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Data Rate, S/N & Spectroscopy for the OBSS-A/B/C Concepts

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ABSTRACT

I estimate approximate signal-to-noise ratios as a function of apparent magnitude for the OBSS-A, OBSS-B and OBSS-C mission concepts. A "traditional" spectrograph setup is studied. The consequences for velocity accuracy and data rate are also addressed.

1. Introduction

To first order, the accuracy to which one can determine radial velocities from noisy stellar spectra depends on the number of photons collected. Similar scaling relations hold for the attainable astrometric accuracy: the final accuracy scale with the (inverse-square-root of the) total number of photons collected during the mission.

This first-order truth is reflected in the scaling relation valid for the quest for extra-solar planet via radial velocities. For resolutions (R) less than 50,000 the results from Marcy & Butler *et al.* (1996, PASP, 108, 500) may be rewritten (Hajian, private communications, 2000-2004) as:

$$\delta V^{MB} \approx f_\lambda \times \left(\frac{0.4}{S/N}\right) \times \left(\frac{50,000}{R}\right) \times \sqrt{\frac{90}{\Delta\lambda}} \qquad \text{[km/s]} \ , \quad (1)$$

where S/N is the signal-to-noise ratio per resolution element, $\Delta\lambda$ the wavelength range in nanometers, and f_λ is a "constant" of order unity that depends on the density of spectral lines ($f_\lambda \sim 1$ for the 500–590 nm range). This scaling relation makes intuitive sense: at constant S/N, better precision is obtained if more resolution elements are used, which can be obtained by increasing R or $\Delta\lambda$. Note that the $1/\sqrt{\Delta\lambda}$-scaling amounts to a dependence on the total number of photo-electrons (i.e., $\sqrt{\Delta\lambda} \propto \sqrt{N_{photo}}$). Also, the inverse dependence on S/N is as expected: at constant R and $\Delta\lambda$, higher signal-to-noise yield smaller velocity errors.

The linear dependence on the total number of photons is obtained in the *low* S/N regime. When the noise is independent of the signal, *in*creasing the resolution by a factor η *de*creases the number of photons per resolution element by a factor η, so that $S \times R$, and hence δV, is preserved.

In the *high* S/N regime ($S/N = \sqrt{S}$) , δV is given by:

$$\delta V^{MB}(\eta) \;\approx\; f_\lambda \times \left(\frac{0.4}{\sqrt{S/\eta}}\right) \times \left(\frac{50,000}{\eta R}\right) \times \sqrt{\frac{90}{\Delta\lambda}} \;=\; \frac{\delta V(\eta = 1)}{\sqrt{\eta}} \ , \tag{2}$$

so that higher resolution ($\eta > 1$) leads to better velocity accuracy. Thus, for a given wavelength range, the obtainable δV is:

- *independent* of the instrumental resolution in the low S/N regime
- scales with the inverse square-root of the resolution in the high S/N regime.

The data rate scales with the number of pixels to be transmitted, and is hence linearly dependent on the instrumental resolution.

To conclude, to first order, velocity accuracy is independent of resolution, while data rate depends linearly on resolution. Thus, to first order, a low-resolution instrument is to be preferred since it minimizes the data rate.

Below, I will show that the OBSS spectrometer is expected to be operated mostly in the low S/N regime. Thus the question *du ans* is: "What is the minimal resolution at which believable velocities can be determined (to 10 km/s)."

1.1. GAIA Performance

Munari *et al.* (2003, GAIA, Spectroscopy, Science & Technology, ASP, Conf. Series, V. 298, p. 283, table 4) give an equation for the achievable velocity accuracy in the near-IR CaII region, but the results in their table 4 seem to work better over a wider range of S/N. I find:

$$\delta V^G \;\approx\; 10^{\left(1.612 - 1.275 \log S/N' + 0.164 (\log S/N')^2 + 1.167 \log D_A\right)} \times \sqrt{\frac{26.5}{\Delta\lambda}} \ , \qquad \text{[km/s]} \tag{3}$$

where D_A is the spectroscopic dispersion in Å per pixel, and S/N' is the signal-to-noise ratio *per pixel.* Here I have added the ($\Delta\lambda/26.5$)-term to include any wavelength-range scaling. The relation (3) was obtained from ground-based spectra of real stars in the region of the Near-IR CaII triplet (8480–8745 Å). Note that it is likely that the D_A term depends on the "average width" of the spectral lines, and is thus likely wavelength dependent.

1.2. GAIA Versus M&B

Comparing the Marcy & Butler results with the GAIA accuracies, I find that the relations overlap reasonably well when multiplying eqn (2) by a factor that depends on the dispersion per pixel:

$$\delta V^{MBG} \approx f_\lambda \times \left(\frac{0.4}{S/N}\right) \times \left(\frac{50,000}{R}\right) \times \sqrt{\frac{90}{\Delta\lambda}} \times \sqrt{0.862 + 1.415 D_A}\ , \qquad \text{[km/s]} \quad (4)$$

where the D_A-term is of order 1.5, for $D_A = 1$ Å per pixel. For the near-IR CaII triplet, $f_\lambda \sim 1.84$, while f_λ decreases quickly towards the blue as the number of lines increase substantially towards shorter wavelengths. I present a comparison between the GAIA and "Marcy/Butler/GAIA" results in figure 1 below. The difference between the two relations is reasonably small, except at low and high signal-to-noise ratios.

2. Signal-to-Noise Ratios

Since the velocity accuracy depends so strongly upon the signal-to-noise ratio, it is imperative to estimate the expected values, and the resultant δV. As a baseline, I use the design of the OBSS/A mission as proposed to the JPL/Origins Roadmap program. Relevant parameters and assumptions are summarized in Table 1 below.

Using the design parameters for OBSS/A as specified in Table 1, I derive the signal-to-noise ratio's, and the corresponding velocity accuracies listed in Table 2. Here I have used equation (4) with $f_\lambda = 1$. Similarly, the results for OBSS/B are presented in Table 3.

3. Unknowns

The results presented above are valid in the *high* signal-to-noise regime, with $S/N \gtrsim 1$. However, the expected S/N for OBSS/A is of order 0.05 per detection (0.2 per epoch) at $V = 15$. However, the results obtained in section 2 are likely to be rather uncertain for several reasons:

- Extrapolation of eqn. (4) by a factor 5 to 20 is risky
- Can exposures to be added with sufficient accuracy to achieve the S/N gain in accordance with $\sqrt{N_{Epoch}}$ and/or $\sqrt{N_{Mission}}$?
- Can the zero-point of each noisy spectrum be known to sufficient accuracy for the co-adds to work? The accuracy needs to be less than about $1/3^{rd}$ of the required accuracy, or 3 km/s or $1/12^{th}$ of a pixel.
- More

All these issues need be addressed to evaluate the viability of co-adding noisy spectra.

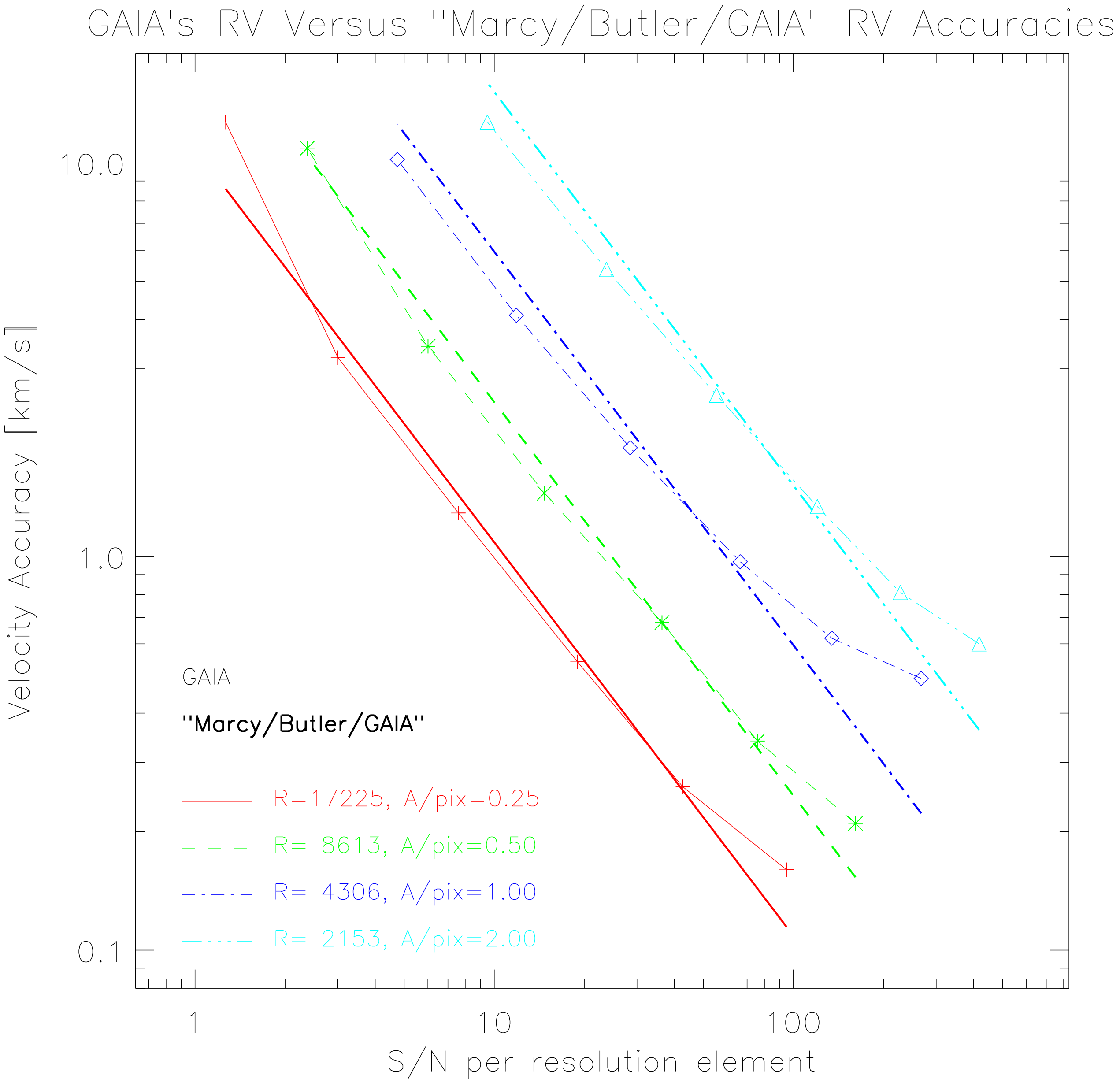

Fig. 1.— The achievable radial velocity errors as a function of signal-to-noise ratio. The thin lines hold for the GAIA results [eqn. (3)], the thick lines for the "Marcy/Butler/GAIA" relation [eqn. (4) with $f_\lambda = 1.84$]. The difference between the two relations is reasonably small, except at low and high signal-to-noise ratios.

Table 1. Instrument parameters for the “classical” OBSS-A, OBSS-B and OBSS-C spectroscopic instruments. Parameters are “strawman” values.

Parameter	OBSS/A	OBSS/B	OBSS/C	unit
Mirror Size (I-scan)	1.50	1.50	2.00	meter
Mirror Size (X-scan)	0.50	1.50	2.00	meter
Mirror Area	0.75	2.25	4.00	m^2
Astrometric Throughput	0.60	0.60	0.60	
Integration Time	0.70	10	30	sec
Astro Band	400–1000	400-1000	400–1000	nm
Band width	600	600	600	nm
$N^{Astro}_{photo-elec}(V=9)$	$3.4\ 10^6$	times 42.9	times 229	A0V star
Wavelength range	450–600	350–900	350–900	nm
$\Delta\lambda$	150	550	550	nm
Resolution	3,000	5,000	10,000	
Pixels per R.element	2.5	2.5	2.5	
Pixels per spectrum	2,143	times 5.1	times 10.2	(1D)
R.elements per spectrum	857	4,371	8,743	(1D)
Spectromter QE*TP	0.54	0.54	0.20	
CCD size	4,096 x 670	8,192 x 4,096	8,192 x 4,096	pixels
Read Rate	1,950	2,237	2,237	kHz
Noise floor per R.element	26.5	27.7	27.7	e$^-$ (1D)
$N^{Spectro}_{photo-elec}(V=9)$	887.3	times 30.8	times 82.3	e$^-$ per resolution element
$(V = 9)$	887.3	27,329	72,202	
$(V = 12)$	56.0	1,725	4,609	
$(V = 14)$	8.9	274	732	
$(V = 15)$	3.5	108	288	
$(V = 16)$	1.4	43	115	
$(V = 18)$	0.2	6	16	
$(V = 20)$	0.0	1	3	
Limiting Magnitude	15	18	20	V-band
#Stars to V$_{Lim}$	40	320	1,280	million
#Spec. Observations	2,207	46	10	Total
#Spec. Observations	18	1.7	1	Per Epoch
Size of 1D spectrum	34.2	205	510	kBit (16 bit/pixel)
Average Downlink rate	19.2	19.2	41.5	MBit/sec (5 yr mission)

Table 2. Expected photon-count, noise levels, S/N and velocity accuracies [$\mathrm{km\,s^{-1}}$] for the OBSS/A mission.

V	$\frac{N_{electrons}}{ResolutionElement}$	$\frac{Noise}{Detection}$	$\frac{S/N}{Detection}$	$\frac{\delta V}{Detection}$	$\frac{S/N}{Epoch}$	$\frac{\delta V}{Epoch}$	$\frac{S/N}{Final}$	$\frac{\delta V}{MissionEnd}$
9	877.2	39.8	22.27	0.3	94.8	0.1	1046.2	0.00
12	56.0	27.5	2.04	3.5	8.7	0.8	95.6	0.07
14	8.9	26.6	0.33	21.1	1.4	4.9	15.7	0.45
15	**3.5**	**26.5**	**0.13**	**52.8**	**0.6**	***12.4***	**6.3**	**1.12**
16	1.4	26.5	0.05	132.4	0.2	31.1	2.5	2.82
18	0.2	26.5	0.01	835.0	0.0	196.0	0.4	17.80
20	0.0	26.5	0.00	5267.0	0.0	1237.0	0.1	795.00

Table 3. Expected photon-count, noise levels, S/N and velocity accuracies [$\mathrm{km\,s^{-1}}$] for the OBSS/B mission.

V	$\frac{N_{electrons}}{ResolutionElement}$	$\frac{Noise}{Detection}$	$\frac{S/N}{Detection}$	$\frac{\delta V}{Detection}$	$\frac{S/N}{Epoch}$	$\frac{\delta V}{Epoch}$	$\frac{S/N}{Final}$	$\frac{\delta V}{MissionEnd}$
9	28,835	172.1	167.0	0.0	216.4	0.0	737.8	0.00
12	1,819	50.9	35.7	0.1	46.2	0.0	157.4	0.01
14	288	32.5	8.9	0.2	11.4	0.2	39.0	0.05
15	**115**	**29.7**	**3.9**	**0.5**	**5.0**	***0.4***	**17.0**	**0.16**
16	46	28.6	1.6	1.3	2.1	1.0	7.0	0.29
18	7	27.8	0.3	7.8	0.3	6.0	1.1	1.77
20	1	27.8	0.0	49.1	0.1	38.0	0.2	11.14