Continuous and Burst Sources

Once we move away from binaries we enter unknown territory. All other types of sources are of unknown strength, which is another way of saying that if they are detected, we can learn a lot of astrophysics.

The first of these uncertain classes of sources that we will treat is continuous sources. A binary increases its frequency as it loses energy, meaning that searching for an unknown binary requires potentially involved data analysis. In contrast, a spinning source can in principle emit gravitational waves at a single frequency for a long time, so the signal builds up in a narrow frequency bin. As a result, particularly for high frequencies observable with ground-based detectors, continuous-wave sources are interesting because they can in principle be seen even at relatively low amplitudes.

What amplitude can we expect? From the first lecture we know that if the moment of inertia is I, then the amplitude is

$$h \sim (G/c^4)(1/r)(\partial^2 I/\partial t^2)\ . \tag{1}$$

For binaries we argued that $I \sim MR^2$, and also had a relation between $\Omega^2 \sim \partial^2/\partial t^2$ and M and R. However, for a spinning source these relations do not have to hold. For a gravitationally bound source (e.g., a neutron star and not a strange star, which is self-bound and can therefore in principle rotate faster), Ω cannot be greater than the Keplerian angular velocity, but it can certainly be less. In addition, unlike for binaries, not the entire moment of inertia is involved in gravitational wave generation (indeed, if the spinning source is axisymmetric, no gravitational radiation is emitted). Let us say that some fraction ϵ of the moment of inertia is nonaxisymmetric. Generically this could be, e.g., a lump or a wave. Therefore, $h \sim (G/c^4)(1/r)\Omega^2\epsilon I$.

The luminosity is then

$$\begin{aligned} L &\sim r^2 h^2 f^2 \\ &= (32/5)(G/c^5)\epsilon^2 I_3^2 \Omega^6\ , \end{aligned} \tag{2}$$

where we have put in the correct factors for rotation around the minor axis of an ellipsoid (here I_3 is the moment of inertia around that axis), and we are now defining ϵ to be the ellipticity in the equatorial plane: $\epsilon = (a-b)/(ab)^{1/2}$, where the principal axes of the ellipsoid are $a \geq b > c$.

Note the extremely strong dependence on Ω. The rotational energy is $E_{\rm rot} = \frac{1}{2}I\Omega^2$, so if the part of the star generating the gravitational waves (e.g., a lump) is coupled to the rest of the star then we have

$$
\begin{aligned}
I\Omega\dot{\Omega} &= -(32/5)(G/c^5)\epsilon^2 I_3^2\Omega^6 \\
\dot{\Omega} &= -(32/5)(G/c^5)\epsilon^2 I_3\Omega^5\ .
\end{aligned}
\tag{3}
$$

For pulsars, we can relate this to the dimensionless period derivative $\dot{P} = -2\pi\dot{\Omega}/\Omega^2$, which is between $\sim 10^{-13}$ for young pulsars and $\sim 10^{-21} - 10^{-22}$ for the most stable of the millisecond pulsars. Therefore, we have

$$\dot{P} = (64\pi/5)(G/c^5)\epsilon^2 I\Omega^3\ . \tag{4}$$

For a typical neutron star moment of inertia $I \approx 10^{45}$ g cm^2 and a young pulsar like the Crab with $\Omega \approx 200$ rad s^{-1} and $\dot{P} \approx 10^{-13}$, this implies $\epsilon < 3 \times 10^{-4}$. The reason for the inequality is that the observed spindown can also be caused by other effects, notably magnetic braking. By the same argument, a millisecond pulsar with $\Omega \approx 2000$ rad s^{-1} and $\dot{P} \approx 10^{-21}$ has $\epsilon < 10^{-9}$.

What strain amplitudes should we expect? When the correct factors are put in, we find that the strain amplitude from a pulsar of period P seconds at a distance r is

$$h_c \approx 4 \times 10^{-24}\epsilon P^{-2}(1\ \mathrm{kpc}/r)\ . \tag{5}$$

For the Crab pulsar, $P = 0.03$ s, $r = 2$ kpc, and $\epsilon < 3 \times 10^{-4}$, so the maximum amplitude is $h_c \approx 6 \times 10^{-25}$. For a millisecond pulsar with $P = 0.003$ s, $r = 1$ kpc, and $\epsilon < 10^{-9}$, the maximum amplitude is $h_c \approx 4 \times 10^{-28}$. These amplitudes seem extremely small, but the coherence of their signal (and the fact that the frequency is known from radio observations) means that searches can go extremely deep. For example, the LIGO sensitivity goal at 60 Hz (the frequency of the Crab signal, or twice the rotation frequency) is $\sim 10^{-22}$ Hz$^{-1/2}$. Therefore, in principle, a coherent signal at the Crab maximum could be detected in a time $[10^{-22}/6 \times 10^{-25}]^2 \approx 3 \times 10^4$ s, or less than a day. For a very stable millisecond pulsar, though, the required integration time would be more than 10^{10} s, which is prohibitively large.

The full sensitivity of initial LIGO has not quite been reached, and the strongest currently reported limits on various pulsars are in the $h <$ few $\times 10^{-24}$ range. Advanced LIGO will have strain sensitivities more than an order of magnitude better, with the prospect of "narrowbanding" to improve sensitivity at a specific frequency if pulsar searches were a high priority. The most stable of the millisecond pulsars would probably still be out of reach, but the anticipated sensitivity of Advanced LIGO will be great enough to, at a minimum, provide interesting limits on the ellipticity of some pulsars.

In a similar vein, some researchers have investigated the possibility that actively accreting neutron stars might balance the accretion torque by gravitational radiation losses of angular momentum. You will investigate this in more detail in the problems; the astrophysical situation is unclear, in that although there is at present no compelling evidence that gravitational radiation plays any role in torque balance in these systems (magnetic torques are an acceptable alternative), it might be that these systems produce significant radiation that could be detected by future ground-based instruments.

Let us now consider continuous-wave radiation from another perspective. What is it, exactly, that could produce the required nonaxisymmetry? We will divide the possibilities into two categories. "Lumps" are nonaxisymmetries that are fixed relative to the star. "Waves" are nonaxisymmetries that move relative to the star.

Lumps first. What if the neutron star is a perfect fluid with no magnetic field? Then, as analyzed in the late 19th century, the equilibrium shape of the star below some critical rotation frequency is a spheroid that is axisymmetric around the rotation axis. Above this critical frequency, however, the shape that minimizes the energy for a given angular momentum is a triaxial ellipsoid. Rotation of this ellipsoid will therefore generate gravitational radiation. The critical rotation frequency is approximately 80% of the frequency of the "mass-shedding limit", at which corotating matter is flung away from the star. For neutron stars, the mass-shedding limit is at $\sim 1500 - 2000$ Hz, depending on mass and equation of state. Therefore, a neutron star rotating at $> 1200 - 1600$ Hz is a potential source of gravitational radiation. No neutron stars are known at frequencies this high; in fact, in 2005 a new record of 716 Hz was set by PSR J1748–2446ad (gotta love the naming convention; this one is in the globular cluster Terzan 5, which has lots of other pulsars, thus the "ad" at the end). Therefore, there are no sources expected to emit gravitational radiation in this way. If there were, the radiation would slow the star down very quickly, so in any case these would be transient sources. It is conceivable that such rapid rotation could be produced in the core collapse that produced the neutron star, in which case the gravitational radiation would have the character of a burst.

Another possibility is that the star has a substantial magnetic field that is misaligned with the rotation axis. The magnetic stresses would produce triaxiality, which would then lead to gravitational radiation. Is there evidence that such misalignment happens? Yes! At the simplest level, rotation-powered pulsars have to be somewhat misaligned, otherwise we wouldn't see pulsations. More recently, another piece of evidence has been uncovered. The pulsar PSR 1828-11 has been shown to precess nonsinusoidally with a period of about 500 days. Various ideas have been proposed, but the most promising appears to be a misaligned magnetic field (Stairs, Lyne, & Shemar 2000). As discussed in detail by Wasserman (2003), steady rotation is only possible if the rotation is along the axis of one of the principal moments of inertia, and for a given angular momentum the lowest energy state is attained when the rotation is around the axis with the largest moment of inertia. If the magnetic field is neither aligned nor orthogonal relative to the rotation axis, then precession can occur for sufficiently strong fields. Over a long time, comparable to or longer than the spindown time, the magnetic axis will presumably drift towards an aligned or orthogonal state (because this is the state of global minimum energy), but in the meantime precession can occur, and with it gravitational radiation can be produced. In fact, if the magnetic axis is orthogonal to rotation then gravitational radiation can be produced even without precession. In addition, an inherently triaxial field will produce gravitational radiation, no matter what the orientation. The precession rate and spin frequency of PSR 1828-11 are, unfortunately, much too low for detection by currently planned instruments.

The last possibility we will discuss relates to accreting neutron stars. Neutron stars in so-called low-mass X-ray binaries (LMXBs, in which the companion star has mass $M < M_\odot$) accrete at a high rate, roughly $10^{-10} - 10^{-8}\, M_\odot$ yr^{-1} for the brighter sources. Various phenomena, including regular pulsations during the accretion-powered emission (for six sources), regular pulsations during thermonuclear X-ray bursts (for about a dozen sources), and quasi-periodic oscillations in the brightness of the accretion-powered emission (for more than twenty sources) can be used to infer the spin frequencies of these sources. Until 2002 most researchers (myself included) interpreted these in such a way that it appeared there was a clustering of spin frequencies at around 300 Hz. In 2002 observations of the particular source SAX J1808–3658 changed this picture, and it now appears that there is a broad range of spin frequencies, from 45 Hz to 620 Hz. There is no particular evidence of clustering, but no high-frequency sources are seen. This suggests that some braking torque is probably operating to offset the spin-up produced by the accreting matter (the other possibility is that there simply has not been enough time to spin up sources to higher frequencies). Magnetic torques appear to be able to explain all the observations, but people have also explored the possibility that gravitational radiation can play a role.

For example, Bildsten (1998) and Ushomirsky, Cutler, & Bildsten (2000) investigated one particular model, in which a nonaxisymmetric density profile could be maintained if (1) the accretion onto the surface was nonaxisymmetric and persistent in its orientation over tens of thousands of years, and (2) electron capture reactions deep in the crust were able to maintain density asymmetries. Persistent nonaxisymmetric accretion suggests that the magnetic field is playing a dynamically important role, but perhaps the field is buried and can therefore produce asymmetry as the matter settles, even if the external field is weak. The calculations of Ushomirsky et al. (2000) suggested that equilibrium at ~ 300 Hz was possible if the critical breaking strain of the crust at densities of $\sim$few$\times 10^{13}$ g cm^{-3} was $\sim 10^{-2} - 10^{-1}$, because then a large enough quadrupolar asymmetry could be maintained to balance accretion torques by gravitational radiation torques. This critical strain is uncomfortably close to the $\sim 10^{-1}$ maximum possible for a perfect crystal, particularly because at those densities most of the mass is believed to be in a neutron fluid, with nuclei distributed irregularly. However, with the understanding that the spin frequencies can be much higher (e.g., that limiting frequencies might be more like 700–800 Hz), the required quadrupolar asymmetry is reduced by an order of magnitude, making this picture more reasonable.

All of the "lump" mechanisms would imply a gravitational wave frequency of twice the spin frequency. However, it has also been proposed that there are wave instabilities that can produce gravitational radiation. The most discussed of these are Rossby waves, or r-modes.

The basic idea behind r-modes is as follows. Suppose that we have a rotating neutron star, and we go into the rotating frame. If a wave is produced that moves in the direction of rotation, as seen in the rotating frame, then energy losses to gravitational radiation will cause the wave to move more slowly. This, therefore, is stable. However, if a wave is produced that moves backwards compared to the rotation, then the energy losses that make it move more slowly as seen in the static frame will cause it to move more rapidly backwards as seen in the rotating frame. Therefore, this is unstable. In a perfect fluid with no magnetic field, this instability operates for *all* angular velocities!

In more detail, the actual modes in question are related to weather patterns on Earth. Fluid that moves in latitude experiences a Coriolis restoration force. The result is circular patterns of fluid movement, centered on the rotational equator. The lowest-order (and thus probably highest-amplitude) mode has a frequency as seen at infinity that is 2/3 of the rotation frequency, hence the gravitational waves would appear at 4/3 of the rotational frequency. Therefore, if for some source we know the spin frequency (e.g., by measurement of coherent pulsations) and can measure periodic gravitational waves, we can determine whether it is lumps or Rossby waves that are present.

However, we have to be careful. If neutron stars were really perfect fluids with no complications, and if these modes could reach high amplitudes, then we'd never see isolated neutron stars with frequencies of hundreds of Hertz. Therefore, something else is going on. For example, some calculations suggest that there is nonlinear saturation of the modes at low amplitude because of coupling between large numbers of modes. Other ideas have included viscous effects that damp the modes (the viscosity is interestingly temperature-dependent, and might in some circumstances lead to limit-cycle behavior), effects related to the interface between the liquid core and solid crust, and magnetic couplings. It is fair to say that at this point there is no consensus about the strength of Rossby waves or the role they could play in neutron star spindown and gravitational radiation.

Burst Sources

The next category of gravitational wave sources is burst sources. These refer to events of very limited duration that do not have to have any special periodicity. Data analysis for these will be very challenging indeed, but since they are by definition associated with violent events, we could potentially learn a great deal from detection of gravitational radiation. Let's consider a few of the more commonly discussed possibilities.

Core-collapse supernovae. When the core of a massive star collapses, it will not do so in a perfectly symmetric fashion. For example, convection will introduce asymmetries. What fraction of the mass-energy will therefore be released as gravitational radiation? This is a question that has to be answered numerically, but it is an extraordinarily challenging problem. Convection is important, so simulations have to be done in three dimensions. Radiation transfer is also essential, as is a good treatment of neutrino transport. To make things even worse, it seems likely that magnetic fields will play a major role, and a wide range of scales could influence each other! Nonetheless, the current best guess is that only a very small fraction of the total mass-energy will come out in gravitational radiation, perhaps $\sim 10^{-6}$. If so, supernovae outside our galaxy will be undetectable. However, the rate of core-collapse supernovae in our Milky Way is estimated to be one per few decades, which means that there is a probability of tens of percent per decade that a supernova will occur within ~ 10 kpc. Current calculations suggest that the strain amplitude at 10 kpc could be $h \sim 10^{-20}$ for a few milliseconds, which would be detectable with advanced ground-based instruments. There have also been proposals that a much higher fraction of energy is emitted during the collapse, which brings us to the next topic.

Gamma-ray bursts. These are short (milliseconds to minutes), high intensity bursts of gamma rays. After a long and interesting history (starting with their detection with US spy satellites!), it has been established that there are two categories of GRBs, the long (tens of seconds) and the short (less than a second, typically). The long bursts are convincingly associated with a type of supernova, but the detailed mechanism for their production is uncertain. Some people believe that GRBs are the birth events for rapidly rotating black holes. If so, the rapid rotation could be a path to much more substantial gravitational wave production. For example, in a massive disk there are bar instabilities that could produce rotating nonaxisymmetric structures. If these emit a lot of gravitational radiation and can be identified with particular bursts, then we have a wonderful situation: extremely bright events at cosmological distances whose redshift can be determined based on the electromagnetic signal, and whose luminosity distance can be determined based on the gravitational wave signal. The difficulty is that to be detectable at cosmological distances (at least 3 Gpc is needed to be interesting), a truly enormous fraction of the mass-energy needs to emerge in gravitational waves (at least tens of percent). This currently seems unlikely, but it is obviously worth pursuing from the observational standpoint.