See Chapter 4 in Longair for photon interactions, chapters 2, 3, 5 for particle interactions

Photon-photon pair production

Ask class: consider a single photon propagating in a vacuum. Given that energy and momentum must be conserved in all interactions, is it possible for the photon to spontaneously produce an electron and positron, i.e., to produce a pair? No, it is not. The energy-momentum relations for photons and for particles with nonzero rest mass are different, and this ends up meaning that single photon pair production is impossible in a vacuum. Pair production requires the presence of something else to take up momentum: another photon, a nearby nucleus, or a magnetic field, for example.

Let's consider photon-photon pair production of an electron and a positron. Clearly, if there is not enough energy available, there can be no pair production, so there is an energy threshold. **Ask class:** consider two photons moving head-on towards each other. What would be a reasonable energy condition for pair production to be allowed? You might think it would be that the sum of the energies exceed $2m_ec^2$, which is the rest mass energy of the electron and positron produces. However, we then remember that the physics (in particular whether there is pair production) should look the same in all Lorentz frames. So, consider a radio photon (low energy) moving head-on towards a gamma-ray photon with energy just above $2m_ec^2$. Can this produce a pair? Go into a Lorentz frame such that the radio photon's energy is doubled, meaning that the gamma-ray photon's energy is halved. Then the total energy in that frame is only m_ec^2 instead of $2m_ec^2$, so it would be stunning if there were any pair production! Indeed, the condition has to be that in *all* Lorentz frames the $\hbar\omega_1 + \hbar\omega_2 > 2m_ec^2$ criterion is satisfied. For head-on collisions, this turns out to mean that $(\hbar\omega_1)(\hbar\omega_2) > (m_ec^2)^2$.

The cross section for this process is (Berestetskii *et al.* 1982)

$$\sigma_{\gamma\gamma} = \frac{3}{16}\sigma_T(1-v^2)\left[(3-v^4)\ln\left(\frac{1+v}{1-v}\right) - 2v(2-v^2)\right] \ , \tag{1}$$

where

$$v = \sqrt{1-(mc^2)^2/\hbar^2\omega_1\omega_2} \ , \tag{2}$$

This has a maximum value of $\sigma_{max} \approx \frac{3}{16}\sigma_T$, and in the ultrarelativistic limit, $\hbar^2\omega^2 \equiv \hbar^2\omega_1\omega_2 \gg (mc^2)^2$, reduces to

$$\sigma_{\gamma\gamma} \approx \frac{3}{8}\sigma_T\left(\frac{mc^2}{\hbar\omega}\right)^2\left(\ln\frac{2\hbar\omega}{mc^2} - 1\right) \ . \tag{3}$$

Will consider single-photon pair production and photon splitting when we come to neutron stars

Photon-nucleon interactions

Ask class: why haven't we talked about interactions of photons with protons or other nuclei? Because protons are much tougher to affect with the oscillating electomagnetic fields of photons. In particular, since they're more massive and e/m is smaller, the resulting acceleration is less and the radiation (hence cross section) is tiny by comparison to protons. For comparison, though, the scattering cross section off of protons is $\approx m_e^2/m_p^2$ less than off of electrons. That's a factor of almost 4 million. So, for most purposes we can ignore photon-nucleon interactions.

Particle interactions

Previously we considered interactions from the standpoint of photons: a photon travels along, what happens to it? Now, we'll think about interactions of particles: an electron, proton, or nucleus zips along, what happens to it?

Ask class: generically, what could, say, an electron interact with? Photons, protons or nuclei, magnetic fields, neutrinos. Let's first consider interactions of electrons with photons. **Ask class:** for a low-energy electron interacting with low-energy photons, what is the cross section (not a trick question)? Just the Thomson cross section. In fact, since this is exactly the same process as we considered before, the cross section for general energies is again the Klein-Nishina value.

Compton scattering

In the absence of a magnetic field, the cross section for the interaction of a photon with an electron is just the Thomson cross section ($\sigma_T = 6.65 \times 10^{-25}\text{cm}^{\ 2}$) for low energies, but becomes more complicated at higher energies. The general Klein-Nishina expression, valid at all energies, is

$$\sigma = \frac{3}{4}\sigma_T \left\{ \frac{1+x}{x^3} \left[\frac{2x(1+x)}{1+2x} - \ln(1+2x) \right] + \frac{1}{2x}\ln(1+2x) - \frac{1+3x}{(1+2x)^2} \right\} , \qquad (4)$$

where $x = \frac{\hbar\omega}{mc^2}$ is the energy of the photon in the rest frame of the electron. For $x \ll 1$ this reduces to the Thomson value, whereas for $x \gg 1$

$$\sigma \approx \frac{3}{8}\sigma_T x^{-1}(\ln 2x + 0.5) . \qquad (5)$$

Radiation can exert a force on matter, via scattering or other interactions. Radiation force is often referred to as radiation pressure in the literature. However, let's give some thought to this. Suppose that an electron is in an isotropic bath of radiation. The radiation pressure is nonzero; $P_r = aT^4$, in fact. **Ask class:** is there any net radiation force on the electron? No, because the bath is isotropic. In this situation it would be more accurate to say that the force is due to a pressure *gradient*. This is the same reason why we're not currently being crushed by the atmosphere, despite the pressure of about 1 kg per square centimeter (many tons over the whole body). Even more accurately, it's the net radiation flux that matters.

By balancing radiation force with gravitational force we can define the *Eddington* luminosity, which is very important for high-energy astrophysics. For a flux F on a particle of mass m and scattering cross section σ around a star of mass M, the balance implies

$$\frac{GMm}{r^2} = \frac{\sigma}{c}F = \frac{\sigma}{c}\frac{L}{4\pi r^2} \ , \tag{6}$$

where L is the luminosity. The r^{-2} factors cancel, leaving us with the Eddington luminosity L_E:

$$L_E = \frac{4\pi GcMm}{\sigma} \ , \tag{7}$$

For fully ionized hydrogen, we assume that the electrons and protons are electrically coupled (otherwise a huge electric field would be generated), so the light scatters off the electrons with cross section σ_T and the protons provide the mass m_p. Then $L_E = 4\pi GcMm_p/\sigma_T = 1.3 \times 10^{38}(M/M_\odot)$ erg s^{-1}. If the luminosity of the star is greater than this, radiation will drive matter away. This is also the maximum luminosity for steady spherical accretion.

Ask class: does the dependence on m and σ make sense, that is, should m be in the numerator and σ in the denominator? Yes, because if gravity is stronger (m is higher) then more luminosity is needed; if σ is greater, radiation couples more effectively and the critical luminosity is less. This means that for a fixed density, large things are less affected by radiation than small things (because for a size a, the mass goes like a^3 whereas the area goes like a^2. So, asteroids are not affected significantly by radiation!

Curvature radiation

We know that any accelerated charge radiates. If there is a magnetic field around, there are two ways the charge can be acclerated. One is if it moves perpendicular to the field (synchrotron radiation). The other is if it moves along the field, but the field is curved (curvature radiation). We'll just state a couple of results here.

The power emitted by curvature radiation is (see, e.g., Jackson 1975)

$$P \approx \frac{2}{3}\frac{e^2c}{R^2}\gamma^4 \tag{8}$$

where $\gamma = (1-\beta^2)^{-1/2}$ and $\beta = |\mathbf{v}|/c$. **Ask class:** is the dependence on R reasonable? Yes, because for smaller radius of curvature and a fixed energy, the acceleration is greater and hence so is the radiation.

The spectrum for curvature radiation is

$$\frac{dI}{d\omega} \propto \frac{e^2}{c}\left(\frac{R}{c}\right)^{1/3}\omega^{1/3} \tag{9}$$

up to a limiting frequency of

$$\omega_c \approx 3\gamma^3\frac{c}{R}\ . \tag{10}$$

Synchrotron radiation

If a particle of charge e and energy E is moving perpendicular to a static magnetic field of strength B, the frequency of its orbit around the field is

$$\omega_c = \frac{eBc}{E}\ . \tag{11}$$

If the particle has velocity v, this means that its orbital radius is $d = v/\omega_c$, so for highly relativistic particles with $v \approx c$, $d = E/eB$.

A particle may acquire a nonnegligible motion perpendicular to the magnetic field if, e.g., it is created from a photon which was moving with some angle to the magnetic field. If $\gamma\frac{B}{B_c} \ll 1$, then a classical treatment of synchrotron radiation is approximately valid. **Ask class:** recalling that synchrotron radiation is due to acceleration of a charge, suppose you have an electron and a proton with the same Lorentz factor. Which would you expect to lose energy on a faster time scale? The electron, because the proton is not as easily accelerated, hence the proton does not lose energy as rapidly. That's why circular accelerators accelerate protons or ions instead of electrons, and why electron accelerators are straight: the radiation losses are too significant otherwise. However, when the magnetic fields are weak, relativistic electrons have a long synchrotron cooling time. In fact, radio emission from many AGN is dominated by synchrotron radiation.

In the non-relativistic limit, the energy loss rate and rate of angular change due to synchrotron radiation are

$$\frac{\dot{E}}{E} = -\frac{2}{3}r_0^2B^2\gamma\beta^2\sin^2\alpha/mc \approx 2\times10^{15}\gamma\beta^2B_{12}^2\sin^2\alpha \mathrm{s}^{-1} \tag{12}$$

and

$$\dot{\alpha} = -\frac{2}{3}r_0^2B^2\sin\alpha\cos\alpha/mc\gamma \ \approx 2\times10^{15}\gamma^{-1}B_{12}^2\sin\alpha\cos\alpha\ \mathrm{s}^{-1}\ , \tag{13}$$

where $r_0 = \frac{e^2}{mc^2} \approx 2.8\times10^{-13}$cm is the classical radius of the electron and α is the angle between the magnetic field and the direction of motion. In this limit, the average energy of the synchrotron photons is

$$\hbar\omega_{\mathrm{ave}} \approx 0.46\frac{B}{B_c}\gamma^2\sin^2\alpha mc^2\ . \tag{14}$$

Pair annihilation, $e^- e^+ \rightarrow \gamma\gamma$

In the extreme relativistic limit, the cross section for two-photon annihilation is

$$\sigma_{\rm an} \approx \frac{3}{8}\sigma_T \frac{\ln 2\gamma}{\gamma}\ . \tag{15}$$

Will consider one-photon annihilation in neutron star section

Bremsstrahlung

The energy lost per unit length to bremsstrahlung radiation by an electron or positron traversing a region of number density n fixed charges per unit volume is

$$\frac{dE}{dx} = n \int_0^{\omega_{max}} \frac{d\chi(\omega)}{d\omega} d\omega\ , \tag{16}$$

where ω_{max} is the maximum frequency of the radiation, and in the extreme relativistic limit

$$\frac{d\chi}{d\omega} \approx \frac{16}{3}\frac{e^2}{c}\left(\frac{e^2}{mc^2}\right)^2 \ln\left(\frac{2EE'}{mc^2\hbar\omega}\right) \tag{17}$$

(e.g., Jackson 1975). In this expression, m is the mass of the electron, E is the original energy of the electron, E' is the energy after interaction, and it is assumed that $\hbar\omega \ll E$ and $E,\ E' \gg mc^2$. We'd like to know approximately how far an electron of energy $E \gg m_e c^2$ can travel before it loses a significant amount of energy. We could do the integral, but we can get a rough answer by making a couple of approximations. First, logarithms vary slowly, so we can take it to be about constant and take it out of the integral. Second, "all logarithms are 10" to within a factor of a few, so we'll just call that factor 10. Third, we need to determine $\omega_{\rm max}$. **Ask class:** can $\hbar\omega > E$? Of course not! So, let's say that $\hbar\omega_{\rm max} = E$. Then we can write the energy loss per distance as

$$\frac{dE}{dx} \approx n\frac{160}{3}\frac{e^2}{c}\left(\frac{e^2}{m_e c^2}\right)^2 \omega_{\rm max} = n\frac{160}{3}\frac{e^2}{\hbar c}\left(\frac{e^2}{m_e c^2}\right)^2 (\hbar\omega_{\rm max}) \Rightarrow \frac{d(\ln E)}{dx} \approx n\frac{160}{3}\left(\frac{e^2}{\hbar c}\right)\left(\frac{e^2}{m_e c^2}\right)^2\ . \tag{18}$$

This expression contains two combinations of symbols that are useful to remember. $e^2/\hbar c = 1/137$ is the fine structure constant, and is dimensionless. $e^2/m_e c^2 = 2.8 \times 10^{-13}$ cm is the classical radius of the electron. Evaluating this, we find the column depth for significant interaction, where $d(\ln E) \approx 1$, is about 3×10^{25} cm^{-2}, roughly constant for large E.

Electron-neutrino interactions

See http://www.sns.ias.edu/~jnb/ (John Bahcall's home page) for many details about neutrino astrophysics.

Neutrinos interact very weakly; in fact, their existence is the hallmark of the weak force. Typically, a neutrino of energy E_ν has an electron scattering cross section of

$$\sigma_\nu \approx 10^{-44} \left(\frac{E_\nu}{m_e c^2}\right)^2 \text{ cm}^2 \ . \tag{19}$$

This is what is technically known as an itsy bitsy cross section. Now, particle physicists have a lot of time and a fondness for alcohol, leading to interesting terminology and names for units. In this case, they've dubbed 10^{-24} cm^2 a "barn" and 10^{-48} cm^2 a "shed", so a typical neutrino cross section is some ten thousand sheds! This compares with the Thomson cross section, which is close to one barn; indeed, hitting an electron with a photon is like hitting the broad side of a barn compared to hitting an electron with a neutrino. For people without a sense of humor, 10^{-44} cm^2=10^{-48} m^2 is one square yoctometer. Pretty small, no matter how you slice it.

Let's figure out the fraction of neutrinos interacting in certain circumstances. First, the Sun. **Ask class:** to order of magnitude, what is the density of the Sun? About 1 g cm^{-3}. That means that the number density is about 10^{24} cm^{-3}. **Ask class:** so, what is the mean free path of $\sim$ 1 MeV neutrinos? About 10^{20} cm. The Sun is about 10^{11} cm in radius, so only about 10^{-9} of the neutrinos interact.

Now let's think about the dense core in the center of a star just prior to a supernova. **Ask class:** if you crush the Sun down to a radius 1000 times less than it actually has, what happens to the optical depth to neutrinos? Density is $1000^3 = 10^9$ times greater, but the length traveled is 1000 times less, so optical depth is 10^6 times greater. That suggests an optical depth of about 10^{-3}. The neutrinos in supernova are actually somewhat more energetic as well, about 3–5 MeV, so a fraction $\sim 10^{-2}$ of the energy is absorbed. This seems to be enough to be the crucial driver of the supernova, since a good 10^{53} erg is released in neutrinos.

Proton-proton interactions

Because of their relatively large mass, protons do not interact significantly in the ways discussed above. However, at high energies proton-proton collisions may produce photons, neutrinos or other products through strong interactions. At TeV energies or higher, more than 99% of the interactions are of the form

$$p + p \rightarrow \pi + X \ , \tag{20}$$

where π indicates a pion (charged or neutral), and X indicates the other products. At a few TeV, the interaction length is roughly 20g/cm^2, or a column depth of $\approx 10^{25}$cm^{-2}. The pions can decay to produce photons or neutrinos. Slane and Fry (1988) found the optimum column depth for photon production is $\approx$ 50g/cm^2. At this column depth, a proton will typically produce about 10 photons of average energy $\approx \frac{1}{30}$ that of the proton.