Rocket science

As there is no air in space, the kind of engine that runs your car simply won't work. Fortunately, the principle of conservation of momentum tells us that if we can eject something out of the back of a rocket, the rocket will go forward as a result. To understand this concept, suppose that you and a friend are floating together in the middle of space. If you give your friend a push, you move backwards as a result. Similarly, if you were to throw a baseball in space then you would go backwards, although not as fast as the baseball moves forwards because you have much greater mass than the baseball.

It is *not* necessary for the material that is ejected to have something like air to "push" against. This was not understood by the New York Times when on January 13, 1920 they derided the pioneering rocket experiments of Robert Goddard: "That Professor Goddard, with his 'chair' in Clark College and the countenancing of the Smithsonian Institution, does not know the relation of action to reaction, and of the need to have something better than a vacuum against which to react - to say that would be absurd. Of course he only seems to lack the knowledge ladled out daily in high schools."

Ahem. No, in fact a rocket works just fine in a vacuum; better, in fact, than it does in air because air resistance would slow down the rocket. Those who get their science exclusively from the mass media are doomed to frequent disappointment.

In any case, it is clear that the faster you can expel your fuel from the back of the rocket, the faster your rocket will go. You might also anticipate that the more fuel you have, the faster you will go eventually. That is true, but not nearly to the extent we might hope, because the fuel also has mass and has to be accelerated as well. This is encapsulated in the rocket equation, which was first derived in the early 1800s as part of weapons research. Suppose that we have a rocket whose total mass including fuel starts out as m_0. Let the fuel be ejected at a speed v_e, and let the final mass of the rocket after all the fuel has been ejected be m_1. If we also assume that the fuel comes out one small bit at a time rather than all at once, then the final speed of the rocket is

$$v_{\text{final}} = v_e \ln(m_0/m_1)\ . \tag{1}$$

Here "ln" means the "natural log" to base $e = 2.71828\ldots$. For example, suppose that the fuel makes up 90% of the total mass, so that when it is spent $m_1 = 0.1m_0$. Then the final speed is not ten times v_e but just $\ln(10) \approx 2.3$ times v_e. If the fuel makes up 99% of the total mass then the final speed is $v_{\text{final}} = 4.6v_e$, not $100v_e$. Again, the problem is that we have to accelerate all the fuel as well as the payload or passengers. If we could burn all the fuel at once we could do better. For example, if the fuel makes up 99% of the total mass and is all ejected at once with the speed v_e, then the final speed of the payload would be $99v_e$ instead of just $4.6v_e$. However, when the payload involves people, such enormous acceleration would turn them into piles of quivering sludge, which is why this is not an option!

The typical speed of rocket exhaust with current technology is about 4 km/s. To get this up to the 30 km/s that we mentioned earlier (with a journey time of 100,000 years to get 10 light years away) would require a fuel:payload mass ratio of $\exp(30/4) \approx 1800$. That's huge, and suggests reason for pessimism for very fast trips, but in our main lecture we'll examine some specific suggestions for how to improve our thrust options.