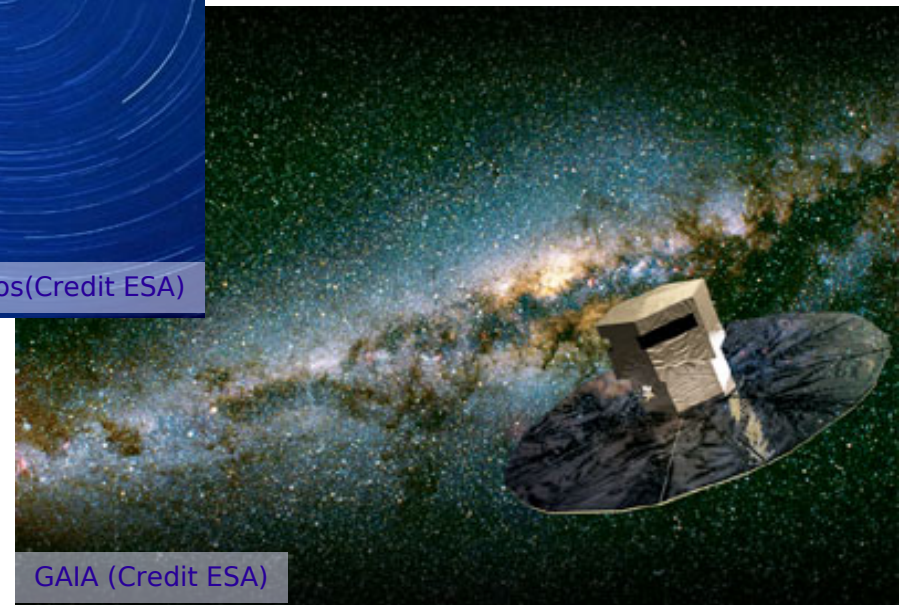
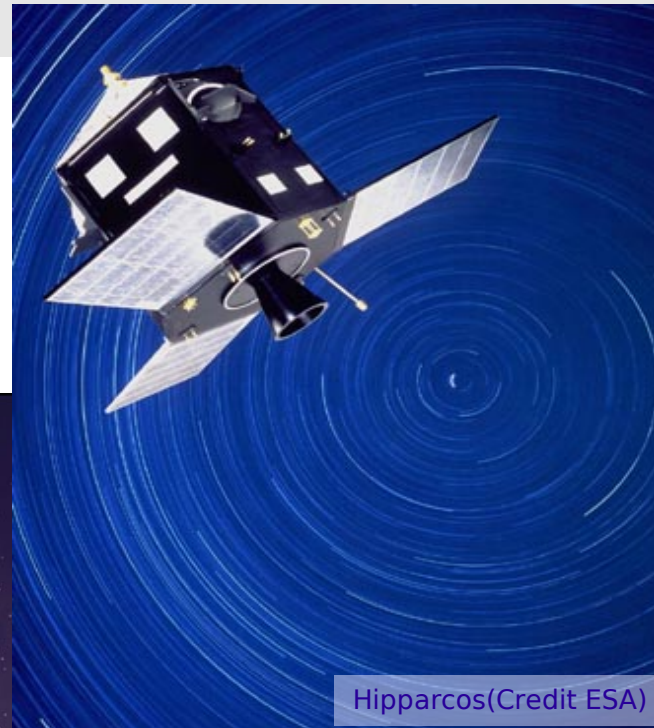


# Astrometry, Precision Astrophysics, $H_0$ & (some) Cosmology

A Connection between Stars, Galaxies and the Universe

Rob Olling (UMd)



# Outline

## 1) Astrometric Missions

## 2) Future of Astrophysics: ✓

- Precision & Accuracy
- Astrometry & Stellar Ages
- Astrometry & Cosmology

## 3) $H_0$ , the CMB & Dark Energy ✓

## 4) Calibrating the Distance Scale

## 5) Rotational Parallax ✓

## 6) Conclusions

## 7) GAIA, SIM & ~~DARWIN~~/TPF-I

Most of this talk is based on a paper earlier this year [Olling, 2007, MNRAS, 378, 1385] & a contribution to the SIM review paper

OR: [http://www.astro.umd.edu/~olling/Papers/RP\\_H0\\_2007\\_Colloquium.pdf](http://www.astro.umd.edu/~olling/Papers/RP_H0_2007_Colloquium.pdf)

# Setting Some Scales

## Parallaxes, in $\mu\text{as}$

- $\alpha$  Cen: 742,000
- RR Lyra: 4,380
- $\delta$  Cep: 3,320
- 
- 1 kpc: 1,000
- Gal. Center: 125
- LMC: 20
- M 31: 1.5

## Proper Motions, in $\mu\text{as}/\text{yr}$

- $\alpha$  Cen: 3,600,000
- RR Lyra: 200,000
- $\delta$  Cep: 16,500
- 
- 10 km/s @ 1 kpc: 2,110
- 200 km/s @ 8 kpc: 5,275
- 50 km/s @ LMC: 211
- 200 km/s @ M 31: 60

**USA @ 10 pc** **2.9** ; **2  $M_{\text{EARTH}}$  @ 10 pc: 1  $\mu\text{as}/\text{yr}$**

# Some Astrometric Missions

- Hipparcos:  $\Delta\pi \leq 1000 \mu\text{as}$  ( $V \leq 7$ );  $\Delta\pi \leq 3000 \mu\text{as}$  ( $V \leq 12$ )
- Twice better calibration of systematics available [van Leeuwen, 2007]
- GAIA:  $\Delta\pi \leq 7 \mu\text{as}$  ( $V \leq 11$ );  $\Delta\pi \leq 230 \mu\text{as}$  ( $V \leq 20$ )
- Accuracy is at MISSION END for  $\sim 400$  Observations per coordinate
- Spatial variation by  $\pm 50\%$  due to scanning law
- Observations are split in  $\sim 36$  epochs of  $\sim 3.5$  hr, with 22 measures  $\rightarrow 17 \mu\text{as/epoch}$
- GAIA saturates at  $V \sim 11$ : excess charge is dumped (anti-blooming drain, TDI gating)
- Saturation/dumping ( $\geq 300,000 e^-$ ) implies best possible accuracy of  $4 \mu\text{as}$
- These are special effects that are not calibrated as well as for stars with  $V \geq 11$
- SIM:  $\Delta x \sim 1 \mu\text{as}$  ( $V \leq 6$ );  $\Delta\pi \leq 5 \mu\text{as}$  ( $V \leq 20$ )
- OBSS: Origins Billion Star Survey
- Goal, Survey:  $V$ : [9,20 ],  $\Delta\pi$ : [15, 100 ]  $\mu\text{as}$
- Pointed:  $V$ : [9,20,24],  $\Delta\pi$ : [15, 15, 100]  $\mu\text{as}$
- 16-channel photometry [320, 1100] nm
- Status: Possible Origins Probe: Ready to go ahead (after  $\sim 2014$ )

# Some Comments on the Future of Astrophysics & Astrometry

- Astrophysics is slowly transitioning from:  
Exploration                      to                      Understanding

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# Some Comments on the Future of Astrophysics & Astrometry

- from **“Astronomy”** to **“Physics”**
  - from Precision  $\Rightarrow$  Accuracy
  - from Model-Dependent  $\Rightarrow$  “True”

- **The Universe is finite:**

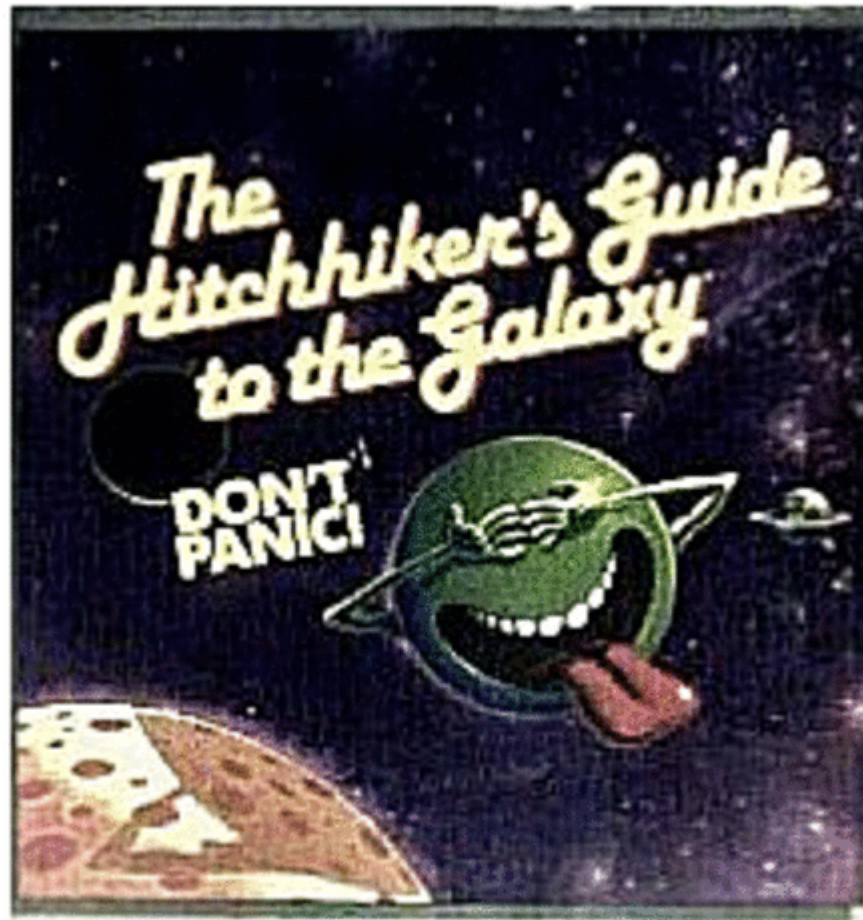
*Eventually there will be nothing new to explore, But plenty to discover (laboratory physics)*

JWST looks back to the 1<sup>st</sup> stars: in a decade or so we will have “seen it all”

- 
- **If this is scary ....**



# Don't Panic !



# Stellar Ages & Astrometry

- Astrophysics of stars is primarily based on:  
**THE SUN**
  - Fundamental parameters well determined:  
**Mass, Radius, Luminosity, He-abundance (Y), [Fe/H]**
  - ~100 Binaries with M and R better than 1%
    - but see: Kurucz' "**Some things we do not know about stars**" (2002nqsa.conf....3K)
- **M, R, [Fe/H] and Y “set” the rate of evolution**
  - Precise Age Determination of Individual Stars
- **⇒ Detailed Formation History of Galaxy**
  - Star Formation + Oldest Stars (< age of Universe?)
  - **⇒ Galaxy Formation & Cosmology**

# Stellar Ages & Astrometry (cntd)

## • Rate of Evolution

- Luminosity:  $(\Delta L/L)^{\text{theory}} \sim (10 + 2 L/L_{\odot}) \pm 5$  [% / Gyr]

Age:  $\Delta\tau^{\text{theory}} = (\Delta L/L)^{\text{obs}} / (0.1 + 0.02 L/L_{\odot})$  [Gyr]

$= (2\Delta\pi/\pi)^{\text{obs}} / (0.1 + 0.02 L/L_{\odot})$  [Gyr]

• **Mid G-type stars :  $(\Delta L/L) \sim 10$  %/Gyr**

• **Hipparcos:  $(\Delta\pi/\pi)^{\text{obs}} \sim (1 \text{ mas} / 100 \text{ pc}) \Rightarrow \Delta\tau^{\text{theory}} \sim 1,800 \text{ Myr}$**

• **SIM :  $(\Delta\pi/\pi)^{\text{obs}} \sim (5 \mu\text{as} / 100 \text{ pc}) \Rightarrow \Delta\tau^{\text{theory}} \sim 9 \text{ Myr}$**

**SIM @ 1 kpc  $\Rightarrow \Delta\tau^{\text{theory}} \sim 90 \text{ Myr}$**

**SIM @ 5 kpc  $\Rightarrow \Delta\tau^{\text{theory}} \sim 450 \text{ Myr}$**

**SIM @ 10 kpc  $\Rightarrow \Delta\tau^{\text{theory}} \sim 900 \text{ Myr}$**

• **Ages are model-dependent**

**Will be calibrated with highly accurate GAIA/SIM,  
Seismology & Ground-Based data**

# Stellar Ages & Astrometry (cntd)

- Radio Example: NRAO Press Release, Oct 8<sup>th</sup>
- “VLBA Changes Picture of Famous Star-Forming Region” -- the Orion Nebula --
  - Based on work by Bower, Sandstrom, Peek, Bolatto and Plambeck [2007, ApJ, Oct 10]
  - "Knowing the **accurate distance to this region is vitally important** to properly understanding ... star-formation processes there," Sandstrom said.
  - The new [VLBA] distance to the region ... is 1270 light-years, compared with the best previous measurement of 1565 light-years.
  - Because the ... **distance to the region is 20 percent closer** than the earlier measurement, **the stars ... are intrinsically fainter by a factor of 1.5.** This has a major impact ... their ages. **"These stars are nearly twice as old as previously thought,"** said Bower.

# Stellar Ages: GAIA and SIM

- **NEED RADIUS**  $\Rightarrow$  **Eclipsing Binaries** ( $\sim 1\%$  of Population)

- Photometry  $\Rightarrow R_*$ ,  $m_V$ ,  $A_V$
- Spectroscopy  $\Rightarrow V_{\text{ORB}}$ ,  $M_*$  &  $[\text{Fe}/\text{H}]$
- Astrometry ( $\pi$ ) +  $m_V$  &  $A_V \Rightarrow$  **Luminosity**

## **TWO stars on same Isochrone $\Rightarrow$ Age & Helium**

[ Ribas (2006ASPC..349...55R), Lebreton (2005tdug.conf..493L),  
Lastennet (2002A&A...396..551L); Metcalfe et al (2006ASPC..349...55R) ]

- **GAIA** is survey mission will determine overall SF History  
 $\leq 600$  pc has  $\Delta\pi/\pi \leq 1\%$  for G7 star ( $V \sim 14.5$ )
  - $\sim 6,254,000$  thin-disk stars  $\Rightarrow \sim 190,000$  EBs  $\Rightarrow \Delta\tau \sim 4$  Myr
  - $\sim 385,000$  thick-disk stars  $\Rightarrow \sim 12,000$  EBs  $\Rightarrow \Delta\tau \sim 5$  Myr
  - $\sim 12,000$  spheroid stars  $\Rightarrow \sim 300$  Ebs  $\Rightarrow \Delta\tau \sim 9$  Myr

- **SIM** should do the rare **Special Cases** at larger distances  
Binary cousins of **Old Uranium Stars** with  $[\text{Fe}/\text{H}] \sim < -3$

$\tau \sim 13.2 \pm 2.7$  Gyr HE 1523-0901,  $d \sim 1$  kpc,  $V \sim 11$  [Frebel et al 2007]

$\tau \sim 14.9 \pm 3.0$  Gyr CS 22892-0529,  $d \sim 1.5$  kpc,  $V \sim 12$  [Snedden et al 1996; Hill et al 2002]

$\tau \sim 13.2 \pm 2.7$  Gyr HE 1424-0241,  $d \sim 8$  kpc,  $V \sim 14$  [Cohen et al 2007]

[See Beers & Christlieb 2004, ARA&A for a review]

# Astrometry & Cosmology

- CMB, high-z galaxy data, Ly- $\alpha$  forest & BBN yield:

- WMAP & Other yield:

Hubble Constant	= $H_0$	= 71	$\pm 2$	$\pm 7$	[km/s/Mpc]
Age	= $t_0$	= 13.7	$\pm 0.2$		[Gyr]
Matter Density	= $\Omega_m$	= 0.27	$\pm 0.02$		[ $\rho_{\text{CRIT}}$ ]
Total/Baryon Matter	= $\Omega_m / \Omega_b$	= 6.1	$\pm 1.1$		
Primordial Helium	= $Y_p$	= 0.2482	$\pm 0.0004$		

- **Astrometry** of *M31, M33*  $\Rightarrow$  strong limits on  $H_0$
- **Astrometry** of *Galactic Objects* can set relevant limits on  $t_0$ ,  $Y_p$  and Star Formation History

[Spergel *et al*, 2003, 2006; Freedman *et al* 2001; Mathews *et al*, 2005; Madau *et al*, 1996, this talk]

# H<sub>0</sub>, the CMB & Dark Energy

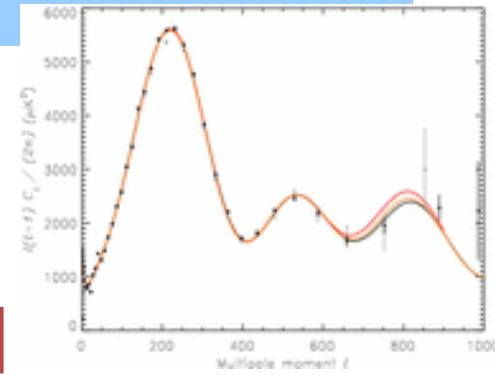
- From the shape of the power spectrum, WMAP “directly” [e.g., Hu 2005] measures the:

- *photon-to-baryon ratio*  $[R_* \sim \frac{\omega_b}{0.0223} \times \frac{1089}{1+z_*}]$

controls amplitude of odd/even peaks

- *radiation-to-matter ratio*  $[r_* \sim \frac{0.126}{\omega_m} \times \frac{1+z_*}{1089}]$

controls amplitude of envelope



- $z_*$  is known *redshift of recombination*

(weakly cosmology dependent)

- $\omega_b = \Omega_b h^2$  is the *physical baryon density*

[g/cm<sup>3</sup>]

- $\omega_m = \Omega_m h^2$  is the *physical matter density*

[g/cm<sup>3</sup>]

- $h$  is  $H_0 / 100$

$$\Omega_b = \frac{\rho_b}{\rho_{crit}} \quad \text{and} \quad \Omega_m = \frac{\rho_b + \rho_{DM}}{\rho_{crit}}$$

[km/s/Mpc]

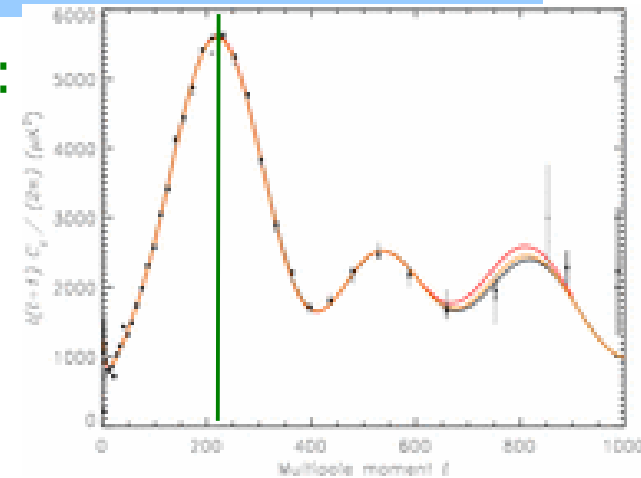
- $\rho_{crit} = 3 H_0^2 / (8\pi G)$  is the *critical density of Universe*

# $H_0$ , CMB and Dark Energy (cntd)

- WMAP yields: location ( $l_A$ ) of the acoustic peak:

$$l_A = D_A \pi / s_* \quad (\pm < 1\%)$$

- Cosmology yields:
  - size of the acoustic oscillation



$$s_* \approx 140 \left( \frac{R_*}{0.854} \right)^{-0.252} \left( \frac{r_*}{0.338} \right)^{-0.083} \quad (\pm 1\%) \quad [\text{Mpc}] \quad l \rightarrow$$

- the angular-size distance ( $D_A$ ) relation

$$D_A = a_* \int_{a_*}^1 \frac{1}{a^2 H(a)} da \quad \text{with} \quad \frac{H(a)}{100} = \sqrt{\frac{\omega_m}{a^3} + \frac{h^2 - \omega_m}{a^{3(1+w)}}} \quad \text{and} \quad \Omega_{tot} = 1$$

where:  $a = 1/(1+z)$  the scale factor,

An assumed flat Universe ( $\Omega_{TOT} = \Omega_\Lambda + \Omega_m = 1$ )

$\Omega_\Lambda$  the Dark Energy, and

“ $w$ ” the “Equation of State” (EOS) of Dark Energy



# $H_0$ , CMB and Dark Energy (cntd)

- IF the Cosmological Constant is the Dark Energy, THEN  $w=-1$ 
  - The DE-candidates have different  $w$ 's:
    - (strings=-1/3; domain walls -2/3, ...) or are variable

- For a flat Universe, current constraints on  $\omega_m$  and  $H_0$  yield:

$$\Omega_\Lambda = 1 = \Omega_m = 1 - \frac{\omega_m}{h^2} \approx \mathbf{0.770 \pm 0.022}$$

- Alternatively: the relation for the location of the acoustic peak yields:

$$D_A = \frac{l_A s_*}{\pi} = \mathbf{100} a_* \int_{a_*}^1 \left( \frac{\omega_m}{a^{-1}} + \frac{h^2 - \omega_m}{a^{3w-1}} \right)^{-1/2} da$$

- The observables are:  $l_A$ ,  $s_*$  and  $\omega_m$
- Assuming a cosmological constant ( $w=-1$ ), then: **unknown is  $h$** 
  - **$\Rightarrow$  the 1 unknown,  $h$  ( $H_0$ )**

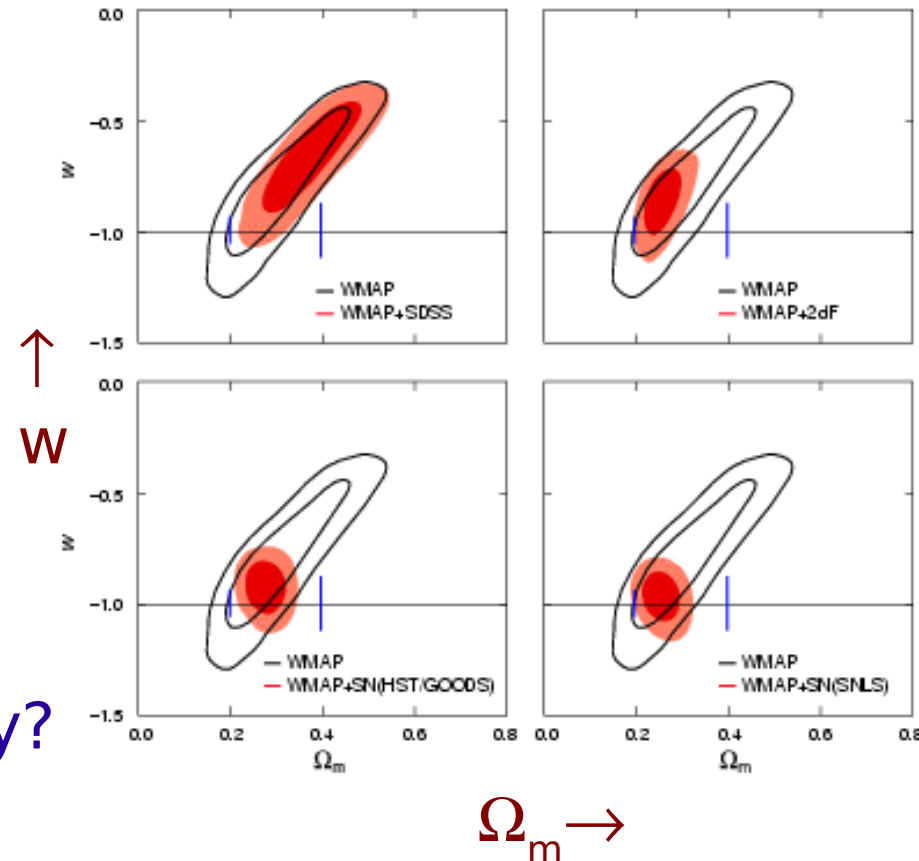
# $H_0$ , CMB and Dark Energy (cntd)

- IF one wants to determine  $w$ , THEN one needs to know  $h$ :
- A fit to the current WMAP data [Spergel et al 2006] yields:

$$-w \approx 1.59 - 2.56 \Omega_m$$

- Where  $\Omega_m$  is unknown, but can be determined from:
  - Large-Scale Structure [SDSS]
  - Large-Scale Structure [2dF]
  - Luminosity-distance( $z$ ) [SN: HST/GOODS]
  - Luminosity-distance( $z$ ) [SN: SNLS]
- But why not from  $\omega_b$  and  $H_0$  directly? currently:

$$-w \approx 1.59 - 2.56 \frac{\omega_m}{h^2} \approx 0.985 \quad (\pm 7\%)$$



# $H_0$ , CMB and Dark Energy (cntd)

- That is to say: EOS of Dark Energy is known with a **precision of ~7%**

$$\epsilon_w \approx 2.56 \frac{\omega_m}{h^2} \sqrt{\left(\frac{\epsilon_{\omega_m}}{\omega_m}\right)^2 + \left(2 \frac{\epsilon_h}{h}\right)^2}$$

- And Dark Energy follows directly from:
  - Better (x 8) determination of  $\omega_m$  with *PLANCK*,
  - Better (x10) determination of  $H_0$  (with *SIM?*)
- However, this is ***not very accurate***. The Assumptions were:
  - Flat Universe
  - Dark Energy has constant EOS
  - Dark Matter does not cluster, no tensor modes, no quintessence, no running spectral index, no strings, no domain walls, no non-Gaussian fluctuation, no deviations from GR, et cetera

# $H_0$ , CMB and Dark Energy (cntd)

- Allowing for a variable EOS of Dark Energy, Hu (2005) concludes that: "... the Hubble constant is the single most useful complement to CMB parameters for dark energy studies ... [if  $H_0(z)$  is] ... accurate to the present level ... ."
- Basically due to the angular-size distance relation:

$$D_A = \frac{l_A s_*}{\pi} = \frac{1}{\chi} F \left( \frac{\chi a_*}{100} \int_{a_*}^1 \frac{1}{a^2 h(a)} da \right) \text{ and } h^2(a) = \frac{\omega_v}{a^4} + \frac{\omega_m}{a^3} + \frac{\omega_K}{a^2} + \frac{\omega_\Lambda}{e^{3 \int [1+w(a)] d \ln(a)}}$$

where F is a function that depends on the curvature of the Universe with  $\chi = 1/(H_0 |\Omega_K|^{1/2})$   
[e.g.  $F(y) = y$  for  $\Omega_K = 0$ ; e.g., Carroll 2001]

# Alternatively, one can (try to) determine the **ages**:

$$- \tau = \int_0^1 da / [a H(a)] \Leftrightarrow \text{ages of oldest stars}$$

$$- \tau(z) = \int_0^{a(z)} da / [a H(a)] \Leftrightarrow \text{ages of high-z galaxies}$$

[e.g., Bothum et al 2006, Jimenez et al 2003, Simon 2005]

Summarized in Figure 4 of Spergel et al, 2004

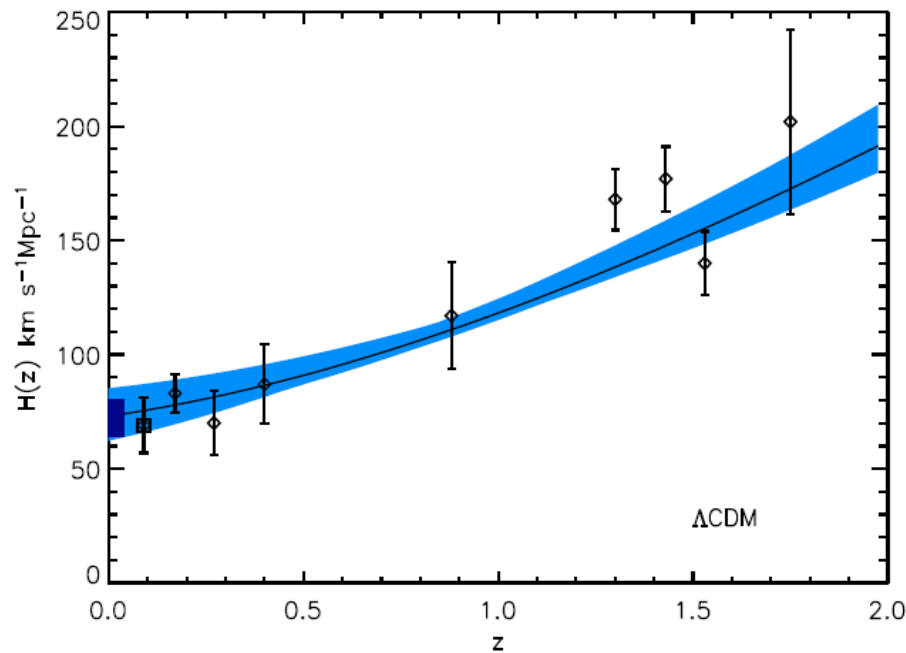
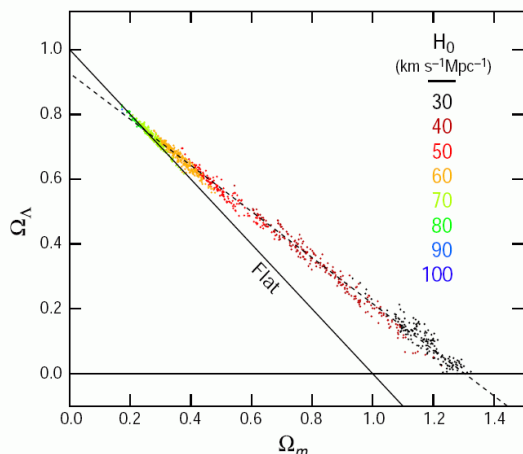


Fig. 4.— The  $\Lambda$ CDM model fit to the WMAP data predicts the Hubble parameter redshift relation. The blue band shows the 68% confidence interval for the Hubble parameter,  $H$ . The dark blue rectangle shows the HST key project estimate for  $H_0$  and its uncertainties (Freedman et al. 2001). The other points are from measurements of the differential ages of galaxies, based on fits of synthetic stellar population models to galaxy spectroscopy. The squares show values from Jimenez et al. (2003) analyses of SDSS galaxies. The diamonds show values from Simon et al. (2005) analysis of a high redshift sample of red galaxies.

# $H_0$ , CMB and Dark Energy (cntd)

- Many groups pursue other methods to determine some (combination of) parameter(s) that constrain the “integral”
  - **Luminosity-Distance relation from Supernovae Ia**
    - $D_L(z) = D_A(z) / a(z)^2$
  - **Baryon Oscillations (sensitive to local galaxy density)**
    - $Volume(z) = [D_A(z) / a(z)]^2 / H(z) * \Omega_{sky} \Delta z$
  - **Galaxy Cluster Abundance**
    - Depends on Volume(z) and non-linear structure growth
  - **Weak Lensing**
    - Depends on:  $D_A(z)$ ,  $H(z)$  and structure growth
    - [e.g., Albrecht et al 2006 = DETF]

# H<sub>0</sub> , CMB and Dark Energy (cntd)



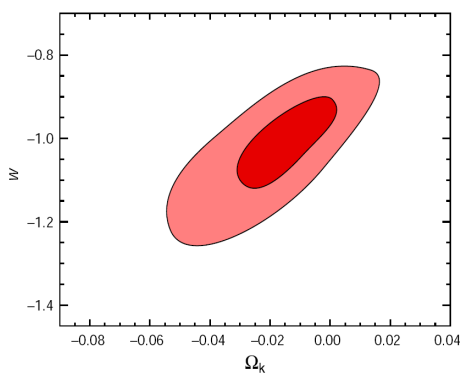
- We use the Spergel et al (2006/7) WMAP & “other data” to approximate the relations between the various parameters ( $P_i = a_{ij} + b_{ij} P_j$ ):

$$\Omega_\Lambda = a_{\Lambda m} + b_{\Lambda m} \Omega_m$$

$$\Omega_K = a_{K\Lambda} + b_{K\Lambda} \Omega_\Lambda$$

$$w = a_{wK} + b_{wK} \Omega_K$$

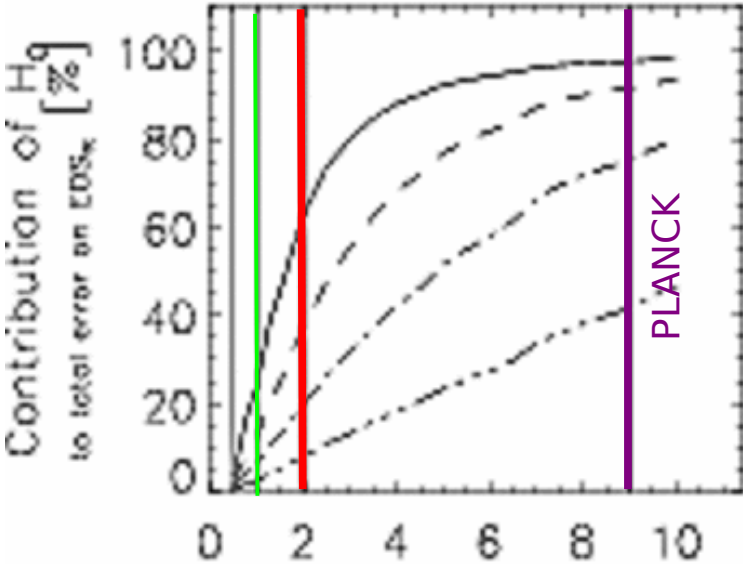
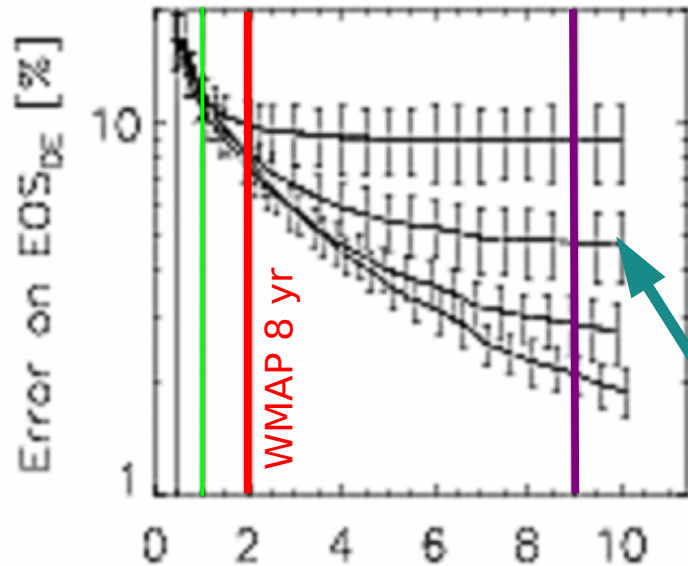
Data Set	$\Omega_K$	$\Omega_\Lambda$
WMAP + $h = 0.72 \pm 0.08$	$-0.003^{+0.013}_{-0.017}$	$0.758^{+0.035}_{-0.058}$
WMAP + SDSS	$-0.037^{+0.021}_{-0.015}$	$0.650^{+0.055}_{-0.048}$
WMAP + 2dFGRS	$-0.0057^{+0.0061}_{-0.0088}$	$0.739^{+0.026}_{-0.029}$
WMAP + SDSS LRG	$-0.010^{+0.011}_{-0.015}$	$0.728^{+0.020}_{-0.028}$
WMAP + SNLS	$-0.015^{+0.020}_{-0.016}$	$0.719^{+0.021}_{-0.029}$
WMAP + SNGold	$-0.017^{+0.022}_{-0.017}$	$0.703^{+0.030}_{-0.038}$



- For a constant EOS, but a Universe of general curvature**

# Dark Energy Equation of State:

CMB and  $H_0$  Error Dependencies



$\epsilon(\omega_m; \text{NOW}) / \epsilon(\omega_m) \rightarrow$

# To arrive at:

- $$\mathbf{w} = \mathbf{a}_{wK} + \mathbf{b}_{wK} (\mathbf{a}_{K\Lambda} + \mathbf{a}_{\Lambda m} \mathbf{b}_{K\Lambda}) + \mathbf{b}_{\Lambda m} \mathbf{b}_{K\Lambda} \mathbf{b}_{wK} * \omega_m / h^2$$

$$= (-0.83 \pm 0.11) - (0.56 \pm 0.06) \omega_m / h^2$$
- Error on EOS as a function of  $\epsilon(\omega_m)$ :

$$\epsilon_w^2 = \dots + \mathbf{b}_{\Lambda m} \mathbf{b}_{K\Lambda} \mathbf{b}_{wK} \left[ \left( \frac{\epsilon_{\omega_m}}{h^2} \right)^2 + \left( \frac{2 \omega_m \epsilon_h}{h^3} \right)^2 \right]$$

- In Figure: curves from top to bottom for  $\epsilon(H_0) = \epsilon(H_0; \text{now}) * [1, 1/2, 1/4, 1/10]$

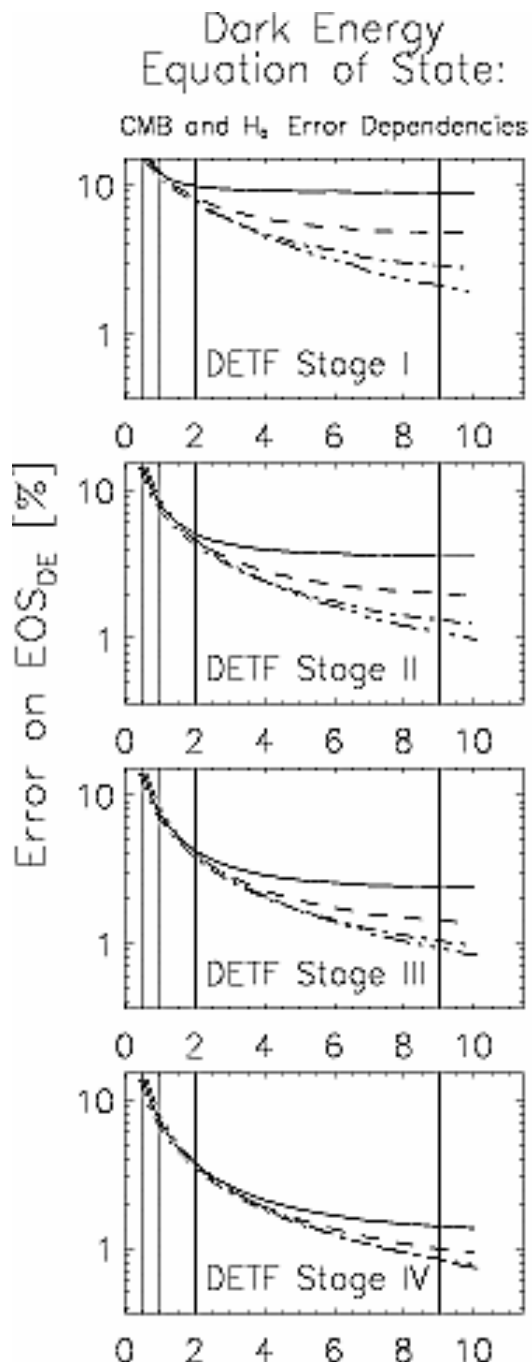
[Olling, 2007, MNRAS, 378, 1385]



# $H_0$ , CMB and Dark Energy (cntd)

- The Dark Energy Task Force [\[Albrecht et al 2006\]](#) recommends several approaches to determine the “evolution” of the EOS:
  - Stage I: Current knowledge
  - STAGE II: Projects finishing soon (including *PLANCK*)
  - STAGE III: Photo- (spectro-) redshifts on 4<sup>m</sup> (8<sup>m</sup>) telescopes
  - STAGE IV: *Large Synoptic Telescope,*  
*Joint Dark Energy Mission,*  
*Square Kilometer Array*
    - At Stage IV, accurate  $H_0$  knowledge matters  $< \sim 50\%$
- **Unpublished Minority Opinion** (Freedman & Hu):  
**Spend effort on determination of  $H_0$**

# $H_0$ , CMB and Dark Energy (cntd)



• At intermediate stages, small  $H_0$  errors matter more:

– Stage I:  $\epsilon(H_0) = 10\% \Rightarrow \epsilon_w \sim 8.9\%$   
 $\epsilon(H_0) = 1\% \Rightarrow \epsilon_w \sim 2.3\%$

– Stage II:  $\epsilon(H_0) = 10\% \Rightarrow \epsilon_w \sim 3.6\%$   
 $\epsilon(H_0) = 1\% \Rightarrow \epsilon_w \sim 1.2\%$

– Stage III:  $\epsilon(H_0) = 10\% \Rightarrow \epsilon_w \sim 2.4\%$   
 $\epsilon(H_0) = 1\% \Rightarrow \epsilon_w \sim 1.0\%$

– Stage IV:  $\epsilon(H_0) = 10\% \Rightarrow \epsilon_w \sim 1.5\%$   
 $\epsilon(H_0) = 1\% \Rightarrow \epsilon_w \sim 0.9\%$

$\epsilon(\omega_m; \text{NOW}) / \epsilon(\omega_m) \rightarrow$

# Calibrating the Extragalactic Distance Scale

- I review several methods in my 2007 paper

- “Standard Candle Methods:”

Extinction may be greatest difficulty. For known Galactic Cepheids:  $\langle A_V \rangle \sim 1.7$  mag  
GAIA expects:  $\epsilon(A_V) \sim 0.1$  mag [Jordi et al, 2006MNRAS.367..290]

- More promising, “geometric” methods:

- Velocity Gradient, [Applied to LMC by GAIA]
- (H<sub>2</sub>O) Masers in extra-galactic star formation regions  
[Few systems per galaxy: depends on external velocity-field data]
- Extra-galactic (nuclear) Mega masers [Just 2 lines of sight: sensitive to systematics]
- “Licht Echo” method; X-ray scattering of background sources;  
Expanding Photospheres of SNe (non=LTE) [Special events]
- (Detached) Eclipsing Binaries; Gravitational Waves Close WDs  
[No calibrators in HIPPARCOS (fixed by GAIA?)]

- [e.g., Gould 2000; Argon et al 2004, Brunrhaller et al 2005, Braatz et al 2006; Panagia et al 1991, Gould 2000, Sparks 1994, Sugerman 2006; Draine & Bond 2004; Nugent et al 2006; Paczynski & Sasselov 1997, Fitzpatrick et al 2004, Stanek et al 1998; Cooray & Seto 2005 ]

# Rotational Parallax

- Distance (D) to *Local Group Spirals* can be determined via the [Rotational Parallax Method](#)  
[Peterson & Shao, 1997; Olling & Peterson, APH/0005484; Olling, 2007, MNRAS, 378, 1385]
- Principle **very** straightforward:
  - Measure circular rotation via radial-velocities ( $V_C$ )
  - Measure circular rotation via proper motions ( $\mu_C \propto V_C / D$ )
  - Distance  $\propto V_C / \mu_C$ 
    - **Expected Results:**

## Unbiased Distances with

## Accuracy of several % out to ~1 Mpc

- Requires:
  - Large-scale ordered motions (rotation)
  - Ground-based radial velocities and
  - Space-based proper motions at the  $< \sim 10 \mu\text{as/yr}$  level

# The Rotational Parallax Method (cntd)

- **Order of magnitude Estimates:**

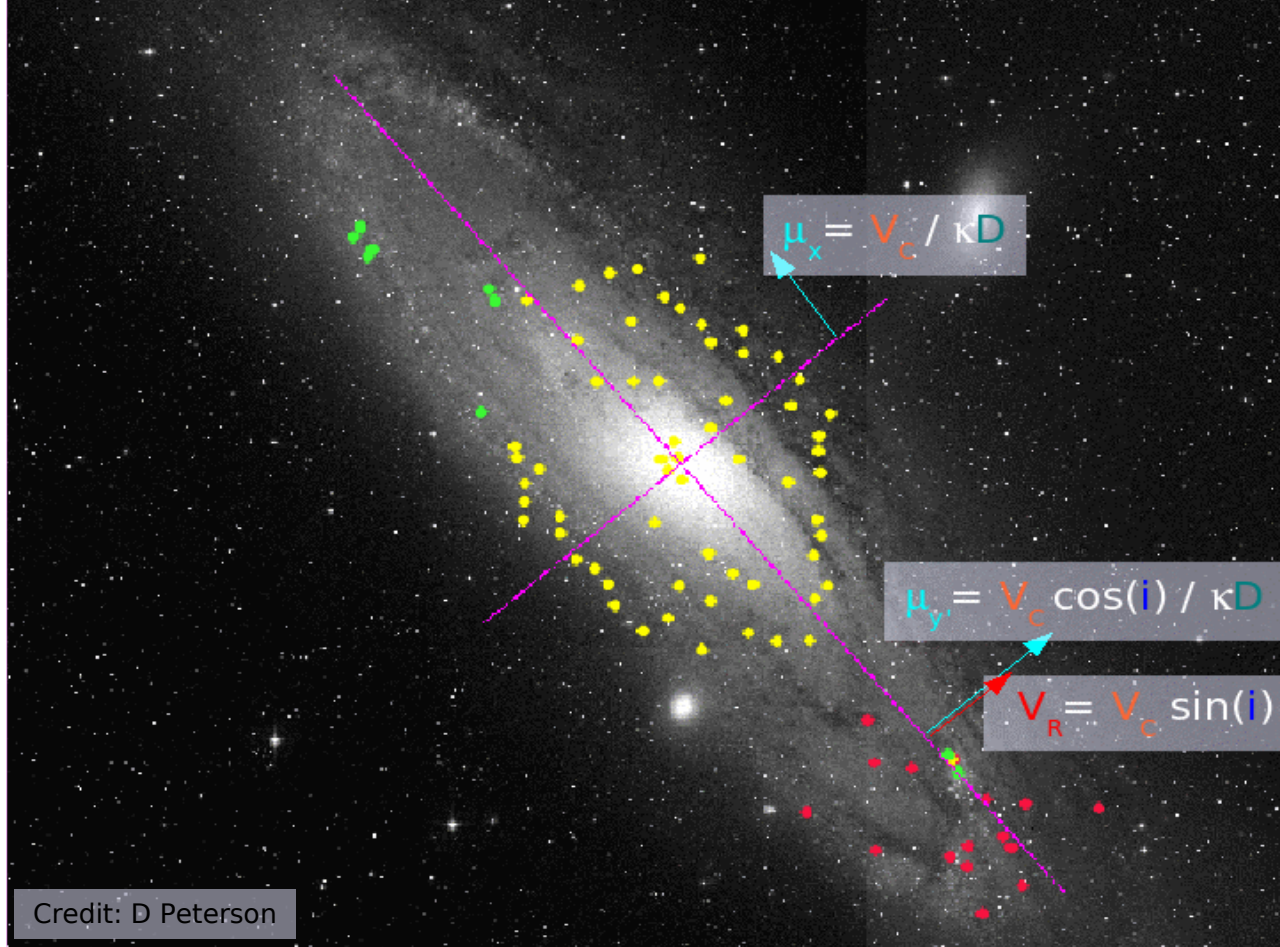
- **M 33:**  $i \sim 56^\circ$ ,  $D \sim 0.84$  Mpc,  $V_c \sim 97$  km/s  $\Rightarrow \mu_c \sim 24 \mu\text{as/yr}$
- **M 31:**  $i \sim 77^\circ$ ,  $D \sim 0.84$  Mpc,  $V_c \sim 270$  km/s  $\Rightarrow \mu_c \sim 74 \mu\text{as/yr}$
- **LMC:**  $i \sim 35^\circ$ ,  $D \sim 0.055$  Mpc,  $V_c \sim 50$  km/s  $\Rightarrow \mu_c \sim 192 \mu\text{as/yr}$

- **Importance of Random Motions ( $\sigma$ )**  
~ “measurement errors”

- **M 33:**  $V_c/\sigma = 9.7 \Rightarrow \varepsilon_{D,HI} \sim (\sqrt{2})/9.7 \sim 14.5\%$  (per star)
- **M 31:**  $V_c/\sigma = 27.0 \Rightarrow \varepsilon_{D,HI} \sim (\sqrt{2})/27.0 \sim 5.2\%$  (per star)
- **LMC:**  $V_c/\sigma = 2.5 \Rightarrow \varepsilon_{D,HI} \sim (\sqrt{2})/2.5 \sim 56.5\%$  (per star)

- **Real errors are ~twice larger**

# Rotational Parallax Illustrated



## • For Circular Orbits:

- minor axis:  $\mu_x = V_c / (\kappa D)$
- Major axis:  $\mu_y = V_c \cos(i) / (\kappa D)$
- Major axis:  $V_R = V_c \sin(i)$   
with  $\kappa \sim 4.74$  [km/s] / [AU/yr]

- **Three equations,**

- **Three unknowns,**

- **Three solutions**

- **Several Approaches**

# The Rotational Parallax Method

- Flat Rotation Curve, Circular Orbits, HI Inclination

$$D_{iHI} = V_R(\text{major axis}) / [ \kappa * \sin(i) * \mu_x(\text{minor axis}) ]$$

$$\epsilon(D_{iHI})^2 = D^2 [ (\epsilon(V_R) / V_R)^2 + (\epsilon(\mu_x) / \mu_x)^2 ]$$

- Flat Rotation Curve, Circular Orbits, Unknown Inclination

$$\cos(i) = |\mu_{y'}(\text{major axis})| / |\mu_x(\text{minor axis})|$$

$$D_{mM} = V_R * [ (\mu_{y'}(\text{major axis}))^2 - (\mu_x(\text{minor axis}))^2 ]^{-1/2}$$

- GENERAL CASE, any position in galaxy (except principle axes)

$$\cos^2(i) = -(y' \mu_y) / (x \mu_x)$$

$$D_G = V_R / \kappa * [ -(y' / \mu_y) / (x \mu_x + y' \mu_y) ]^{-1/2}$$

# The Rotational Parallax Method (cntd)

- **How About?**

- Space-motion of the galaxy
- Non-circular motions
  - Spiral-arm streaming motions
  - Bar-induced motions
- Tidal distortions
- Et cetera

**How Robust is the RP method?**

**!!! Very Much So !!!**



# General Rotational Parallaxes

- Unknowns:

- Total Space Velocity:

- $\mathbf{V}_{\text{total}} = \mathbf{V}_{\text{sys}} + (V_c + \mathbf{V}_p) + \mathbf{V}_\sigma$

- = systemic velocity + “orbital” + random

⇒ 3+1+3+3 = **10 unknowns**

- Coordinate system:

- Origin of coordinate system

⇒ **2 unknowns**

- Position angle of major axis ( $\phi$ )

⇒ **1 unknown**

- Distance and Inclination

⇒ **2 unknowns**

- Star position in galaxy

⇒ **3 unknowns**

- **TOTAL: 18 unknowns**

- OBSERVABLES (per star):

2 positions, 2 proper motions,  $V_{\text{rad}}$  = **5 knowns**

# General Rotational Parallaxes (cntd)

- **However:**

- Many unknowns are shared between test particles:

- Center of galaxy + PA: 3 shared vars.
- Systemic velocity: 3 shared vars.
- Rotation Speed: 1 shared var.
- Distance: 1 shared var.
- Inclination: 1 shared var.
- Velocity dispersion: 3 shared vars.
- **TOTAL** **12 shared variables**

- Left with: 3  $V_p$ 's & x,y,z: **6 star-dependent unknowns**

- No solution because we have **5 observables per star**

- **No solution  $\Rightarrow$  Eliminate 2 more variables**

- e.g., assume  $\langle V_{p;z} \rangle = 0$  and  $\langle z \rangle = 0$

# General Rotational Parallaxes (cntd)

- **Then:**  $(4 N_* + N_{sv})$  unknowns  
 $5 N_*$  observables
- $\Rightarrow$  **Solution if:**  $5 N_* \geq (4 N_* + N_{sv})$   
 $N_* \geq N_{sv}$
- **In our example, if  $N_* \geq N_{sv} = 12$**
- **Alternatively,** allow for corrugations
  - $z(\theta) = z_0 + \sum_1^{n_z} A_n \cos(2n\pi\theta) + B_n \cos(2n\pi\theta)$
  - $V_{p;z}(\theta) = V_{p;z;0} + \sum_1^{n_{vpz}} C_n \cos(2n\pi\theta) + D_n \cos(2n\pi\theta)$
  - $\Rightarrow$  **Increase  $N_{sv}$  &  $N_*$  by:  $2 * (n_z + n_{vpz} + 1)$**
- $\Rightarrow$  **Measure/determine  $V_c$ ,  $D$ ,  $i$  and:**
  - Could add:  $V_c(R) = V_c(R_0) + dV/dR * (R-R_0)$   $[N_{sv} = N_{sv} + 2]$
  - $i(R) = i(R_0) + di/dR * (R-R_0)$   $[N_{sv} = N_{sv} + 2]$
- $\Rightarrow$  **4 measured & 2 modeled phase-space cmpnts per star**

# General Rotational Parallaxes (cntd)

- **Alternatively:**

- **Model in-plane peculiar motions,**

- Either physically from observed light distribution & “M/L”
  - Including:  $i(R)$ ,  $V_c(R)$ ,  $\phi(R)$ , Bar, Spiral Arms, Tides
    - Adds more variables [ $i$ : >1;  $V_c$ : >1;  $\phi$ : >1; Bar: >3; Spiral: >3)
- Or as Fourier series to check the z-assumptions
  - Iterate between  $(z; V_z)$  and  $(V_{p;x}; V_{p;y})$

- **Similar Procedures are/will-be employed for:**

- Distance determination with maser-regions in galaxies
  - ~17 H<sub>2</sub>O Masers in M31 & M33 at SKA sensitivity
  - Barely exceeds the minimum number of shared variables
- Velocity-field/Rotation Curve determination of Milky Way

# Rotational Parallaxes: Accuracy

## The equations to solve

$$\begin{aligned}\kappa D \mu_x &= V_{sys,x} + V_{\sigma,x} + V_{c,x} + V_{p,x} \\ \kappa D \mu_{y'} &= V_{sys,ry'} \sin i_s - (V_{p,z} + V_{\sigma,z}) \cos(i) + (V_{c,y} + V_{p,y} + V_{\sigma,y}) \sin(i) \\ V_r &= V_{sys,ry'} \cos i_s + (V_{p,z} + V_{\sigma,z}) \sin(i) + (V_{c,y} + V_{p,y} + V_{\sigma,y}) \cos(i)\end{aligned}$$

are mildly non-linear with reasonably well-known initial conditions

**Good solutions expected**

### • Problem investigated by Olling & Peterson [2000, [aph/0005484](#)]

- Solve  $V_r$  relation for  $(V_{p,z} + V_{\sigma,z})$  and substitute in  $\mu_{y'}$
- Or solve  $V_r$  relation for  $(V_{c,y} + V_{p,y} + V_{\sigma,y})$  and substitute in  $\mu_{y'}$
- Or solve  $\mu_{y'}$  relation for  $V_{c,y} = V_{c,x} * x/y$  and substitute in  $\mu_x$

# Rotational Parallaxes: Accuracy (cntd)

## Rewrite equations employing observables $x, y'$

- $\mu_{y'}(V_R) = \alpha_{y'r} * V_R + \gamma_{y'r}$
- $\mu_x(V_R; y'/x) = \alpha_{xr} * V_R * y'/x + \beta_{xr} * y'/x + \gamma_x$
- $\mu_x(\mu_{y'}; y'/x) = \alpha_{xy'} * \mu_{y'} * y'/x + \beta_{xy'} * y'/x + \gamma_x$
- Solve for unknown  $\alpha$ ,  $\beta$  and  $\gamma$  coefficients
- The  $\alpha$  and  $\gamma$  coefficients yield the desired parameters
  - $\cos^2(i) = -1 / \alpha_{xy'}$
  - $D = 1 / [ \alpha_{y'r} \kappa \tan(i) ]$
  - Non-circular motions and  $V_{\text{SYS}}$  appear only in  $\gamma_{y'r}$  and  $\beta$ s
- Accuracy of fitted parameters follows from back-substitution and Fourier analysis of velocity field

# Rotational Parallax: Observability

- **Need bright sources:**
  - Minimize confusion & Maximize observing speed
    - All stars share (almost) the same proper motion
  - More than enough available”
    - M 33: 1,000 ( $\pm 200$ ) 2MASS stars ( $K_s \leq 15$ )
    - M 31: 2,000 ( $\pm 270$ ) 2MASS stars ( $K_s \leq 15$ )
    - LMC: 23,000 UCAC stars ( $V \leq 16$ )
- Need least disturbed galaxy
  - M33, M31, LMC**
- **My Preference for SIM: M33, M31, LMC**

# Probing the Hubble Flow:

- **Need to go to  $>100$  Mpc**

$$\varepsilon(H_0) \sim V_{\text{pec}}/V_{\text{Hubble}} \sim 200 \text{ km/s} / (100 \text{ Mpc} * 75 \text{ km/s/Mpc}) \sim 2.6\%$$

- **The only known geometric method that probes that far:**

- **Extra-galactic H<sub>2</sub>O Masers**

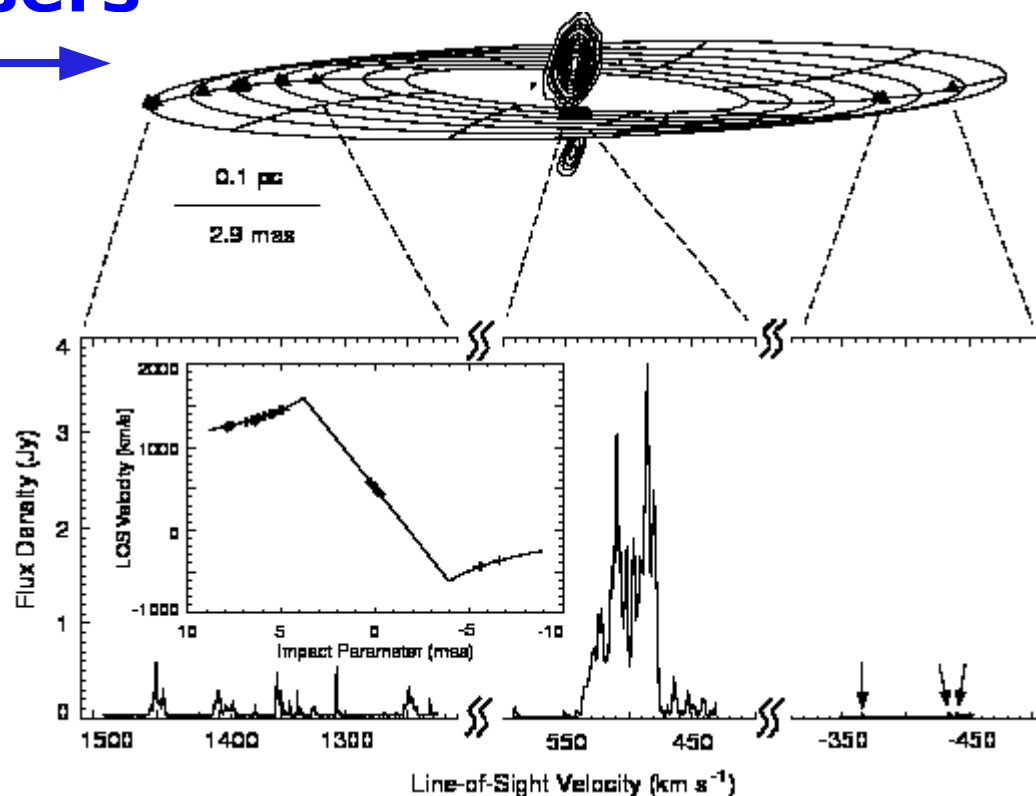
Herrstein et al, 1999, Nature

Thin, edge-on disks  $\longrightarrow$

- NGC 4258:  $D \sim 7.3$  Mpc  
 $\Delta D/D \sim 5\%$

- NGC 1068  $D \sim 14$  Mpc

- ....  $D \sim 200$  Mpc  
[e.g., Argon et al, 2007, ApJ, 659, 1040]





# Mega Maser Distance Uncertainties:

- N 4258 Distance:  
 $7.2 \pm 0.3$  (random)  
 $\pm 0.4$  (systematic)
- mostly due to orbital eccentricity [Argon 2007],
- Up to  $e \sim 0.3$  due to, e.g., binary black holes

[Eracleous, etal 1995]

- But ruled out by monitoring

[Gezari, Halpren, Eracleous, 2007]

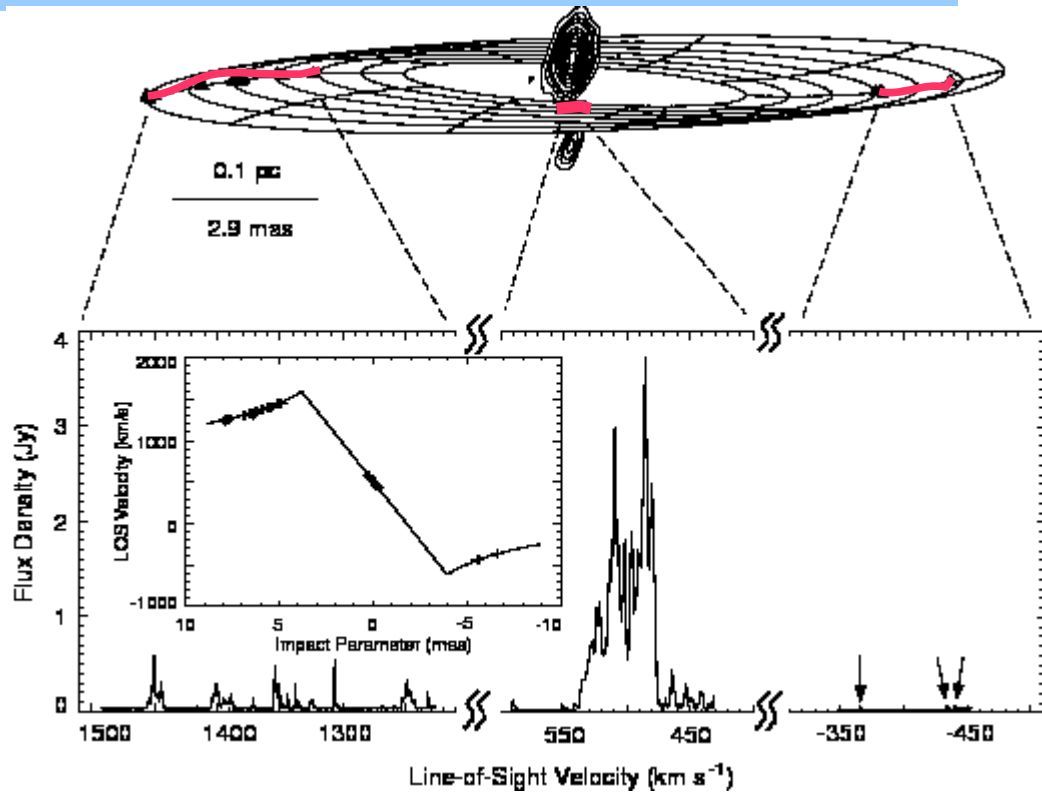
- Not clear that elliptical orbits exist,

if not >60% has emissivity variations [Storchi-Bergmann etal, 2003]

- **Distance error in case of unmodeled eccentricity:**

$$D_{\text{CIRC}} = D_{\text{TRUE}} \left[ \frac{(1 \mp e)^3}{(1 \pm e)} \right]^{1/2} \sim D_{\text{TRUE}} [1 \mp 2e]$$

[Olling 2007]



Herrstein etal, 1999, Nature

# Conclusions

- **$H_0$  is important for Cosmology**
- **1% Galaxy Distances will be possible**
  - **SIM should do M33, M31**
  - **GAIA will do LMC, SMC**
- **1% Distance to LG galaxies will calibrate secondary calibrators (Cepheids, TRGB, EBs, ...) to determine  $H_0$**
- **Other methods will also become available for cross-checks: very important**

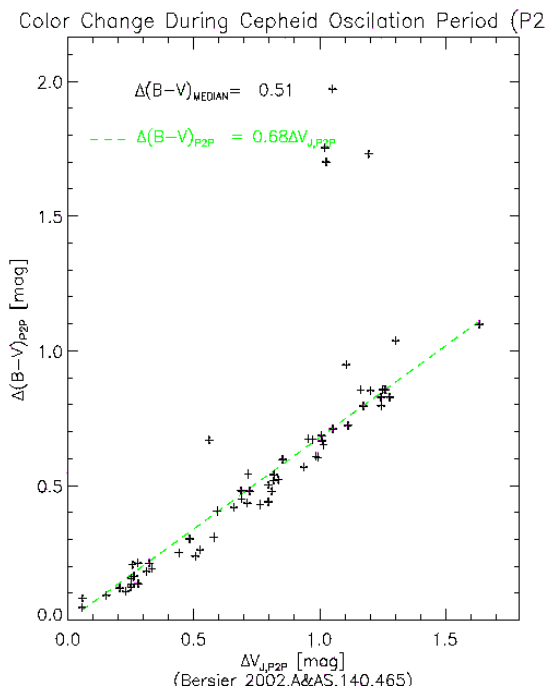
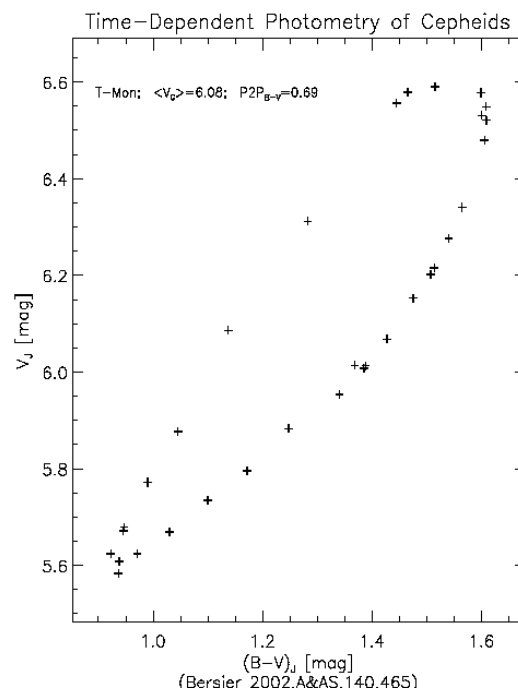
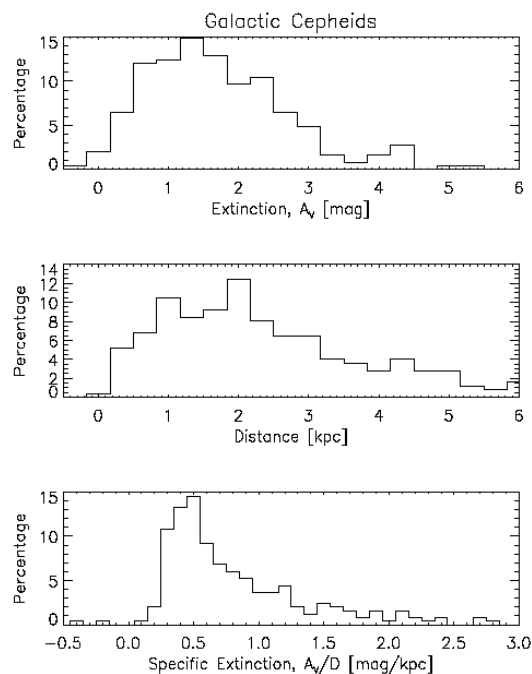
# Backup Slides

# The Future of Astrometry-enabled Astrophysics in the US

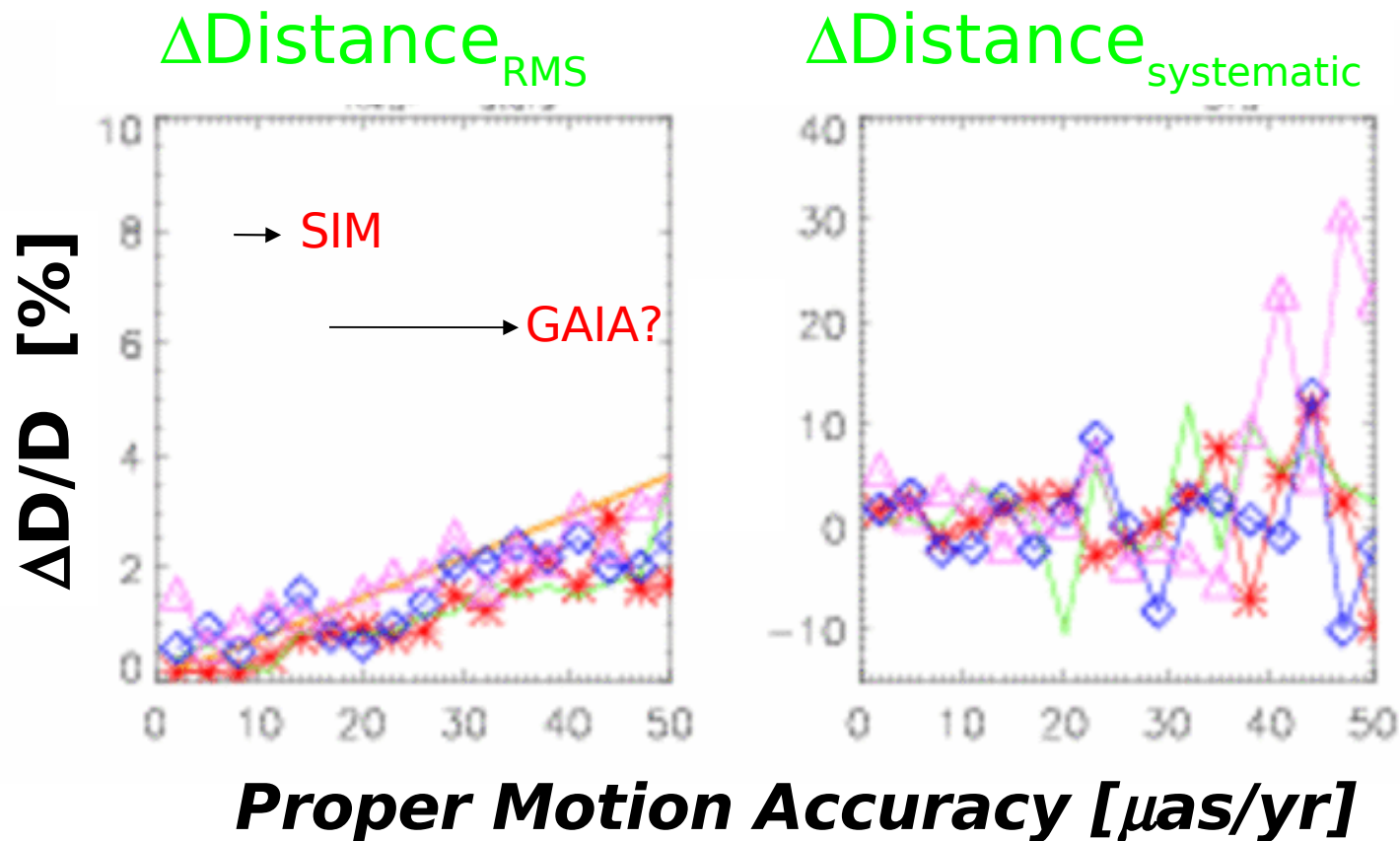
- **Is GAIA going to go before SIM for sure?**
  - If so, then there will be many (10,000's) interesting objects too look at with SIM to get better data
- **Is there going to be dedicated US funding to work with GAIA data?**
  - This would be required to prepare GAIA-follow-up SIM programs
- **Would it not be useful to upgrade SIM,**
  - Make a larger difference with GAIA
  - Deal with the extra-source “burden” (faster of faint sources)
  - Make *definite detections* of Earth-mass planets
  - Improve extra-galactic capabilities
- **How about ~~DARWIN~~ & TPF-I ?**

# Calibrating the Extragalactic Distance Scale

- I review several methods in my 2007 (MNRAS) paper
- “Standard Candle Methods:”
  - No great fan, but will be calibrated with GAIA
  - Extinction may be greatest difficulty. For known Galactic Cepheids:  $\langle A_V \rangle \sim 1.7$  mag
  - GAIA expects:  $\epsilon(A_V) \sim 0.1$  mag [Jordi et al, 2006MNRAS.367..290]



# Rotational Parallax: Expected Results.

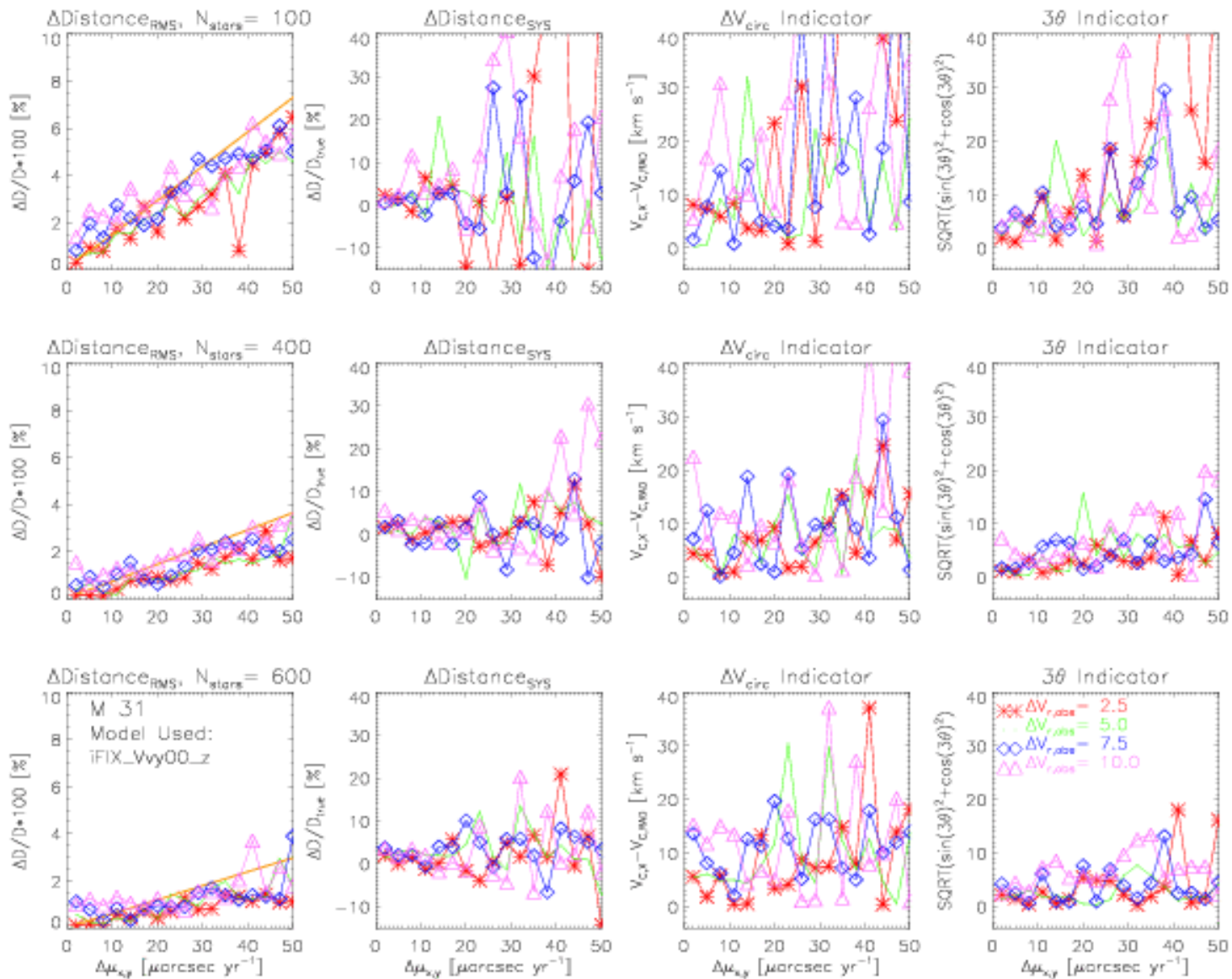


Achievable distance errors as a function of proper motion errors:  
LEFT: random errors, RIGHT: systematic component

Symbols: accuracy of radial velocity data (2.5 – 10 km/s)

400 Stars used

from: Olling & Peterson (2000)



Distance accuracy (LEFT panel) and Systematic Effects (3 RIGHT panels) as a function of proper motion accuracy

# “A Few Things We do not Know About Stars”

R.L. Kurucz (2002nqsa.conf....3K)

- We do not know how to make realistic model atmospheres:
  - we do not understand convection
  - we do not consider the variation in micro-turbulent velocity
- We do not understand spectroscopy:
  - we do not have good spectra of the Sun or any star
  - We do not have energy distributions for the Sun or any star
  - We do not know how to determine abundances:
    - **we do not know the abundances of the Sun or any star**
  - We do not have good atomic and molecular data:
    - 50% of the lines in the solar spectrum are not identified



# “A Few Things We do not Know About Stars”

R.L. Kurucz (2002nqsa.conf....3K), cntd

- Cepheids have convective pulsation but the models do not:
  - we do not have high quality spectra over phase for any Cepheid
- We do not understand abundance evolution in early type stars
- Many early type stars are oblate fast rotators
  - e.g., Vega [\[Peterson et al, 2006 Natur. 440..896P\]](#)

# Kurucz: Optimism and Pessimism

*“People sometimes complain that I am too pessimistic and that I criticize too much. In fact I am the most optimistic person. I believe that the human race is tremendously improvable and that humans can solve any problem. **But the most important step in solving a problem is to realize that the problem exists.** When I identify a problem I tell, or try to tell, the people who are capable of doing something about it. I also work on correcting the problem myself, if I am capable. A pessimist does not believe that problems can be solved so does not question the present and does not search for errors. A pessimist acts so “optimistically” about the present that a pessimist prevents progress. **Why worry about basic physics when everything is fine as it is?”***

See also: [kurucz.harvard.edu](http://kurucz.harvard.edu).