

1. Suppose you were to observe the spin frequency of a pulsar with high precision over a long time. If you can measure the “braking index”  $\Omega\ddot{\Omega}/\dot{\Omega}^2$ , then you can in principle discriminate between spindown due to gravitational radiation and spindown due to magnetic torques. Recall that gravitational radiation gives  $\dot{\Omega} \propto \Omega^5$ , and magnetic dipole torques give  $\dot{\Omega} \propto \Omega^3$ . Use these to compute the braking indices for pure gravitational radiation and for pure magnetic dipole torques.

Suppose that a neutron star with zero magnetic field accretes matter from a companion at a rate  $\dot{M}$ . The rate at which angular momentum is accreted is  $\sqrt{12}\dot{M}GM/c$ . In the following, assume that the mass  $M$  and moment of inertia  $I$  of the star do not change significantly in the period of interest. In addition, we are working under the assumption that a lump, rather than a wave, produces gravitational radiation.

2. Suppose that torques from gravitational radiation balance the accretion torques at a frequency  $\Omega_{\text{equil}}$ . As a function of  $\dot{M}$ ,  $M$ ,  $I$ , and  $\Omega_{\text{equil}}$ , calculate the required ellipticity  $\epsilon$ . Approximately what is the value of  $\epsilon$  if  $\dot{M} = 10^{17} \text{ g s}^{-1}$ ,  $M = 3 \times 10^{33} \text{ g}$ ,  $I = 10^{45} \text{ g cm}^2$ , and  $\Omega_{\text{equil}} = 2\pi \times 300 \text{ Hz}$ ?

3. Suppose that an accreting nonmagnetic neutron star has an observed electromagnetic flux  $F$ , which we assume is produced by accretion onto the surface: the luminosity is  $L = 0.2\dot{M}c^2$ , where the 0.2 is the efficiency of energy release. Show that, if the torque from the accreting matter is balanced exactly by gravitational radiation, that the flux in gravitational radiation has a simple relation to the electromagnetic flux, and find that relation. Assume isotropic emission of both.

4. The highest-flux accreting neutron star in the sky is Sco X-1, with a flux of about  $3 \times 10^{-8} \text{ erg cm}^{-2} \text{ s}^{-1}$ . If other torques are negligible and its gravitational radiation emerges at 1000 Hz, roughly what strain amplitude would we measure? At that frequency, the target sensitivity in standard configuration for LIGO 1 is  $\sim 10^{-22} \text{ Hz}^{-1/2}$  and for Advanced LIGO is  $\sim 10^{-23} \text{ Hz}^{-1/2}$ . Approximately how long would it take each of them to measure the signal?

5. One burst source some people have proposed is pulsar glitches. In a glitch, the spin frequency of the pulsar changes suddenly, due (we think) to a sudden coupling between the crust and the underlying superfluid core. The energy release is  $I\Omega\Delta\Omega$ , but  $I$  is the moment of inertia of the crust, which is perhaps  $10^{43} \text{ g cm}^2$ , or 1% of the moment of the inertia of the star (because the crust exists only at low densities). In a really big glitch,  $\Delta\Omega \sim 10^{-6}\Omega$ . Let’s say that such a glitch happens to a star with  $\Omega = 100 \text{ rad s}^{-1}$ , and that *all* the energy

comes out in gravitational waves with frequency 2000 Hz (comparable to double the sound crossing frequency), in a period of only 1 second. If this is a very close source, at 1 kpc (or about  $3 \times 10^{21}$  cm), could this be seen with Advanced LIGO (sensitivity  $\sim 2 \times 10^{-23}$  Hz $^{-1/2}$  at 2000 Hz)?

6. Dr. Sane doesn't understand all this focus on binary compact object mergers. He thinks that direct collisions of single neutron stars in clusters with each other will make wonderful burst sources. He has requested that you work out the numbers for him. Suppose that you consider a dense globular cluster, such that in the center the number density of neutron stars is  $10^6$  pc $^{-3}$  and there are 1000 total neutron stars per cluster. Suppose that each neutron star has a radius of 10 km and mass of  $1.5 M_{\odot} = 3 \times 10^{33}$  g, and that the typical random speed in the cluster is 10 km s $^{-1}$ . To within an order of magnitude, calculate how often two neutron stars in a given cluster will hit each other. If there are  $10^{10}$  such clusters in the universe, how often will this happen in the universe? **Note:** be careful when you calculate the cross section for collisions, because gravitational focusing is important.