeed, but since they are by definition associated with violent events, we could potentially learn a great deal from detection of gravitational radiation. Let's consider a few of the more commonly discussed possibilities.

Core-collapse supernovae. When the core of a massive star collapses, it will not do so in a perfectly symmetric fashion. For example, convection will introduce asymmetries. What fraction of the mass-energy will therefore be released as gravitational radiation? This is a question that has to be answered numerically, but it is an extraordinarily challenging problem. Convection is important, so simulations have to be done in three dimensions. Radiation transfer is also essential, as is a good treatment of neutrino transport. To make things even worse, it seems likely that magnetic fields will play a major role, and a wide range of scales could influence each other! Nonetheless, the current best guess is that only a very small fraction of the total mass-energy will come out in gravitational radiation, perhaps $\sim 10^{-6}$. If so, supernovae outside our galaxy will be undetectable. However, the rate of core-collapse supernovae in our Milky Way is estimated to be one per few decades, which means that there is a probability of tens of percent per decade that a supernova will occur within ~ 10 kpc. Current calculations suggest that the strain amplitude at 10 kpc could be $h \sim 10^{-20}$ for a few milliseconds, which would be detectable with advanced ground-based instruments. There have also been proposals that a much higher fraction of energy is emitted during the collapse, which brings us to the next topic.

Gamma-ray bursts. These are short (milliseconds to minutes), high intensity bursts of gamma rays. After a long and interesting history (starting with their detection with US spy satellites!), it has been established that there are two categories of GRBs, the long (tens of seconds) and the short (less than a second, typically). The long bursts are convincingly associated with a type of supernova, but the detailed mechanism for their production is uncertain. Some people believe that GRBs are the birth events for rapidly rotating black holes. If so, the rapid rotation could be a path to much more substantial gravitational wave production. For example, in a massive disk there are bar instabilities that could produce rotating nonaxisymmetric structures. If these emit a lot of gravitational radiation and can be identified with particular bursts, then we have a wonderful situation: extremely bright events at cosmological distances whose redshift can be determined based on the electromagnetic signal, and whose luminosity distance can be determined based on the gravitational wave signal. The difficulty is that to be detectable at cosmological distances (at least 3 Gpc is needed to be interesting), a truly enormous fraction of the mass-energy needs to emerge in gravitational waves (at least tens of percent). This currently seems unlikely, but it is obviously worth pursuing from the observational standpoint.

Stochastic Backgrounds

A background due to processes in the early universe (say, before the production of the cosmic microwave background) would be very exciting because it would contain information that is unavailable otherwise. In principle, one could see gravitational waves from very early in the universe, because gravitons have a very small interaction cross section. We need to state clearly that, even by the standards of gravitational wave astronomy, these processes are *highly* speculative. One consequence of this is that although it would be extremely exciting to detect a background of early-universe gravitational radiation, a nondetection would not be surprising. For more details, I suggest you consult notes from Alessandra Buonanno (gr-qc/0303085).

As our first contestant for a gravitational wave background, let us consider simply a thermal background of gravitons. It is possible that in the very early universe (roughly the Planck time, or about 10^{-43} s after the Big Bang!) gravitons were in thermal equilibrium with the rest of the matter and energy. At some point not long after the Planck time, the gravitons decoupled and streamed freely. What would their temperature be now? As a guide, we can consider the cosmic microwave background, at a temperature of 2.7 K. Would the graviton background have a larger or smaller temperature? It would have to be smaller. The way to see this is to realize that at any given epoch, the energy is shared among all the relativistic species that are coupled tightly. At the time of the neutrino background, for example, in addition to photons there were neutrinos, electrons, and positrons. As a result, the temperature that we see now would be less, a bit under 2 K. Since neutrino scattering cross sections scale as the temperature squared, the probability of scattering is some 20 orders of magnitude less than the normal $\sim$MeV neutrinos detected with enormous pools of water or other substances. No chance.

For gravitons, at the Planck time the energy would be shared with the entire zoo of particles in the Standard Model, leading to a present-day temperature of a bit under 1 K. The frequency of the photons would therefore be $\nu \sim kT/h$, or in the $\sim 10^{10}$ Hz range. No current or planned detectors could see this. The whole line of argument leading to this background is in any case dubious, because Planck-era physics is unknown and gravitons might never have been strongly coupled to other particles.

What about generic possibilities from later in the universe? Suppose that a graviton of frequency f_* is emitted when the scale factor of the universe is a_*. Then we observe the frequency at $f = f_*(a_*/a_0)$, where a_0 is the current scale factor. This gives

$$f \approx 10^{-13} f_*(1\ \mathrm{GeV}/kT_*)\ , \tag{10}$$

modulo a factor of order unity related to the number of relativistic degrees of freedom at time t_*. Here T_* is the temperature. What value of f_* should we take? Based on general causality considerations we know that the *lowest* frequency possible is the Hubble constant H_* at that epoch. The frequency could be higher, though, so we'll take $f_* = H_*/\epsilon$, with

$\epsilon < 1$. In the radiation-dominated epoch of the universe (valid for $z > 10^4$, which will be true for almost all processes we consider), $H_* \sim T_*^2$, so $f \sim T_*$, or with the constants put in,

$$f \approx 10^{-7}\ \mathrm{Hz}\epsilon^{-1}(kT_*/1\ \mathrm{GeV})\ . \tag{11}$$

If ϵ is not too much less than unity, this tells us the energy scale probed at a given present-day gravitational wave frequency. For example, LISA (at 10^{-4} Hz) would probe the TeV scale and ground-based detectors would probe the EeV scale.

What limits are there to the overall strength of the gravitational wave background? One comes from Big Bang nucleosynthesis (BBN). This is the very successful model that relates the overall density of baryons in the universe to the abundances of light elements. The idea is that in the first few minutes of the universe, after the temperature had dropped below the level when photodisintegration of nuclei was common but before free neutrons decayed, protons and neutrons could merge to form heavier elements. George Gamow, who originally proposed this, had hoped that this process would explain all the elements in the universe, but the lack of stable elements at mass 5 and mass 8 prevents this. Instead, just the light elements are formed. These include hydrogen, deuterium, helium-3 and helium-4, and trace amounts of lithium and beryllium. The relative abundances of each depend only on the overall entropy of the universe (i.e., ratio of photons to baryons) and the number density of baryons. Measurements of primordial abundances of the light elements are in excellent agreement with the baryon fraction $\Omega_b \approx 0.04$ measured independently from the microwave background.

If the current energy density of gravitational waves were too high, this would mess up BBN. The constraint is

$$\int_{f=0}^{f=\infty} d\ln f\, h_0^2 \Omega_{\rm GW}(f) < 5\times 10^{-6}\ , \tag{12}$$

where $h_0 \equiv H_0/100\mathrm{km\ s^{-1}Mpc^{-1}}$. In principle you could imagine that $\Omega_{\rm GW}(f) \gg 10^{-5}$ in some narrow frequency interval, but this seems unlikely.

Observations of the cosmic microwave background (CMB) also limit the GW background. This is because gravitational waves would produce fluctuations in the CMB, hence the measured fluctuations give an upper limit at low frequencies:

$$\Omega_{\rm GW}(f) < 7\times 10^{-11}(H_0/f)^2\ , \qquad H_0 = 3\times 10^{-18}\ \mathrm{Hz} < f < 10^{-16}\ \mathrm{Hz}\ . \tag{13}$$

Yet another constraint comes from pulsar timing. The tremendous stability of millisecond pulsars means that we would know if a wave passed between us and the pulsar, because the signal would vary. Roughly eight years of timing has led to the bound

$$\Omega_{\rm GW}(f) < 5\times 10^{-9}(f/f_{\rm PSR})^2\ , \qquad f > f_{\rm PSR} \equiv 4.4\times 10^{-9}\ \mathrm{Hz}\ . \tag{14}$$

What are some specific mechanisms by which gravitons can be generated in the early universe, after the Planck time? The two primary mechanisms that have been explored are production during inflation, and production during a phase transition.

Various models of inflation have been discussed, but one that is considered relatively realistic is slow-roll inflation. In this model, the universe had a scalar field that, at the beginning of the inflationary period, was not at its minimum. The field value "rolls" towards the minimum and as it does so it drives rapid expansion of the universe. The rolling process means that the Hubble parameter is not constant during inflation. Therefore, fluctuations that leave the Hubble volume during inflation and re-enter later have a tilt with respect to other fluctuations. The net result of calculations is that if standard inflation is correct then, unfortunately, there is no hope of detecting a gravitational wave background, because the amplitude is orders of magnitude below what current or planned detectors could achieve. Variants of or substitutes for standard inflation have been proposed that might lead to detectable gravitational radiation, including bouncing-universe scenarios and braneworld ideas, but whether these encounter reality at any point is anyone's guess!

If phase transitions in the early universe (e.g., from a quark-gluon plasma to baryonic matter) are first-order, then by definition some variables are discontinuous at the transition. If the transition occurs in localized regions ("bubbles") in space, collisions between the bubbles could produce gravitational radiation. In addition, turbulent magnetic fields produced by the fluid motion could generate secondary gravitational radiation, but these are weaker. The most optimistic estimates put the contribution at $h_0^2\Omega_{\rm GW} \sim 10^{-10}$, peaking in the millihertz range. This would be detectable with LISA, but don't bet on it.

A more recent suggestion has been that gravitational radiation could be produced by cosmic strings. Cosmic strings, if they exist, are one-dimensional topological defects. Assuming a network of cosmic strings exists, it would have strings of all sizes and therefore contribute gravitational radiation at a wide range of frequencies. Recently, some work has been done on the possibility that cusps or kinks in cosmic strings could produce beams of gravitational radiation.

If any of these scenarios comes true and in fact there is a cosmological background of gravitational waves detected with planned instruments, this will obviously be fantastic news. However, what if it isn't seen? That won't be a surprise, but there has been discussion about missions to go after weaker backgrounds. It is often thought that the 0.1-1 Hz range is likely to be least "polluted" by foreground vermin (i.e., the rest of the universe!). This may be, but it is worth remembering that there are an enormous number of sources out there in even that frequency range, and that to see orders of magnitude below them will required *extremely* precise modeling of all those sources. Either way, whether we see a background or "merely" detect a large number of other sources, gravitational wave astronomy has wonderful prospects to enlarge our view of the cosmos.