Black Holes

In this lecture and the next one we will discuss the properties of the two types of objects whose frequencies fall in the range of ground-based detectors: black holes and neutron stars.

Black holes are interesting for many reasons: they are one of only three possible endpoints of stellar evolution (the others being white dwarfs and neutron stars), they are the powerhouses of the most luminous things in the universe (quasars and active galactic nuclei), and they are the simplest macroscopic objects in the universe, with only two parameters important for their astrophysical properties. They are also way cool. Their simplicity means that it is possible to study them in a way impossible for any other object: with mathematical rigor. There was, for example, a flurry of activity in the late 1960s and early 1970s about proving theorems related to black holes, something which is mightily difficult to do with a star! However, our main interest is in astrophysics, and specifically in explaining observed phenomena. We will therefore describe and use some of the derived results, but will not derive them (this would take overwhelmingly too much time).

Let us start by defining "black hole". A black hole is an object with an event horizon instead of a material surface. Events inside that horizon cannot be seen by any external observer. This is the fundamental property of black holes that distinguishes them from all other objects. It should be noted that although there is compelling evidence for the existence of black holes in the universe, never has the existence of the horizon itself been demonstrated. An observation that unambiguously indicates the presence of a horizon would be a major advance. From time to time there are press releases announcing proofs of event horizons based on theoretical arguments, but so far these have not been rigorously convincing. There is, however, hope that within the next few years high-resolution radio observations will resolve the shadow of the event horizon of the black hole in the center of our Galaxy.

In the rest of this lecture we will (1) discuss some of the aspects of black holes by themselves, then (2) talk about some of the astrophysics of black holes, including supernovae (which might be the only way that black holes are produced in the universe), and accretion disks (which allow electromagnetic radiation to be produced from near black holes). We then list some current puzzles about black holes that might be resolved with observations of gravitational radiation. At the end we have some text about other issues related to black holes that we don't expect to be able to cover in the time allowed, but that might be of interest.

Properties of the Schwarzschild Geometry

It could be reasonably said that what an average astrophysicist needs to know about black holes is encapsulated by a few properties of the Schwarzschild and Kerr spacetimes (or, equivalently, geometries). By the way, although it is a somewhat pedantic point, I note that the metric is not the same thing as the geometry; the metric is a particular representation of the geometry using specific coordinates. To be correct, you would talk about the spacetime or the geometry when you mean the solution in general, and the metric when you have particular coordinates in mind.

We'll start with the Schwarzschild geometry, which is the uniquely correct geometry outside of a spherically symmetric, nonrotating, uncharged, object. This geometry thus describes the exterior spacetime around a nonrotating planet or star, not just a black hole.

The coordinates most used by astrophysicists to describe the Schwarzschild geometry are the Schwarzschild coordinates. Written using these coordinates, the line element is

$$ds^2 = -(1 - 2M/r)dt^2 + dr^2/(1 - 2M/r) + r^2(d\theta^2 + \sin^2\theta d\phi^2) \ . \tag{1}$$

Here we use geometrized units in which $G = c = 1$; written without that simplification, for example, the dt^2 term would be $-(1 - 2GM/(rc^2))dt^2$. Note that you have a choice about your "metric signature", i.e., the sign of the dt^2 term (which must be the opposite of the sign of the spatial terms). Here we use the $- + ++$ signature, but others use the $+ - --$ convention. Just make sure you're self-consistent.

ds^2 is the square of the *invariant interval*; any observers will agree that two infinitesimally separated spacetime events have a squared spacetime distance equal to ds^2. If the two events are along the worldline of a particular observer, then that observer sees $dr = d\theta = d\phi = 0$ and therefore $ds^2 = c^2 d\tau^2$, where $d\tau$ is the time interval seen by the observer (τ is called the proper time).

In this line element θ and ϕ are the usual colatitude and azimuthal coordinate that we normally use to describe a spherically symmetric system. r is the *circumferential* radius; if you determined the circumference of a circle at a fixed r and divided by 2π, you would get r. It is therefore *different* from the radius you would get by starting at the coordinate origin and integrating straight out; that quantity (the "proper radius") would be the integral of $(1 - 2M/r)^{-1/2}$ times r, which has the odd consequence that the circumference divided by the proper radius is less than 2π. The coordinate t is the time at infinity, which means that if two colocated events have Schwarzschild coordinate times that differ by dt, then an observer at infinity would see them separated in time by dt. These coordinates therefore have the nice property that many of the things measurable to us at infinity (say, the angular velocity of a rotating star) are easily expressible using (t, r, θ, ϕ).

So what are the special properties of this spacetime?

- The event horizon. When $r = 2M$ something odd obviously happens. Among other things, contemplation of the line element above ultimately makes it clear that as seen from infinity, a clock outside of but close to $r = 2M$ runs very slowly, and would actually stop at $r = 2M$. This means that you can never see something fall through the event horizon from anywhere outside. Early on this was misinterpreted by most people (including Einstein!) as meaning that stars wouldn't collapse inside $r = 2M$, but in 1958 David Finkelstein showed otherwise: if you fall freely into a black hole you *will* cross the event horizon and hit the central singularity in a time that will be all too finite from your perspective!

Note, though, that the tidal acceleration does *not* become infinite at the horizon. We can get a sense for this in the Newtonian limit, where the relative tidal acceleration across an object of length ℓ a distance $R \gg \ell$ from a mass M is $\Delta a \approx GM\ell/R^3$. Letting $R = 2M$ gives $\Delta a \sim M^{-2}$, which means that the tidal acceleration becomes lower for larger black hole masses. You would be killed by the tides well outside a black hole of mass $M = 10\ M_\odot$, but if $M = 10^8\ M_\odot$ (typical of the black holes that power quasars) you wouldn't even feel the tides at the horizon, although you'd still plummet to your doom at the singularity. This is relevant to tidal disruption events: solar-type stars can be disrupted outside the horizon if $M \lesssim 10^8\ M_\odot$, but are swallowed whole if the black hole is more massive.

- The innermost stable circular orbit, or ISCO. In Newtonian gravity, all circular orbits are stable. This means that if such an orbit were perturbed slightly, the orbit would just become somewhat elliptical but nothing else would happen. But in general relativity there is an innermost stable circular orbit. For the Schwarzschild geometry in Schwarzschild coordinates, $r = 6M$ is that innermost radius. Inside $r = 6M$, a circular orbit is unstable; a slight perturbation inward leads to a rapidly-opening inspiral to the hole. Thus it is common to assume that accretion disks of gas spiraling onto a black hole are effectively truncated at the ISCO; there is gas inside the ISCO, but no longer is there a disk which moves with slow inward radial motion. Gas pressure gradients change somewhat the location of the ISCO, but this is a decent approximation that allows you to compute approximately the radiative efficiency of a geometrically thin disk. For a nonrotating star, the ISCO radius is $r_{\rm ISCO} = 8.85$ km $(M/M_\odot)$. Neutron stars, which have thus far measured masses between 1.25 $M_\odot$ and 2.01 $M_\odot$, are likely to be able to fit inside the ISCO, especially at the higher-mass end. It is therefore possible that signatures of the ISCO have been seen in some sources; indeed, my colleagues and I have written several papers about this, but the implications would be important enough that there is justified caution in the community about our suggestions.

  From the standpoint of gravitational waves, the ISCO is a marker for when the coalescence of two black holes transitions from a slow inspiral to a more radially rapid plunge. It is *not*, however, a clear line in spacetime! Radiation of gravitational waves means that, strictly speaking, *no* circular orbit is really stable; such orbits decay. But, in the order of magnitude sense, it does give us an idea of when the binary is close to merger. The orbital frequency at the ISCO of a nonrotating black hole of mass M is 2200 Hz $(M_\odot/M)$. The quadrupolar nature of gravitational waves means that for a circular orbit we see twice the orbital frequency (for an elliptical orbit, all harmonics of the orbital frequency contribute, peaking more or less at twice the orbital frequency at the pericenter distance). Thus a rough measure, for nonrotating black holes, of the transition between inspiral and merger is 4400 Hz $(M_\odot/M)$.

- Pericenter precession. Again using Newtonian gravity for comparison, we note that if we have an elliptical orbit around a point source then the orbit exactly traces over itself every time. In contrast, in the Schwarzschild spacetime the orbit precesses, in the same direction as the orbit (thus this is prograde precession). The angle precessed per orbit is

$$\Delta\phi = \frac{6\pi GM}{ac^2(1-e^2)} \tag{2}$$

for a semimajor axis $a \gg GM/c^2$ and eccentricity e. This was the first test of general relativity: Einstein was able to explain the anomalous 43" per century precession of Mercury, after the influences of the other planets had been taken out. At the ISCO, the precession is formally infinite; another way of saying this is that the radial epicyclic frequency (the frequency at which a radially perturbed orbit will return to its original radius; this exactly equals the orbital frequency in Newtonian gravity) goes to zero at the ISCO.

The Kerr-Newman Spacetime

It was discovered in 1963 that an exact spacetime exists for a black hole with just mass and angular momentum (Kerr geometry), and in 1965 a solution including charge was found (Kerr-Newman geometry). The most common coordinates used to express this spacetime are generalizations of Schwarzschild coordinates called Boyer-Lindquist coordinates, and for the record the metric line element is then

$$ds^2 = -(\Delta/\rho^2)[dt - a\sin^2\theta\, d\phi]^2 + (\sin^2\theta/\rho^2)[(r^2+a^2)d\phi - a\, dt]^2 + (\rho^2/\Delta)dr^2 + \rho^2\, d\theta^2 \ . \tag{3}$$

There are several definitions here. The parameter $a = J/M$ describes the angular momentum, and it has dimensions of mass. $\Delta = r^2 - 2Mr + a^2 + Q^2$, where Q is the electric charge (in cgs units Q^2 has the units of erg-cm, which can then be converted to grams in the usual geometrized units way). Finally, $\rho^2 = r^2 + a^2\cos^2\theta$. As before, θ and ϕ are the usual spherical polar coordinates, and t is the time at infinity, but r is no longer the circumferential radius; there are corrections of order a^2.

The most important new feature of this geometry, compared to Schwarzschild, is the $d\phi dt$ terms. These indicate a relation between time and azimuthal angle, and correspond to frame dragging: spacetime is "twisted" in the direction of rotation of the black hole.

This geometry has a horizon (and therefore descibes a black hole) only if $Q^2 + a^2 \leq M^2$. If equality holds, this is called an extremal black hole. If this condition is violated, centrifugal acceleration or electrostatic repulsion will halt the collapse. You cannot, however, spin up a black hole or feed charge to it so that it loses its horizon.

Let's see if we even need the charge term, astrophysically. **Ask class**: how should we determine whether Q can ever be gravitationally significant? Suppose that $Q^2 = M^2$, the maximum possible. Converting M^2 into erg-cm units means $Q^2 = (Mc^2)(GM/c^2) = GM^2$. Suppose we compare the electrical and gravitational forces on a particle of mass m and charge q at a distance $r \gg M$, so that the Newtonian force law is accurate. The electrical force is qQ/r and the gravitational force is GMm/r, so the ratio is $f_e/f_g = qQ/(GMm) = qG^{1/2}M/(GMm) = q/(G^{1/2}m)$. For example, for a proton $f_e/f_g \approx 10^{18}$ and for an electron $f_e/f_g \approx 2 \times 10^{21}$. This shows that (as always!) the electromagnetic force is overwhelmingly stronger than gravity if there is a lot of unbalanced charge. The result is that if Q is anything remotely significant gravitationally, the black hole will sweep up every stray charge within parsecs until it is almost electrically neutral. That's why we can ignore the charge, and consider just the mass and angular momentum when thinking about the spacetime. With angular momentum but no charge this is called the Kerr spacetime. It is also common to use the dimensionless quantity $j = a/M$ instead of a.

Properties of the Kerr Spacetime

Unlike the Schwarzschild spacetime, which is the correct exterior spacetime for *any* uncharged spherically symmetric matter, the Kerr spacetime is only the exact exterior spacetime for black holes. Thus, for example, rotating neutron stars have an exterior spacetime that must be computed numerically.

The major new component to this spacetime compared to Schwarzschild is *frame-dragging*, which becomes more important as one gets closer to the hole. One of many bizarre consequences is that if you were to drop a particle from infinity radially at the hole, then as it got closer it would acquire a nonzero angular velocity (but still have zero angular momentum!). The angular velocity of a zero angular momentum particle, which can be thought of as the angular velocity of spacetime, is

$$\omega = \frac{2Mar}{(r^2 + a^2)^2 - a^2\Delta \sin^2\theta} \; . \tag{4}$$

For almost all applications of interest, the r^4 term dwarfs the others and $\omega \approx 2Ma/r^3 = 2jM^2/r^3$.

Frame-dragging has many implications. One is that, near enough to the hole, a particle *must* orbit in the same direction as the hole. This is even true outside the horizon, so there is a region, called the *ergosphere*, in which no static observers can exist; nonetheless, they could escape from that region, so it isn't like the horizon. The radius of the ergosphere is $r_{\rm ergo} = M + (M^2 - a^2 \cos^2\theta)^{1/2}$. In addition, the black hole itself shrinks; the radius of the horizon is $r = M + \sqrt{M^2 - a^2}$, so for an extremally rotating black hole ($a = M$), $r = M$. The radius of the innermost stable circular orbit shrinks for prograde orbits (to a minimum of $r_{\rm ISCO} = M$ for $a = M$) and increases for retrograde orbits (to a maximum of $r_{\rm ISCO} = 9\,M$ for $a = M$). That means that if gas spirals in to the hole on prograde orbits, the energy emitted and hence the accretion efficiency increases with increasing spin (from 5.7% for $a = 0$ to 42% for $a = M$, or 40% if you discount energy that goes down the hole; note, though, that as Kip Thorne pointed out, when a black hole rotates close to its maximum, photons with negative angular momentum that are captured by the hole slows down the rotation, to an "astrophysical maximum" of $a = 0.998\ M$ at most). Yet another consequence is that a particle in a circular orbit that is tilted with respect to the spin plane will precess in its orbit, at the rate ω. This means that a nonaxisymmetric warp in an accretion disk has a tough time surviving unless it is confined to a small radial range, because the strong dependence of ω on r means that there would be a lot of shear otherwise. Also, a gyroscope with an axis tilted from the spin axis will precess at ω. This is an effect which some people think has already been seen from some neutron star and black hole sources (although I'm highly skeptical). Finally, Kepler's Third Law (angular velocity of a particle in a circular orbit at r) takes the simple form

$$\Omega = \pm\frac{M^{1/2}}{r^{3/2} \pm aM^{1/2}} \tag{5}$$

where $+$ is for prograde and $-$ is for retrograde orbits.

I recommend that you read Bardeen, Press, and Teukolsky 1972 for more details about the Kerr spacetime.

The Formation of Black Holes: Core Collapse of Massive Stars

Various other black hole origins are sometimes suggested, including collapses at special times in the early universe, but there is only one way that we are confident that they can be formed. This way involves the collapse of the core of a massive star, and it is also the way that neutron stars are formed, so it deserves our attention.

The ultra-brief summary is:

1. Stars such as the Sun are on the "main sequence" of stellar lifetimes in which the energy that helps support them against gravity is supplied by the fusion of hydrogen into helium.
2. When enough of the core hydrogen has been used up, the star swells into a giant (with $\sim 100\times$ its main-sequence radius). If the star is massive enough, it then settles into a

phase in which it fuses helium to carbon. However, because the difference in nuclear binding energy per nucleon between carbon and helium is much less than it is between helium and hydrogen, this phase lasts a much shorter time than the main sequence.

3. For sufficiently massive stars (ones that start their lives with $\gtrsim 8\ M_\odot$), this sequence continues with shorter and shorter lifetimes in each phase: carbon to oxygen to neon to magnesium to silicon and finally to ^{56}Fe. At that stage, no further energy can be derived from fusion; ^{56}Fe isn't *quite* at the peak of the binding energy curve for zero-pressure matter (that honor belongs to ^{62}Ni), but so little energy remains available that it is actually quantum degeneracy pressure, not thermal pressure, that has to hold up the core.

4. But degeneracy pressure can't hold the core up forever. As Chandrasekhar showed, and as we will discuss in the next lecture, for cold matter with roughly two baryons (i.e., neutrons or protons) per electron, electron degeneracy pressure can only hold the star up to around a baryonic rest mass of $1.35 - 1.4\ M_\odot$. After that, it collapses.

5. The fate of the star then depends on how much mass falls back. If the mass is less than the limit for neutron stars, you get a neutron star. If the mass is greater than that limit, you get a black hole. This limit is still being debated, and it depends on the detailed and unknown properties of matter beyond nuclear density.

Accretion Disks

Black holes by themselves are nearly invisible loners. Their small size (a $5\ M_\odot$ nonrotating black hole would fit inside Bengaluru!) means that they are extremely difficult to detect, and if they don't accrete matter they don't radiate. Thus evidence for black holes comes from their effect on other things. One such effect is they way that supermassive black holes at the centers of galaxies dictate the motion of the nearby stars. Another effect is the way that gas spirals onto black holes and releases energy. By the way, it is worth pointing out that although we expect that there are about 10^8 black holes in our Galaxy, only in ~ 25 cases do we have clear evidence of specific black holes, and only in < 100 cases do we even have suspicion of stellar-mass black holes. The circumstances that make a black hole visible are rare!

There are too many details to go into depth, but suffice it to say that because most matter in the universe has angular momentum, and black holes are small, the matter that gets to black holes usually does so in a disklike arrangement (there are some exceptions that we won't address). The ultimate energy release is gravitational, but there is an interesting surprise that is worth examining. To derive this, we'll use an approach in some of Roger Blandford's notes, which focuses on three conserved quantities: rest mass, angular momentum, and energy.

First, we assume the equation of continuity: the mass accretion rate is constant as a function of radius, so

$$\dot{M} = 2\pi r \Sigma v_r = \text{const} \tag{6}$$

where Σ is the surface density and v_r is the inward radial velocity.

Second, we treat angular momentum conservation. Assume for simplicity that the radial velocity is small and that the Newtonian form for angular momentum holds. Assume also that there is an inner radius r_I to the disk, and that no more angular momentum is lost inside that (for example, this might be thought a reasonable approximation at the ISCO). Then angular momentum conservation implies that the torque exerted by the disk inside radius r on the disk outside that radius is

$$N = \dot{M}\left[(Mr)^{1/2} - (Mr_I)^{1/2}\right] . \tag{7}$$

Third, energy conservation. The release of gravitational binding energy per unit time is $\dot{E}_g = -\dot{M}d(m/2r)$. In addition there is a term due to the transport of angular momentum. At a radius r where the angular velocity is $\Omega = (M/r^3)^{1/2}$, the rate of work done on the inner surface of an annulus is $-N\Omega$, and the net energy per time deposited in a ring is $\dot{E}_v = -d(N\Omega)$. The sum of the two is the luminosity released in the ring, $dL = \dot{E}_g + \dot{E}_v$. Evaluating this and replacing the Newtonian constant G we have

$$\frac{dL}{dr} = \frac{3G\dot{M}M}{2r^2}\left[1 - \left(\frac{r_I}{r}\right)^{1/2}\right] . \tag{8}$$

Whoa! Hold on here! This is different than what we might have expected. Far away from the inner edge r_I, this means that the local energy dissipation rate is *three times* the local release of gravitational energy. Where is the extra energy coming from? If we integrate $L(r)$ over the whole disk, we find that it gives $G\dot{M}m/2r_I$, as expected if the matter ends up in a circular orbit at radius r_I. However, close to r_I the energy dissipation rate is *less* than the local gravitational release. Therefore, what is happening is that matter near the inner part of the disk has much of its energy going into transport of angular momentum rather than release of energy, and the extra energy is released further out. This factor of three was missed at first, but was pointed out by Kip Thorne.

Note, by the way, that this expression does not specify the angular momentum transport mechanism. It is thought that this mechanism is magnetic in nature, and indeed there are many groups around the world who are doing detailed simulations of this process.

Puzzles Related to Black Holes

Black holes remain largely mysterious despite decades of study. I expect that gravitational wave observations will yield tremendous insight about these objects. Some of the ways in which ground-based detections could help are:

- Is general relativity correct in strong gravity? This basic question is not yet answered. General relativity works very well in the weak-gravity environments we've been able to explore, and it has mathematical beauty that some consider to be another type of confirming evidence, but direct strong-gravity tests don't really exist. Imaging of the event horizon of our Galactic center's black hole might be possible in a few years, and there are other electromagnetic tests out there (e.g., the evidence I mentioned of the ISCO), but these involve extra assumptions about the behavior of the gas. In principle, agreement of observed gravitational waveforms with what we see will be the best test.
- What is the mass distribution of black holes? We are sure that stellar-mass black holes ($\sim 5-15\ M_\odot$) exist, and that supermassive black holes ($\sim 10^6-10^{10}\ M_\odot$) exist. What about the range in the middle? Do intermediate-mass black holes exist? My opinion is that there is strong circumstantial evidence that they do, but the evidence is not as direct as it is for the other mass ranges. LIGO-India and similar instruments could detect gravitational waves from black holes up to a few hundred solar masess, so they might be able to provide the first definitive evidence of this type of black hole.
- What is the spin distribution of black holes? It has been understood for probably three decades that stellar-mass black holes are probably born with close to the mass and spin magnitudes that they have now (the mass accreted from a stellar companion is likely to be too little to do much). Spin measurements from accretion disk spectra have been reported for several stellar-mass black holes, but these are continuum spectra, which unfortunately means that there are no definite features to fit. Thus we have to understand the possibility that the spin measurements could be systematically off. Gravitational wave measurements will provide independent ways to estimate the spins, which will give us new insight into the formation processes of black holes.

Black Hole Thermodynamics

There is a remarkable black hole analogy with thermodynamics. If one computes the area of the horizon, it is

$$A = 8\pi M\left[M + (M^2 - a^2)^{1/2}\right] \ . \tag{9}$$

For Schwarzschild, $a = 0$, the area is $A = 16\pi M^2$ as expected. Hawking proved that in any interaction of a black hole or between black holes, the sum of the areas can never decrease. This leads one to a possible computation of the maximum amount of energy that can be radiated in a collision between black holes. For example, if two Schwarzschild black holes of mass M hit head-on, then you know that $16\pi M_{\rm tot}^2 \geq 32\pi M^2$, so $M_{\rm tot} > M\sqrt{2}$ and no more than 29% of the total mass-energy can be radiated away. The best case would be two extremal Kerr black holes of the same mass and opposite angular momentum, for which the theoretical maximum is 50%. However, the *actual* amount radiated is much less than this, and must be computed numerically. For head-on Schwarzschild the efficiency is more like 0.1%.

The area theorem is awfully reminiscent of the second law of thermodynamics. But this would require that black holes have finite temperature, so that they radiate. When Bekenstein suggested the thermodynamic analogy, most people (including Hawking) were dubious, but then Hawking showed that black holes *do* radiate! This happens because virtual pairs of particles and antiparticles can be made real by the tidal acceleration near the event horizon, and on occasion one escapes while the other is sucked in; the effect is that the black hole "radiates" even though nothing escapes from inside the event horizon. This is an astrophysically unimportant effect because the effective temperature is $T \approx 10^{-7}\,\mathrm{K}(M/M_\odot)$, so a 10 solar mass black hole would last about 10^{70} years. We'll never see a black hole radiate unless tiny ones (mass of a mountain) were formed in the early universe. Nonetheless, Hawking radiation does have importance in other ways. For example, several years ago great excitement was produced when it was shown that the rate and spectrum of Hawking radiation from special black holes (Schwarzschild and extremally rotating) could be reproduced in M-theory, which is a candidate for the theory of everything. Hawking radiation also brings up interesting semi-philosophical questions; for example, particles and antiparticles have an equal likelihood of being emitted, whereas the star that formed the black hole and almost anything that fell in it were formed of particles. Thus, lepton and baryon number conservation seem to be violated by Hawking radiation. However, for context, particles of nonzero rest mass can only be emitted in significant numbers when the temperature is within a factor of a few of mc^2/k, where m is the rest mass of the particle (below those temperatures it is essentally photons). Maybe neutrinos could be emitted at moderately low temperatures, but baryons only come out when $M \lesssim 10^{-19}\ M_\odot$.

Inevitability of Collapse

One astrophysically relevant result to be stated is that once a star has compacted within a certain radius, formation of a black hole is inevitable. A basic reason for this is that in general relativity, all forms of energy gravitate. This includes pressure in particular. In a normal star, the pressure makes a tiny contribution to the total mass-energy, but in a very compact star the pressure is substantial. Normally, hydrostatic balance is produced by the offset of gravity by a pressure gradient, but in this case squeezing the star only increases the gravity (by increasing the pressure), so in it goes. The minimum stable radius for a spherically symmetric star is not the Schwarzschild radius $R_s = 2M$ as you might expect, but is $\frac{9}{8}R_s$.

No Hair Theorem

How relevant are the Schwarzschild and Kerr spacetimes? **Ask class**: thinking about Newtonian gravity, what are some factors other than the total mass that could influence the gravitational field outside a normal star? Quadrupole terms, fluid motions, asymmetries, et cetera. What happens when collapse into a black hole occurs? An amazing set of theorems proved in the early 1970's shows that the final result is a black hole that has only three qualities to it at all. These are mass, angular momentum, and electric charge. Everything else (quadrupole terms, magnetic moments, weak forces, etc.) decays away. This is a remarkable result that simplifies treatment of black holes greatly. One heuristic way to think about this relates to what you would see if you dropped a lightbulb into the black hole. As the lightbulb falls, light from it diminishes more and more in apparent intensity. **Ask class**: suppose we have a lightbulb with rest-frame specific intensity $I_{\nu 0}$ (the specific intensity is the energy per area per frequency per solid angle per time in a bundle of photons). How do we compute the specific intensity seen at infinity when the bulb is at radius r, if the bulb falls radially from rest at infinity? The key here is to note that $I_\nu \propto \nu^3$ (derivations exist in many places); tracking the frequency thus allows us to compute the specific intensity. As seen by a distant observer, as the light bulb falls into the hole, its frequency decreases because of the Doppler redshift (it's moving away from us) and because of the gravitational redshift.

Very soon, the intensity drops to undetectable levels; in fact, the luminosity seen by a distant observer goes like

$$L \propto \exp\left(-\frac{t}{3\sqrt{3}M}\right) \ . \tag{10}$$

For a solar mass black hole the time constant is a few tens of microseconds. Therefore, in the blink of an eye the black hole really does appear black. In a somewhat analogous fashion, other properties of the infalling matter, such as magnetic field and lumpiness of the matter distribution, decay away on a similar timescale. Only mass, angular momentum, and electric charge are left.

Additional references: For more mathematical details, see "The Mathematical Theory of Black Holes" by Chandrasekhar, or "Black Holes" by Novikov and Frolov.