Coding in advance of the Apr 30, 2018 class

For this coding exercise, we will continue to analyze the same data set as last time: for continuity the set is given as data13_1.txt on the website.

Your task for this week will be to use the affine-invariant MCMC code you developed to fit the Ambrosi et al. (2017) smoothly broken power law model, plus a Gaussian line, to the data. The flux in your model is thus:

$$\begin{aligned}\Phi &= \Phi_0(E/100\ \mathrm{GeV})^{-\gamma_1}\left[1+(E/E_b)^{-(\gamma_1-\gamma_2)/\Delta}\right]^{-\Delta} \\ &+Ae^{-(E-E_0)^2/2\sigma_{\rm line}^2}\ .\end{aligned} \tag{1}$$

Thus the seven parameters in your model are the normalization Φ_0, the power-law indices γ_1 and γ_2, the break energy E_b (those four parameters describe the broken power law; again, Ambrosi et al. (2017) fix the smoothness parameter Δ to $\Delta = 0.1$), and the amplitude A, centroid energy E_0, and width $\sigma_{\rm line}$ of the Gaussian line. Please note again that you need to do a completely new fit; you should not fix the broken power law parameter values to your best fit from last time.

As before our main aim is to get the maximum likelihood for this model: what values of $\Phi_0, \gamma_1, \gamma_2$, E_b, A, E_0, and $\sigma_{\rm line}$ maximize the log likelihood, and what is the value of that maximum log likelihood? Compare the maximum you get here with what you obtained with the broken power law model from last week. Using Wilks' Theorem, do you conclude that the three extra parameters of the line are needed?

Again, as a check on your fit, please plot your best-fit broken power law against the data; is it reasonable?

As a final challenge: suppose that as additional data are taken the statistical uncertainties go down but the systematic errors remain fixed. How much would the statistical uncertainties have to decrease for the evidence for the line to be strongly convincing? Note that this is a bit of a vague question, which is intended to get you to think about the problem more deeply.

Good luck!