Suggested Tracking Update Rate for CARMA

Marc Pound
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1. Calculations

The CARMA Control subsystem contains an Updater component of the control system responsible for ensuring that the distributed components receive updated information to correctly track sources, including RA/Dec coordinates for moving objects and time corrections such as UT1-UTC. The plan is to fix the update rate at the maximum required for a moving object such as a comet.

To find out what that rate needs to be, I used JPL's Horizons ephemeris generator[1] to calculate rates of change in apparent position and velocity during the closest approaches of comets Halley, Hyakutake and Hale-Bopp as viewed from Cedar Flat ($118^\circ 09' 00.0''$W, $37^\circ 16' 48.0''$N, 7000 ft). I used Horizons to generate tables of α, δ, $\dot\alpha$, $\dot\delta$, AZ, EL, $\dot{AZ}$, $\dot{EL}$, and V_{obs}, the velocity with respect to the observer, at hourly intervals for about 1 month around perigee. From these I calculated the $\ddot\alpha$, $\ddot\delta$, and $\dot V_{obs}$ by differencing successive values. For Hyakutake, I also generated a minute-by-minute ephemeris for a few days near perigee.

Figures 1 through 3 show the generated and calculated quantities for each comet. Note the positional rates varied quite a bit between the comets. Hyakutake had a maximum rate $\dot\alpha_{max}$ = 9600 mas/s; Halley had $\dot\alpha_{max}$ = 300 mas/s; and Hale-Bopp peaked at $\dot\alpha_{max}$ = 100 mas/s. They each have small diurnal variations on top of the longer term changes.

CARMA's required positional accuracy is 1 mas. The question before us is how often do we need to update in order to maintain this accuracy? To answer this, we can expand the position function $X(t)$ in a Taylor series around t':

$$X(t') = X(t) + \dot X\ (t'-t) + \frac{1}{2}\ \ddot X\ (t'-t)^2 + \frac{1}{6}\ \dddot X\ (t'-t)^3 + \cdots \tag{1}$$

Clearly, the answer depends on then how many terms we keep in the equation above. We require that any truncation not result in an error more than 1 mas during the interval between updates, $\Delta T = (t'-t)$. If we stop at the $n = 1$ term (first derivative), then we must update on a timescale such that $\frac{1}{2}\ \ddot X \Delta T^2 < 1$ mas or $\Delta T < \sqrt{2/\ddot X}$, where $\ddot X$ is the fastest changing position. Of the example comets, the most extreme case is the right ascension of Comet Hyakutake, which gives $\Delta T < 4$ s. The other comets motions result in ΔT more than an order of magnitude larger (100 s for Halley, 184 s for Hale-Bopp). If instead, we keep the $n = 2$ term, the required update interval increases to $\Delta T < (6/\dddot X)^{\frac{1}{3}}$, or $\Delta T < 80$ s (see panel D of Figure 2).

According to the Antenna API, the antennas will track by interpolating between triplets of positions. If the interpolation is quadratic, that is equivalent to keeping the 2nd order Taylor term. This would suggest an update rate of 80 s.

The rate of change of V_{obs} determined the rate of change in LO frequency, $\dot\nu_{LO} = (\nu_{LO}/c)\dot V_{obs}$, required to Doppler track the comet. From panels C in the figures one can see that $\dot V_{obs}$ is dominated by diurnal variations due to the component of the earth's rotation in the direction of the comet, with a slow drift over many days from the comet's actual motion. The maximum $\dot\nu_{LO}$ is 27 Hz/s (Hyakutake) at $\nu_{LO} = 270$ GHz.

[1] http://ssd.jpl.nasa.gov/cgi-bin/eph

The suggested API is to pass triplets of velocity and fit similarly to position. So we need to check the update time for ν_{LO} as well. We use the radio convention for the Doppler formula,

$$\nu(doppler) = \nu_0(1 - V_{obs}/c) \quad (2)$$

$$\delta\nu \equiv \nu_0 - \nu(doppler) = (\nu_0/c)V_{obs} \quad (3)$$

Again, we Taylor expand

$$\delta\nu(t') = (\nu_0/c)\ [V_{obs}(t) + \dot{V}_{obs}\ (t'-t) + \frac{1}{2}\ \ddot{V}_{obs}\ (t'-t)^2 + \frac{1}{6}\ \dddot{V}_{obs}\ (t'-t)^3 + \cdots] \quad (4)$$

The required accuracy is $\delta\nu < 1$KHz. From comet Hyakutake, the maximum $\ddot{V}_{obs}$ is 2.0E-9 km s^{-3}, which implies an update time $\Delta T < 1054$ s at ν_{LO} = 270 GHz. At 500 GHz, $\Delta T < 774$ s. This is much longer than the positional update rate and suggests that for velocity a linear interpolation would work fine. However, since we are doing quadratic interpolation for other quantities, its easier to do it for velocity too.

Finally, let's look at UT1-UTC. This value changes by about 1 ms per day, exclusive of leap seconds. This corresponds to a 15 mas per day tracking error for sources at the equator. So updating UT1-UTC once per hour would satisfy the 1 mas position requirement.

2. Summary

Based on the analysis above, the update rate is set by positional accuracy when tracking comets. Based on the passage of Comet Hyakutake, the tracking update interval needs to be no more than 4 s if performing linear interpolation and no more than 80 s if doing quadratic interpolation. The LO update rate is much longer, even at the highest frequency at which we might imagine CARMA will operate.

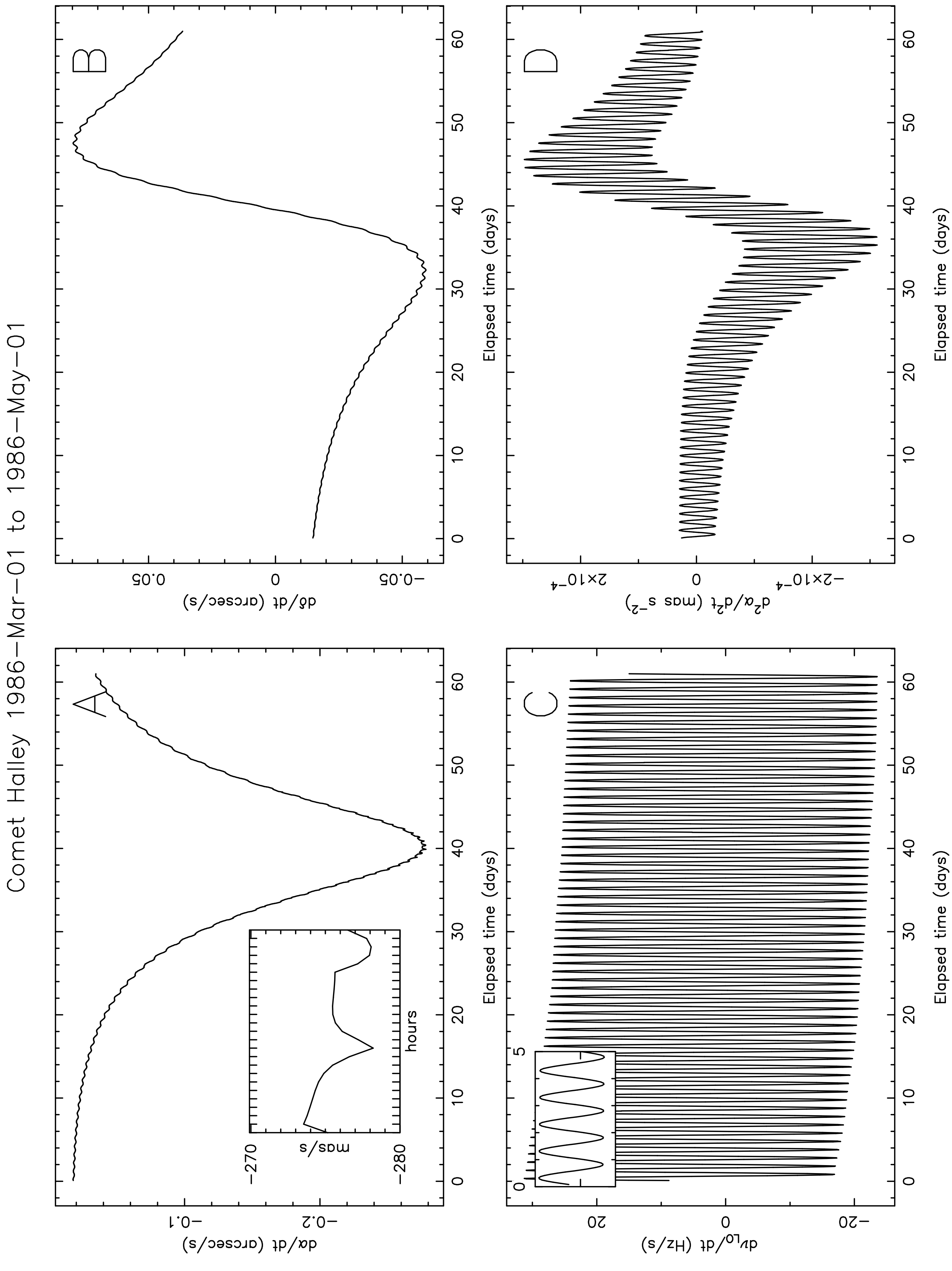

Fig. 1.— Rates for Comet Halley 1986. A) Rate of change of apparent right ascension (inset: expansion of 1 day around perigee in milliarcseconds per second). B) Rate of change of apparent declination. C) Rate of change of LO frequency at ν_{LO} = 270 GHz required to Doppler track the comet (inset: expansion show diurnal variation over 5 days). D) Second derivative of right ascension with respect to time in mas s^{-2}.

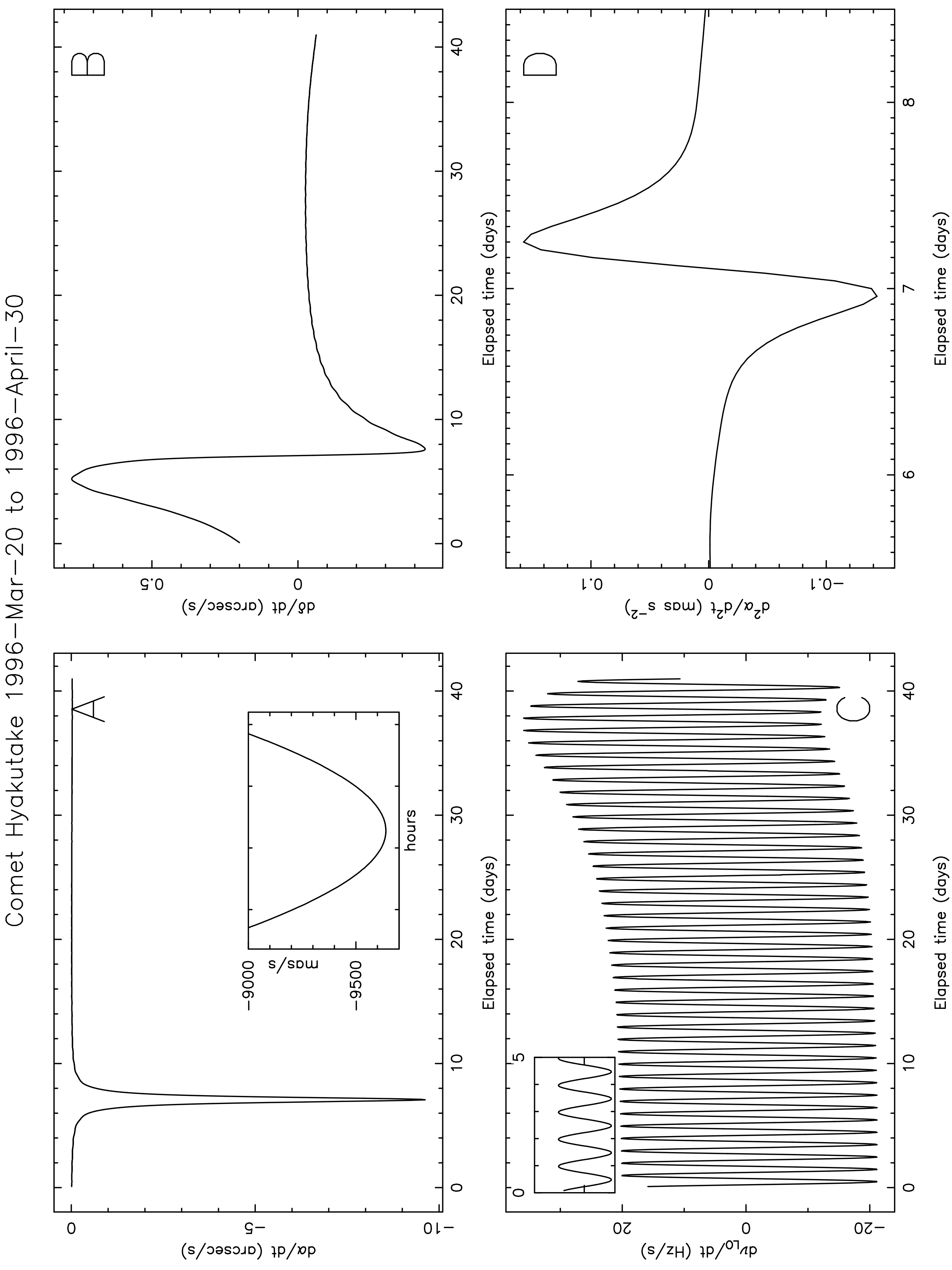

Fig. 2.— Rates for Comet Halley 1986. A) Rate of change of apparent right ascension (inset: expansion of a few hours around perigee in milliarcseconds per second). B) Rate of change of apparent declination. C) Rate of change of LO frequency at ν_{LO} = 270 GHz required to Doppler track the comet (inset: expansion show diurnal variation over 5 days). D) Second derivative of right ascension with respect to time in mas s^{-2}.

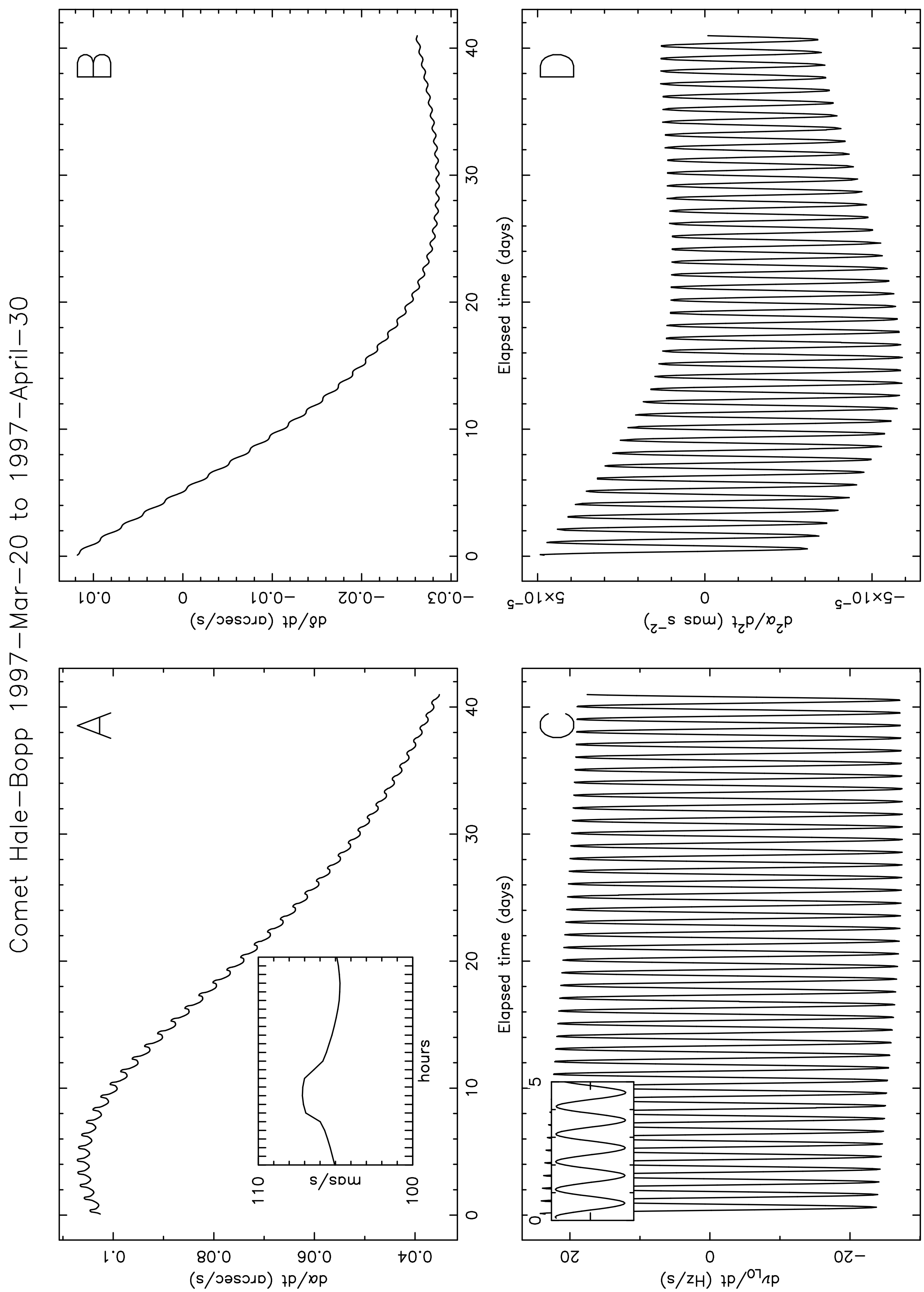

Fig. 3.— Rates for Comet Hale-Bopp 1997. A) Rate of change of apparent right ascension (inset: expansion of 1 day around perigee in milliarcseconds per second). B) Rate of change of apparent declination. C) Rate of change of LO frequency at ν_{LO} = 270 GHz required to Doppler track the comet (inset: expansion show diurnal variation over 5 days). D) Second derivative of right ascension with respect to time in mas s^{-2}.