Continuous and Burst Sources, and Stochastic Backgrounds

Once we move away from binaries we enter unknown territory. All other types of sources are of unknown strength, which is another way of saying that if they are detected, we can learn a lot of astrophysics.

The first of these uncertain classes of sources that we will treat is continuous sources. A binary increases its frequency as it loses energy, and the lifetime of such sources is short (minutes at most) in the frequencies accessible to ground-based detectors. In contrast, a spinning source can in principle emit gravitational waves at a single frequency for a long time, so the signal builds up in a narrow frequency bin. As a result, particularly for high frequencies observable with ground-based detectors, continuous-wave sources are interesting because they can in principle be seen even at relatively low amplitudes.

What amplitude can we expect? From the first lecture we know that if the moment of inertia is I, then the amplitude is

$$h \sim (G/c^4)(1/r)(\partial^2 I/\partial t^2) \ . \tag{1}$$

For binaries we argued that $I \sim MR^2$, and also had a relation between $\Omega^2 \sim \partial^2/\partial t^2$ and M and R. However, for a spinning source these relations do not have to hold. For a gravitationally bound source (e.g., neutron stars and not strange stars, which if they exist are self-bound and can therefore in principle rotate faster), Ω cannot be greater than the Keplerian angular velocity, but it can certainly be less. In addition, unlike for binaries, only a small fraction of the moment of inertia is involved in gravitational wave generation (indeed, if the spinning source is axisymmetric, no gravitational radiation is emitted). Let us say that some fraction ϵ of the moment of inertia is nonaxisymmetric. Generically this could be, e.g., a lump or a wave. Therefore, $h \sim (G/c^4)(1/r)\Omega^2\epsilon I$.

The luminosity is then

$$\begin{aligned} L &\sim r^2h^2f^2 \\ &= (32/5)(G/c^5)\epsilon^2 I_3^2\Omega^6 \ , \end{aligned} \tag{2}$$

where we have put in the correct factors for rotation around the minor axis of an ellipsoid (here I_3 is the moment of inertia around that axis), and we are now defining ϵ to be the ellipticity in the equatorial plane: $\epsilon = (a-b)/(ab)^{1/2}$, where the principal axes of the ellipsoid are $a \geq b > c$.

Note the extremely strong dependence on Ω. The rotational energy is $E_{\rm rot} = \frac{1}{2}I\Omega^2$, so $\dot{E} = I\Omega\dot{\Omega}$ and if the part of the star generating the gravitational waves (e.g., a lump) is coupled to the rest of the star then we have

$$\begin{aligned} I\Omega\dot{\Omega} &= -(32/5)(G/c^5)\epsilon^2 I_3^2\Omega^6 \\ \dot{\Omega} &= -(32/5)(G/c^5)\epsilon^2 I_3\Omega^5 \ . \end{aligned} \tag{3}$$

For pulsars, we can relate this to the dimensionless period derivative $\dot{P} = -2\pi\dot{\Omega}/\Omega^2$, which is between $\sim 10^{-13}$ for young pulsars and $\sim 10^{-21} - 10^{-22}$ for the most stable of the millisecond pulsars. Therefore, we have

$$\dot{P} = (64\pi/5)(G/c^5)\epsilon^2 I\Omega^3 \ . \tag{4}$$

For a typical neutron star moment of inertia $I \approx 10^{45}$ g cm^2 and a young pulsar like the Crab with $\Omega \approx 200$ rad s^{-1} and $\dot{P} \approx 10^{-13}$, this implies $\epsilon < 3 \times 10^{-4}$. The reason for the inequality is that the observed spindown can also be caused by other effects, notably magnetic braking. By the same argument, a millisecond pulsar with $\Omega \approx 2000$ rad s^{-1} and $\dot{P} \approx 10^{-21}$ has $\epsilon < 10^{-9}$.

What strain amplitudes should we expect? When the correct factors are put in, we find that the strain amplitude from a pulsar of period P seconds at a distance r is

$$h_c \approx 4 \times 10^{-24}\epsilon P^{-2}(1 \text{ kpc}/r) \ . \tag{5}$$

For the Crab pulsar, $P = 0.03$ s, $r = 2$ kpc, and $\epsilon < 3 \times 10^{-4}$, so the maximum amplitude is $h_c \approx 6 \times 10^{-25}$. For a millisecond pulsar with $P = 0.003$ s, $r = 1$ kpc, and $\epsilon < 10^{-9}$, the maximum amplitude is $h_c \approx 4 \times 10^{-28}$. These amplitudes seem extremely small, but the coherence of their signal (and the fact that the frequency is known from radio observations if we focus on known pulsars) means that searches can go extremely deep. For example, the LIGO sensitivity goal at 60 Hz (the gravitational wave frequency of the Crab signal, which is twice the rotation frequency) is $\sim 5 \times 10^{-24}$ Hz$^{-1/2}$. Therefore, in principle, a coherent signal at the maximum possible amplitude for the Crab could be detected in a time $[5 \times 10^{-24}/6 \times 10^{-25}]^2 \approx 70$ s, or just over a minute. For a very stable millisecond pulsar, though, the required integration time could exceed 10^{10} s, which is prohibitively large. Indeed, for most millisecond pulsars we know that they will not be detectable, because if their amplitudes were in the advanced LIGO detection range then they would be spinning down faster than they are.

In a similar vein, some researchers have investigated the possibility that actively accreting neutron stars might balance the accretion torque by gravitational radiation losses of angular momentum. It is important to stress that there is no evidence that gravitational radiation plays any significant role in the angular momentum balance of any accreting pulsar, and in a couple of cases the contribution of gravitational radiation to spindown is limited by observations to less than a few tens of percent. Coupling of the stellar magnetic field with the accreting matter explains all of the observations just fine. However, as with so much of gravitational wave astronomy, we are open to surprises!

Let us now consider continuous-wave radiation from another perspective. What is it, exactly, that could produce the required nonaxisymmetry? We will divide the possibilities into two categories. "Lumps" are nonaxisymmetries that are fixed relative to the star. "Waves" are nonaxisymmetries that move relative to the star.

Lumps first. What if the neutron star is a perfect fluid with no magnetic field? Then, as analyzed in the late 19th century, the equilibrium shape of the star below some critical rotation frequency is a spheroid that is axisymmetric around the rotation axis. Above this critical frequency, however, the shape that minimizes the energy for a given angular momentum is a triaxial ellipsoid. Rotation of this ellipsoid will therefore generate gravitational radiation. The critical rotation frequency is approximately 80% of the frequency of the "mass-shedding limit", at which corotating matter is flung away from the star. For neutron stars, the mass-shedding limit is at $\sim 1500 - 2000$ Hz, depending on the mass and equation of state. Therefore, a neutron star rotating at $> 1200 - 1600$ Hz is a potential source of gravitational radiation. No neutron stars are known at frequencies this high; in fact, in 2005 a new record of 716 Hz was set by PSR J1748–2446ad (gotta love the naming convention; this one is in the globular cluster Terzan 5, which has lots of other pulsars, thus the "ad" at the end). Therefore, there are no sources expected to emit gravitational radiation in this way. If there were, the radiation would slow the star down very quickly, so in any case these would be transient sources. It is conceivable that such rapid rotation could be produced in the core collapse that produced the neutron star, in which case the gravitational radiation would have the character of a burst. Indeed, that idea was suggested by Chandrasekhar in 1970, but it turns out that, rather than slowing down as a rigidly rotating ellipsoid, the triaxial star would rapidly develop interior differential rotation, and it seems likely that the amount of emitted gravitational radiation would therefore be small.

Another possibility is that the star has a substantial magnetic field that is misaligned with the rotation axis. The magnetic stresses would produce triaxiality, which would then lead to gravitational radiation. Is there evidence that such misalignment happens? Yes! At the simplest level, rotation-powered pulsars have to be somewhat misaligned, otherwise we wouldn't see pulsations. More recently, another piece of evidence has been uncovered. The pulsar PSR 1828-11 has been shown to precess nonsinusoidally with a period of about 500 days. Various ideas have been proposed, but the most promising appears to be a misaligned magnetic field (Stairs, Lyne, & Shemar 2000). As discussed in detail by Wasserman (2003), steady rotation is only possible if the rotation is along the axis of one of the principal moments of inertia, and for a given angular momentum the lowest energy state is attained when the rotation is around the axis with the largest moment of inertia. If the magnetic field is neither aligned nor orthogonal relative to the rotation axis, then precession can occur for sufficiently strong fields. Over a long time, comparable to or longer than the spindown time, the magnetic axis will presumably drift towards an aligned or orthogonal state (because this is the state of global minimum energy), but in the meantime precession can occur, and with it gravitational radiation can be produced. In fact, if the magnetic axis is orthogonal to rotation then gravitational radiation can be produced even without precession. In addition, an inherently triaxial field will produce gravitational radiation, no matter what the orientation. The precession rate and spin frequency of PSR 1828-11 are, unfortunately, much too low for detection by currently planned instruments.

Burst Sources

The next category of gravitational wave sources is burst sources. These refer to events of very limited duration that do not have to have any special periodicity. Data analysis for these will be very challenging indeed, but since they are by definition associated with violent events, we could potentially learn a great deal from detection of gravitational radiation. Let's consider a few of the more commonly discussed possibilities.

Core-collapse supernovae. When the core of a massive star collapses, it will not do so in a perfectly symmetric fashion. For example, convection will introduce asymmetries. What fraction of the mass-energy will therefore be released as gravitational radiation? This is a question that has to be answered numerically, but it is an extraordinarily challenging problem. Convection is important, so simulations have to be done in three dimensions. Radiation transfer is also essential, as is a good treatment of neutrino transport. To make things even worse, it seems likely that magnetic fields will play a major role, and a wide range of scales could influence each other! Nonetheless, the current best guess is that only a very small fraction of the total mass-energy will come out in gravitational radiation, perhaps $\sim 10^{-10}$. If so, supernovae outside our galaxy will be undetectable. However, the rate of core-collapse supernovae in our Milky Way is estimated to be one per few decades, which means that there is a probability of tens of percent per decade that a supernova will occur within ~ 10 kpc. Current calculations suggest that the strain amplitude at 10 kpc could be $h \sim 10^{-20}$ for a millisecond or so, and maybe 10^{-21} for tens of milliseconds, which would be detectable with advanced ground-based instruments. There have also been proposals that a much higher fraction of energy is emitted during the collapse, which brings us to the next topic.

Gamma-ray bursts. These are short (milliseconds to minutes), high intensity bursts of gamma rays. After a long and interesting history (starting with their detection with US spy satellites!), it has been established that there are two categories of GRBs, the long (tens of seconds or more) and the short (less than a second, typically). The long bursts are convincingly associated with a type of supernova, but the detailed mechanism for their production is uncertain. Some people believe that GRBs are the birth events for rapidly rotating black holes. If so, the rapid rotation could be a path to much more substantial gravitational wave production. That is, as we discussed in the continuous wave section, maybe we could get a triaxial system for a short time. But even these would only be visible to a few tens of megaparsecs, and gamma-ray bursts are much farther than that.

Stochastic Backgrounds

For our last topic, we will focus on stochastic backgrounds, with an emphasis on primordial gravitational waves. To get a handle on these issues, we need to think in terms of broad bands of frequency with many sources, rather than the signal produced by an individual source.

A background due to processes in the early universe (say, before the production of the cosmic microwave background) would be very exciting because it would contain information that is unavailable otherwise. In principle, one could see gravitational waves from very early in the universe, because gravitons have a very small interaction cross section. We need to state clearly that, even by the standards of gravitational wave astronomy, these processes are *highly* speculative. One consequence of this is that although it would be extremely exciting to detect a background of early-universe gravitational radiation, a nondetection would not be surprising. The rest of this part of the lecture is based strongly on notes from Alessandra Buonanno (gr-qc/0303085).

Before examining specific possibilities, let's establish a scaling. Let us define $\Omega_{\rm GW}(f)$ as follows. Suppose that $d\rho_{\rm GW}(f)c^2/df$ is the present-day energy density in gravitational waves of frequencies between f and $f+df$. Let $\rho_c c^2$ be the mass-energy density needed to close the universe. Then

$$\Omega_{\rm GW} \equiv \frac{1}{\rho_c c^2}\frac{d\rho_{\rm GW}(f)c^2}{d\ln f}\ . \tag{6}$$

That is, $\Omega_{\rm GW}$ is the ratio of the present-day energy density in gravitational waves in a logarithmic interval around f to the critical energy density. The current-day critical density is $\rho_c = 3H_0^2/(8\pi G)$, where $H_0 \approx 70$ km s^{-1} Mpc^{-1} is the Hubble constant. We can also define the characteristic gravitational wave amplitude $h_c(f)$ over a logarithmic frequency interval around f; this is related to the one-sided spectral noise density $S_{h,1}(f)$ by $h_c^2(f) = fS_{h,1}(f)$. Dimensional arguments then give

$$\Omega_{\rm GW}(f) \propto H_0^{-2} f^3 S_{h,1}(f)\ . \tag{7}$$

The result of this is that unless in some frequency range $d\ln\Omega_{\rm GW}(f)/d\ln f > 3$, the spectral density decreases with increasing frequency. As a result, for most realistic sources of background, it will be easier to detect the background at lower frequencies. That's why searches in the cosmic microwave background (with effective frequencies $10^{-17} - 10^{-16}$ Hz) are thought to be more promising than searches with ground-based detectors.

What are some specific mechanisms by which gravitons can be generated in the early universe, after the Planck time? The two primary mechanisms that have been explored are production during inflation, and production during a phase transition.

Various models of inflation have been discussed, but one that is considered relatively realistic is slow-roll inflation. In this model, the universe had a scalar field that, at the beginning of the inflationary period, was not at its minimum. The field value "rolls" towards the minimum and as it does so it drives rapid expansion of the universe. The rolling process means that the Hubble parameter is not constant during inflation. Therefore, fluctuations that leave the Hubble volume during inflation and re-enter later have a tilt with respect to other fluctuations. The net result of calculations is that if standard inflation is correct then, unfortunately, there is no hope of detecting a gravitational wave background in the LIGO-India frequency range, because the amplitude is orders of magnitude below what current or planned detectors could achieve. Variants of or substitutes for standard inflation have been proposed that might lead to detectable gravitational radiation, including bouncing-universe scenarios and braneworld ideas, but whether these encounter reality at any point is anyone's guess!

If phase transitions in the early universe (e.g., from a quark-gluon plasma to baryonic matter) are first-order, then by definition some variables are discontinuous at the transition. If the transition occurs in localized regions ("bubbles") in space, collisions between the bubbles could produce gravitational radiation. In addition, turbulent magnetic fields produced by the fluid motion could generate secondary gravitational radiation, but these are weaker. The most optimistic estimates put the contribution at $h_0^2\Omega_{\rm GW} \sim 10^{-10}$, peaking in the millihertz range. This would be detectable with LISA, but don't bet on it.

A more recent suggestion has been that gravitational radiation could be produced by cosmic strings. Cosmic strings, if they exist, are one-dimensional topological defects. Assuming a network of cosmic strings exists, it would have strings of all sizes and therefore contribute gravitational radiation at a wide range of frequencies. Recently, some work has been done on the possibility that cusps or kinks in cosmic strings could produce beams of gravitational radiation.

What limits are there to the overall strength of the gravitational wave background? One comes from Big Bang nucleosynthesis (BBN). This is the very successful model that relates the overall density of baryons in the universe to the abundances of light elements. The idea is that in the first few minutes of the universe, after the temperature had dropped below the level when photodisintegration of nuclei was common but before free neutrons decayed, protons and neutrons could merge to form heavier elements. George Gamow, who originally proposed this, had hoped that this process would explain all the elements in the universe, but the lack of stable elements at mass 5 and mass 8 prevents this. Instead, just the light elements are formed. These include hydrogen, deuterium, helium-3 and helium-4, and trace amounts of lithium and beryllium. The relative abundances of each depend only on the overall entropy of the universe (i.e., ratio of photons to baryons) and the number density of baryons. Measurements of primordial abundances of the light elements are in excellent agreement with the baryon fraction $\Omega_b \approx 0.04$ measured independently from the microwave background.

If the current energy density of gravitational waves were too high, this would mess up BBN. The constraint is

$$\int_{f=0}^{f=\infty} d\ln f h_0^2 \Omega_{\rm GW}(f) < 5\times 10^{-6} , \tag{8}$$

where $h_0 \equiv H_0/100{\rm km~s^{-1}Mpc^{-1}}$. In principle you could imagine that $\Omega_{\rm GW}(f) \gg 10^{-5}$ in some narrow frequency interval, but this seems unlikely. Initial LIGO observations yielded a limit of $\Omega_{\rm GW} < 6.9\times 10^{-6}$ at 100 Hz, which is stronger than the BBN limit in this frequency range.

A constraint at much lower frequencies comes from pulsar timing. The tremendous stability of millisecond pulsars means that we would know if a wave passed between us and the pulsar, because the signal would vary. Roughly 15–20 years of timing have led to the bound

$$\Omega_{\rm GW}(f) \lesssim 10^{-9}(f/f_{\rm PSR})^2 , \qquad f > f_{\rm PSR} \equiv 2.8\times 10^{-9}~{\rm Hz} . \tag{9}$$

Last year there was an announcement from the BICEP2 team that they had detected the signature of gravitational waves from, likely, the inflationary epoch, in polarization patterns in the cosmic microwave background (CMB). The claim was based on the fact that, especially at degree scales and larger, the only source in the early universe that we think can produce polarization patterns with a nonzero curl (i.e., $\nabla\times$) is gravitational waves. See Wayne Hu's outstanding pedagogical pages at http://background.uchicago.edu/∼whu/polar/webversion/node7.html for more details. The team did indeed see these so-called "B mode" polarization patterns, and they interpreted this as a confirmation of a prediction of inflationary theory (which has wide latitude in the amplitude of the predicted spectrum).

However, note that I said that gravitational waves are the only source in the *early* universe that we know can produce B modes. Scattering of light off of dust can perfectly well produce B modes in the current universe. The BICEP2 team thought they had accounted for this by looking at a dust map, but it turns out that this map was incomplete and it is now believed that their signal was dominated by dust effects. Nonetheless, there are many existing or planned experiments that will search for B modes in the CMB in the next few years. A claim that they have detected true primordial gravitational waves rather than dust will hinge on two important checks: (1) multifrequency observations will be needed to distinguish dust from gravitational wave signals, and (2) just as the standard CMB temperature power spectrum has characteristic peaks and dips, so does the predicted B mode power spectrum, so detection of those peaks at the right places, which will necessitate broad angular coverage, will be a critical test of the nature of the signal.

If any of these scenarios comes true and in fact there is a cosmological background of gravitational waves detected with planned instruments, this will obviously be fantastic news. However, what if it isn't seen? That would be disappointing, but there has been discussion about missions to go after weaker backgrounds. It is often thought that the 0.1-1 Hz range is likely to be least "polluted" by foreground vermin (i.e., the rest of the universe!). This may be, but it is worth remembering that there are an enormous number of sources out there in even that frequency range, and that to see orders of magnitude below them will required *extremely* precise modeling of all those sources. Either way, whether we see a background or "merely" detect a large number of other sources, gravitational wave astronomy has wonderful prospects to enlarge our view of the cosmos.