1. Derive the constant of motion associated with inspiral according to the Peters equation. Hint: define $y \equiv e^2$ to get da/dt and dy/dt, then look for a constant in the form $C = af(y)$.

2. Consider a merger of two black holes of arbitrary masses and spins. Suppose that the merger takes place at an *unknown* redshift z. Show that without knowing z, the waveform of the inspiral/merger/ringdown (meaning the observed frequencies, but not the amplitudes) is not sufficient to measure the masses or angular momenta of the black holes uniquely. What combinations of masses, spins, and redshifts can be measured?

3. This problem shows the limits of order of magnitude calculations in some cases. Let's say you'd like to estimate the recoil speed of a merged black hole remnant, due to linear momentum carried away by gravitational radiation. To simplify things, suppose we have two nonrotating black holes of masses M_1 and M_2 that collide head-on, so there is no spin at any point. A theorem from black hole thermodynamics says that the square of the irreducible mass of the final black hole cannot be less than the sum of the squares of the irreducible masses of the initial black holes. For nonrotating black holes, this becomes

$$M_{\rm final}^2 \geq M_1^2 + M_2^2 \ . \tag{1}$$

Like the increase in entropy, this is an *inequality*, but for our order of magnitude estimate we will assume $M_{\rm final}^2 = M_1^2 + M_2^2$.

With that assumption, compute the final speed of the remnant (as a fraction of the speed of light, and as a function of M_1 and M_2) assuming that all the radiated energy is carried away in a single direction. For comparison, the best current estimates are that the speed for $M_1/M_2 \approx 10$ is $\sim 20 - 200$ km s^{-1}.

4. Consider the ringdown produced by two $10\,M_\odot$ black holes. Suppose that the ringdown lasts for 2 cycles and emits a total of 1% of the mass-energy of the final black hole. Assuming a nonrotating black hole ($j = 0$), what would be the frequency of the radiation and how long would it last? The frequency is in the range of human hearing (although, of course, not audible!), and sound amplitude is measured in decibels, where 0 dB has an intensity of 10^{-9} erg cm^{-2} s^{-1}. If the BH-BH merger occurs at the distance of the Virgo Cluster (about 50 million light years), compare the intensity of the ringdown at Earth with the intensity of the loudest scream ever registered (129 dB, by Jill Drake of the UK).

5. Dr. I. M. N. Sane has come to you with a brilliant idea. He has realized that LISA will be the ideal instrument to detect satellites around extrasolar planets. In particular, he

envisions a $m = 6 \times 10^{26}$ g satellite (about 10% of Earth's mass, bigger than any satellite in the Solar System) orbiting with an orbital frequency of $f_{\rm orb} = 5 \times 10^{-5}$ Hz around a planet with mass $M = 2 \times 10^{31}$ g, about ten times Jupiter's mass. At gravitational wave frequencies $f_{\rm GW} < 10^{-3}$ Hz, LISA's spectral density sensitivity at signal to noise of 1 is $10^{-19}(10^{-3}\ {\rm Hz}/f_{\rm GW})^2\ {\rm Hz}^{-1/2}$. Assuming an observing time of 10^8 seconds, evaluate the detection prospects if the system is at a distance of 10 parsecs (about 3×10^{19} cm).

Challenge: Suppose that you are doing radio observations of a double pulsar system, in which both neutron stars are visible as pulsars. We'd like to determine, qualitatively, how overdetermined the system is. That is, we'd like to know how many aspects of the system can be measured, versus how many parameters there are. This is a deliberately vague question to get you thinking about the process of measurement. If more quantites can be measured than there are parameters, the system is overdetermined and the underlying theory can be tested.