Astrometry, Precision Astrophysics, $H_0$ & (some) Cosmology

A Connection between Stars, Galaxies and the Universe

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Outline

1) Astrometric Missions

2) Future of Astrophysics:
   - Precision & Accuracy
   - Astrometry & Stellar Ages
   - Astrometry & Cosmology

3) $H_0$, the CMB & Dark Energy

4) Calibrating the Distance Scale

5) Rotational Parallax

6) Conclusions

7) GAIA, SIM & DARWIN/TPF-I

Most of this talk is based on a paper earlier this year [Olling, 2007, MNRAS, 378, 1385] & a contribution to the SIM review paper
### Setting Some Scales

#### Parallaxes, in μas
- α Cen: 742,000
- RR Lyra: 4,380
- δ Cep: 3,320
- 1 kpc: 1,000
- Gal. Center: 125
- LMC: 20
- M 31: 1.5

#### Proper Motions, in μas/yr
- α Cen: 3,600,000
- RR Lyra: 200,000
- δ Cep: 16,500
- 10 km/s @ 1 kpc: 2,110
- 200 km/s @ 8 kpc: 5,275
- 50 km/s @ LMC: 211
- 200 km/s @ M 31: 60

USA @ 10 pc: 2.9 μas/yr; 2 M_Earth @ 10 pc: 1 μas/yr
Some Astrometric Missions

- **Hipparcos:** $\Delta \pi \leq 1000 \, \mu\text{as}$ ($V \leq 7$); $\Delta \pi \leq 3000 \, \mu\text{as}$ ($V \leq 12$)

- Twice better calibration of systematics available [van Leeuwen, 2007]

- **GAIA:** $\Delta \pi \leq 7 \, \mu\text{as}$ ($V \leq 11$); $\Delta \pi \leq 230 \, \mu\text{as}$ ($V \leq 20$)

- Accuracy is at **MISSION END** for ~400 Observations per coordinate
- Spatial variation by ±50% due to scanning law
- Observations are split in ~36 epochs of ~3.5 hr, with 22 measures --> 17 μas/epoch
- GAIA saturates at $V \sim 11$: excess charge is dumped (anti-blooming drain, TDI gating)
- Saturation/dumping ($\geq 300,000$ e⁻) implies best possible accuracy of 4 μas
- These are special effects that are not calibrated as well as for stars with $V \geq 11$

- **SIM:** $\Delta x \sim 1 \, \mu\text{as}$ ($V \leq 6$); $\Delta \pi \leq 5 \, \mu\text{as}$ ($V \leq 20$)

- **OBSS:** Origins Billion Star Survey
  - Goal, Survey: $V$: [9, 20], $\Delta \pi$: [15, 100] μas
  - Pointed: $V$: [9, 20, 24], $\Delta \pi$: [15, 15, 100] μas
  - 16-channel photometry [320, 1100] nm
  - Status: Possible Origins Probe: Ready to go ahead (after ~2014)
Some Comments on the Future of Astrophysics & Astrometry

- Astrophysics is slowly transitioning from: Exploration to Understanding
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Some Comments on the Future of Astrophysics & Astrometry

- from “Astronomy” to “Physics”
  - from Precision $\Rightarrow$ Accuracy
  - from Model-Dependent $\Rightarrow$ “True”

- The Universe is finite:
  Eventually there will be nothing new to explore, But plenty to discover (laboratory physics)

JWST looks back to the 1st stars: in a decade or so we will have “seen it all”

- If this is scary ....
Don't Panic!

The Hitchhiker's Guide to the Galaxy

DONT' PANIC!
Stellar Ages & Astrometry

• **Astrophysics of stars** is primarily based on: **THE SUN**
  - Fundamental parameters well determined: **Mass, Radius, Luminosity, He-abundance (Y), [Fe/H]**
  - ~100 Binaries with M and R better than 1%
    • but see: Kurucz' *"Some things we do not know about stars"* (2002nqsa.conf....3K)

• M, R, [Fe/H] and Y “set” the rate of evolution
  - Precise Age Determination of Individual Stars

⇒ **Detailed Formation History of Galaxy**
  - Star Formation + **Oldest Stars (< age of Universe?)**

⇒ **Galaxy Formation & Cosmology**
Stellar Ages & Astrometry (cntd)

• Rate of Evolution
  - Luminosity: $(\Delta L/L)^{\text{theory}} \sim (10 + 2 \frac{L}{L_\odot}) \pm 5 \ [\% / \text{Gyr}]
  
  Age: $\Delta \tau^{\text{theory}} = \frac{(\Delta L/L)^{\text{obs}}}{(0.1 + 0.02 \frac{L}{L_\odot})} \ [\text{Gyr}]$
  
  $= \frac{(2\Delta \pi/\pi)^{\text{obs}}}{(0.1 + 0.02 \frac{L}{L_\odot})} \ [\text{Gyr}]$

• Mid G-type stars : $(\Delta L/L) \sim 10 \% / \text{Gyr}$

• Hipparcos: $(\Delta \pi/\pi)^{\text{obs}} \sim (1 \text{ mas} / 100 \text{ pc}) \implies \Delta \tau^{\text{theory}} \sim 1,800 \text{ Myr}$

• SIM: $(\Delta \pi/\pi)^{\text{obs}} \sim (5 \mu \text{as} / 100 \text{ pc}) \implies \Delta \tau^{\text{theory}} \sim 9 \text{ Myr}$
  SIM @ 1 kpc $\implies \Delta \tau^{\text{theory}} \sim 90 \text{ Myr}$
  SIM @ 5 kpc $\implies \Delta \tau^{\text{theory}} \sim 450 \text{ Myr}$
  SIM @ 10 kpc $\implies \Delta \tau^{\text{theory}} \sim 900 \text{ Myr}$

• Ages are model-dependent
  Will be calibrated with highly accurate GAIA/SIM, Seismology & Ground-Based data
Radio Example: NRAO Press Release, Oct 8th

“VLBA Changes Picture of Famous Star-Forming Region” -- the Orion Nebula --

- "Knowing the accurate distance to this region is vitally important to properly understanding ... star-formation processes there," Sandstrom said.
- The new [VLBA] distance to the region ... is 1270 light-years, compared with the best previous measurement of 1565 light-years.
- Because the ... distance to the region is 20 percent closer than the earlier measurement, the stars ... are intrinisically fainter by a factor of 1.5. This has a major impact ... their ages. "These stars are nearly twice as old as previously thought," said Bower.
Stellar Ages: GAIA and SIM

**NEED RADIUS** ⇒ Eclipsing Binaries (≈ 1% of Population)
- Photometry ⇒ $R_*, m_V, A_V$
- Spectroscopy ⇒ $V_{ORB}, M_*, [Fe/H]$
- Astrometry ($\pi$) + $m_V, A_V$ ⇒ Luminosity

**TWO stars on same Isochrone** ⇒ Age & Helium

[ Ribas (2006ASPC..349...55R), Lebreton (2005tdug.conf..493L), Lastennet (2002A&A...396..551L); Metcalfe etal (2006ASPC..349...55R) ]

**GAIA** is survey mission will determine overall SF History
≤ 600 pc has $\Delta \pi/\pi \leq 1\%$ for G7 star ($V \sim 14.5$)
~ 6,254,000 thin-disk stars ⇒ ~190,000 EBs ⇒ $\Delta \tau \sim 4$ Myr
~ 385,000 thick-disk stars ⇒ ~12,000 EBs ⇒ $\Delta \tau \sim 5$ Myr
~ 12,000 spheroid stars ⇒ ~300 Ebs ⇒ $\Delta \tau \sim 9$ Myr

**SIM** should do the rare Special Cases at larger distances
Binary cousins of Old Uranium Stars with [Fe/H] ~< -3
$\tau \sim 13.2 \pm 2.7$ Gyr HE 1523–0901, d ~ 1 kpc, V~11 [Frebel et al 2007]
$\tau \sim 14.9 \pm 3.0$ Gyr CS 22892-0529, d ~ 1.5 kpc, V~12 [Sneden et al 1996; Hill et al 2002]
$\tau \sim 13.2 \pm 2.7$ Gyr HE 1424-0241, d ~ 8 kpc, V~14 [Cohen et al 2007 ]
[See Beers & Christlieb 2004, ARA&A for a review]
Astrometry & Cosmology

- CMB, high-z galaxy data, Ly-α forest & BBN yield:
- WMAP & Other yield:
  - Hubble Constant $= H_0 = 71 \pm 2 \pm 7$ [km/s/Mpc]
  - Age $= t_0 = 13.7 \pm 0.2$ [Gyr]
  - Matter Density $= \Omega_m = 0.27 \pm 0.02$ [$\rho_{\text{crit}}$]
  - Total/Baryon Matter $= \Omega_m / \Omega_b = 6.1 \pm 1.1$
  - Primordial Helium $= Y_p = 0.2482 \pm 0.0004$

- Astrometry of M31, M33 $\Rightarrow$ strong limits on $H_0$
- Astrometry of Galactic Objects can set relevant limits on $t_0$, $Y_p$ and Star Formation History

[Spergel et al, 2003, 2006; Freedman et al 2001; Mathews et al, 2005; Madau et al, 1996, this talk]
H₀, the CMB & Dark Energy

- From the shape of the power spectrum, WMAP “directly” [e.g., Hu 2005] measures the:
  - photon-to-baryon ratio \[ R_* \sim \frac{\omega_b}{0.0223} \times \frac{1089}{1+z_*} \]
    controls amplitude of odd/even peaks
  - radiation-to-matter ratio \[ r_* \sim \frac{0.126 \times (1+z_*)}{\omega_m h^2} \]
    controls amplitude of envelope

- \( z_* \) is known redshift of recombination
- \( \omega_b = \Omega_b h^2 \) is the physical baryon density
- \( \omega_m = \Omega_m h^2 \) is the physical matter density
- \( h = H_0 / 100 \)

\[ \Omega_b = \frac{\rho_b}{\rho_{crit}} \quad \text{and} \quad \Omega_m = \frac{\rho_b + \rho_{DM}}{\rho_{crit}} \]

- \( \rho_{crit} = 3 H_0^2 / (8\pi G) \) is the critical density of Universe

\[ \text{[g/cm}^3\text{]} \]

\[ \text{[g/cm}^3\text{]} \]

\[ \text{[km/s/Mpc]} \]
WMAP yields: location \( l_A \) of the acoustic peak:

\[
l_A = D_A \pi / s_* \quad (\pm < 1\%)
\]

Cosmology yields:
1) size of the acoustic oscillation

\[
s_* \approx 140 \left( \frac{R_*}{0.854} \right)^{-0.252} \left( \frac{r_*}{0.338} \right)^{-0.083} \quad (\pm 1\%) \quad [\text{Mpc}]
\]

2) the angular-size distance \( D_A \) relation

\[
D_A = a_* \int_{a_*}^{1} \frac{1}{a^2 H(a)} \, da \quad \text{with} \quad \frac{H(a)}{100} = \sqrt{\frac{\omega_m}{a^3} + \frac{h^2 - \omega_m}{a^{3(1+w)}}} \quad \text{and} \quad \Omega_{\text{tot}} = 1
\]

where: \( a = 1/(1+z) \) the scale factor,
An assumed flat Universe \( (\Omega_{\text{TOT}} = \Omega_\Lambda + \Omega_m = 1) \)
\( \Omega_\Lambda \) the Dark Energy, and
“\( w \)” the “Equation of State” (EOS) of Dark Energy
**H\(_0\), CMB and Dark Energy** (cntd)

- **IF** the Cosmological Constant is the Dark Energy, **THEN** \(w = -1\)
  - The DE-candidates have different \(w\)'s:
    - (strings=-1/3; domain walls -2/3, ...) or are variable
- **For a flat Universe,** current constraints on \(\omega_m\) and \(H_0\) yield:
  \[
  \Omega_A = 1 = \Omega_m = 1 - \frac{\omega_m}{h^2} \approx 0.770 \pm 0.022
  \]
- **Alternatively:** the relation for the location of the acoustic peak yields:
  \[
  D_A = \frac{l_A s_*}{\pi} = 100 a_* \int_{a_*}^{1} \left( \frac{\omega_m}{a^{-1} + \frac{h^2 - \omega_m}{a^{3w-1}}} \right)^{-1/2} \, da
  \]
  - The **observables** are: \(l_A\), \(s_*\) and \(\omega_m\)
  - **Assuming** a cosmological constant \((w = -1)\), then: **unknown is** \(h\)
    - \(\Rightarrow\) the 1 unknown, \(h\) \((H_0)\)
IF one wants to determine \( w \), THEN one needs to know \( h \):

A fit to the current WMAP data [Spergel et al 2006] yields:

\[
-w \approx 1.59 - 2.56 \Omega_m
\]

Where \( \Omega_m \) is unknown, but can be determined from:

- Large-Scale Structure [SDSS]
- Large-Scale Structure [2dF]
- Luminosity-distance(z) [SN: HST/GOODS]
- Luminosity-distance(z) [SN: SNLS]

But why not from \( \omega_b \) and \( H_0 \) directly?

Currently:

\[
-w \approx 1.59 - 2.56 \frac{\omega_m}{h^2} \approx 0.985 \pm 7\%
\]

Astrometry, \( H_0 \) & Cosmology  Rob Olling (UMd)  NRAO/UVa, Oct 2007
That is to say: EOS of Dark Energy is known with a precision of \( \sim 7\% \)

\[
\epsilon_w \approx 2.56 \frac{\omega_m}{h^2} \sqrt{\left(\frac{\epsilon_{\omega_m}}{\omega_m}\right)^2 + \left(2 \frac{\epsilon_h}{h}\right)^2}
\]

And Dark Energy follows directly from:
- Better (\( \times 8 \)) determination of \( \omega_m \) with \textit{PLANCK},
- Better (\( \times 10 \)) determination of \( H_0 \) (with \textit{SIM}?)

However, this is \textit{not very accurate}. The Assumptions were:
- Flat Universe
- Dark Energy has \textbf{constant} EOS
- Dark Matter does not cluster, no tensor modes, no quintessence, no running spectral index, no strings, no domain walls, no non-Gaussian fluctuation, no deviations from GR, et cetera
Allowing for a variable EOS of Dark Energy, Hu (2005) concludes that: "... the Hubble constant is the single most useful complement to CMB parameters for dark energy studies ... [if \( H_0(z) \) is] ... accurate to the percent level ... ."

Basically due to the angular-size distance relation:

\[
D_A = \frac{l_A s_*}{\pi} = \frac{1}{\chi} F\left(\frac{\chi a_*}{100} \int_a^1 \frac{1}{a^2 h(a)} \, da\right)
\]

and

\[
h^2(a) = \frac{\omega_{\gamma}}{a^4} + \frac{\omega_m}{a^3} + \frac{\omega_k}{a^2} + \frac{\omega_{\Lambda}}{e^{3\int [1 + w(a)] dln(a)}}
\]

where \( F \) is a function that depends on the curvature of the Universe with \( \chi = 1/(H_0|\Omega_k|^{1/2}) \)

[e.g, \( F(y) = y \) for \( \Omega_k = 0; \) e.g., Carroll 2001]
Alternatively, one can (try to) determine the ages:

\[ -\tau = \int_0^1 \frac{da}{a H(a)} \iff \text{ages of oldest stars} \]

\[ -\tau(z) = \int_0^{a(z)} \frac{da}{a H(a)} \iff \text{ages of high-z galaxies} \]

[e.g., Bothum et al. 2006, Jimenez et al. 2003, Simon 2005]

Summarized in Figure 4 of Spergel et al., 2004

Fig. 4.— The \( \Lambda \)CDM model fit to the WMAP data predicts the Hubble parameter redshift relation. The blue band shows the 68% confidence interval for the Hubble parameter, \( H \). The dark blue rectangle shows the HST key project estimate for \( H_0 \) and its uncertainties (Freedman et al. 2001). The other points are from measurements of the differential ages of galaxies, based on fits of synthetic stellar population models to galaxy spectroscopy. The squares show values from Jimenez et al. (2003) analyses of SDSS galaxies. The diamonds show values from Simon et al. (2005) analysis of a high redshift sample of red galaxies.
• Many groups pursue other methods to determine some (combination of) parameter(s) that constrain the “integral”

• **Luminosity-Distance relation from Supernovae Ia**
  
  
  $D_L(z) = D_A(z) / a(z)^2$

• **Baryon Oscillations (sensitive to local galaxy density)**
  
  $Volume(z) = [D_A(z) / a(z)]^2 / H(z) \times \Omega_{sky} \Delta z$

• **Galaxy Cluster Abundance**
  
  Depends on $Volume(z)$ and non-linear structure growth

• **Weak Lensing**
  
  Depends on: $D_A(z)$, $H(z)$ and *structure growth*

  [e.g., Albrecht et al 2006 = DETF]
We use the Spergel et al. (2006/7) WMAP & “other data” to approximate the relations between the various parameters \( P_i = a_{ij} + b_{ij} P_j \):

\[
\begin{align*}
\Omega_\Lambda &= a_{\Lambda m} + b_{\Lambda m} \Omega_m \\
\Omega_K &= a_{K \Lambda} + b_{K \Lambda} \Omega_\Lambda \\
w &= a_{wK} + b_{wK} \Omega_K
\end{align*}
\]

For a constant EOS, but a Universe of general curvature.
To arrive at:

\[ w = a_{wK} + b_{wK} (a_{K\Lambda} + a_{\Lambda m} b_{K\Lambda}) + b_{\Lambda m} b_{K\Lambda} b_{wK} \times \frac{\omega_m}{h^2} \]

\[ = (-0.83 \pm 0.11) - (0.56\pm0.06) \frac{\omega_m}{h^2} \]

Error on EOS as a function of \( \epsilon(\omega_m) \):

\[ \epsilon_w^2 = \ldots + b_{\Lambda m} b_{K\Lambda} b_{wK} \left[ \left( \frac{\epsilon_{\omega_m}}{h^2} \right)^2 + \left( \frac{2 \omega_m \epsilon_h}{h^3} \right)^2 \right] \]

• In Figure: curves from top to bottom for \( \epsilon(H_0) = \epsilon(H_0;\text{now}) \times [1, 1/2, 1/4, 1/10] \)

The Dark Energy Task Force [Albrecht et al. 2006] recommends several approaches to determine the “evolution” of the EOS:

- **Stage I**: Current knowledge
- **STAGE II**: Projects finishing soon (including PLANCK)
- **STAGE III**: Photo- (spectro-) redshifts on 4m (8m) telescopes
- **STAGE IV**: Large Synoptic Telescope, Joint Dark Energy Mission, Square Kilometer Array
  - At Stage IV, accurate $H_0$ knowledge matters $<\sim 50\%$

**Unpublished Minority Opinion** (Freedman & Hu):
Spend effort on determination of $H_0$
At intermediate stages, small $H_0$ errors matter more:

- **Stage I**: $\varepsilon(H_0) = 10\% \Rightarrow \varepsilon_w \approx 8.9\%$
  $\varepsilon(H_0) = 1\% \Rightarrow \varepsilon_w \approx 2.3\%$

- **Stage II**: $\varepsilon(H_0) = 10\% \Rightarrow \varepsilon_w \approx 3.6\%$
  $\varepsilon(H_0) = 1\% \Rightarrow \varepsilon_w \approx 1.2\%$

- **Stage III**: $\varepsilon(H_0) = 10\% \Rightarrow \varepsilon_w \approx 2.4\%$
  $\varepsilon(H_0) = 1\% \Rightarrow \varepsilon_w \approx 1.0\%$

- **Stage IV**: $\varepsilon(H_0) = 10\% \Rightarrow \varepsilon_w \approx 1.5\%$
  $\varepsilon(H_0) = 1\% \Rightarrow \varepsilon_w \approx 0.9\%$
Calibrating the Extragalactic Distance Scale

I review several methods in my 2007 paper

“Standard Candle Methods:”

Extinction may be greatest difficulty. For known Galactic Cepheids: \( <A_V> \sim 1.7 \) mag

GAIA expects: \( \epsilon(A_V) \sim 0.1 \) mag \[Jordi et al, 2006MNRAS.367..290J]\]

More promising, “geometric” methods:

• Velocity Gradient, \[\text{Applied to LMC by GAIA}\]

• (H\(_2\)O) Masers in extra-galactic star formation regions
  [Few systems per galaxy: depends on external velocity-field data]

• Extra-galactic (nuclear) Mega masers \[\text{Just 2 lines of sight: sensitive to systematics}\]

• “Licht Echo” method; X-ray scattering of background sources;
  Expanding Photospheres of SNe (non=LTE) \[\text{Special events}\]

• (Detached) Eclipsing Binaries; Gravitational Waves Close WDs
  [No calibrators in HIPPARCOS (fixed by GAIA?)]

Rotational Parallax

• Distance (D) to Local Group Spirals can be determined via the Rotational Parallax Method

• Principle very straightforward:
  • Measure circular rotation via radial-velocities ($V_c$)
  • Measure circular rotation via proper motions ($\mu_c \propto V_c / D$)
  • Distance $\propto V_c / \mu_c$
    • Expected Results:
      Unbiased Distances with
      Accuracy of several % out to ~1 Mpc

• Requires: - Large-scale ordered motions (rotation)
  - Ground-based radial velocities and
  - Space-based proper motions at the $<~ 10 \mu$as/yr level
• Order of magnitude Estimates:
  - **M 33**: \( i \approx 56^\circ, D \approx 0.84 \) Mpc, \( V_c \approx 97 \) km/s \( \Rightarrow \mu_c \approx 24 \) μas/yr
  - **M 31**: \( i \approx 77^\circ, D \approx 0.84 \) Mpc, \( V_c \approx 270 \) km/s \( \Rightarrow \mu_c \approx 74 \) μas/yr
  - **LMC**: \( i \approx 35^\circ, D \approx 0.055 \) Mpc, \( V_c \approx 50 \) km/s \( \Rightarrow \mu_c \approx 192 \) μas/yr

• Importance of Random Motions (\( \sigma \))
  ~ “measurement errors”
  - **M 33**: \( V_c/\sigma = 9.7 \Rightarrow \varepsilon_{D,HI} \approx (\sqrt{2})/9.7 \approx 14.5 \% \) (per star)
  - **M 31**: \( V_c/\sigma = 27.0 \Rightarrow \varepsilon_{D,HI} \approx (\sqrt{2})/27.0 \approx 5.2 \% \) (per star)
  - **LMC**: \( V_c/\sigma = 2.5 \Rightarrow \varepsilon_{D,HI} \approx (\sqrt{2})/2.5 \approx 56.5 \% \) (per star)

• Real errors are \( \sim \) twice larger
**For Circular Orbits:**

- minor axis: $\mu_x = \frac{V_c}{\kappa D}$
- Major axis: $\mu_y = \frac{V_c \cos(i)}{(\kappa D)}$
- Major axis: $V_R = V_c \sin(i)$

with $\kappa \sim 4.74$ [km/s] / [AU/yr]

- Three equations,
- Three unknowns,
- Three solutions

- Several Approaches
The Rotational Parallax Method

**Flat Rotation Curve, Circular Orbits, HI Inclination**

\[ D_{i\text{HI}} = V_R (\text{major axis}) / \left[ \kappa \times \sin(i) \times \mu_x (\text{minor axis}) \right] \]

\[ \varepsilon(D_{i\text{HI}})^2 = D^2 \left[ (\varepsilon(V_R) / V_R)^2 + (\varepsilon(\mu_x) / \mu_x)^2 \right] \]

**Flat Rotation Curve, Circular Orbits, Unknown Inclination**

\[ \cos(i) = |\mu_y (\text{major axis})| / |\mu_x (\text{minor axis})| \]

\[ D_{mM} = V_R \times \left[ (\mu_y (\text{major axis}))^2 - (\mu_x (\text{minor axis}))^2 \right]^{-\frac{1}{2}} \]

**GENERAL CASE, any position in galaxy (except principle axes)**

\[ \cos^2(i) = -(y' \mu_y) / (x \mu_x) \]

\[ D_G = V_R / \kappa \times \left[ -(y' / \mu_y) / (x \mu_x + y' \mu_y) \right]^{-\frac{1}{2}} \]
The Rotational Parallax Method (cntd)

• How About?
  - Space-motion of the galaxy
  - Non-circular motions
    - Spiral-arm streaming motions
    - Bar-induced motions
  - Tidal distortions
  - Et cetera

How Robust is the RP method?

!!! Very Much So !!!
General Rotational Parallaxes

- **Unknowns:**
  - **Total Space Velocity:**
    - \( \mathbf{V}_{\text{total}} = \mathbf{V}_{\text{sys}} + (\mathbf{V}_c + \mathbf{V}_p) + \mathbf{V}_\sigma \)
    - = systemic velocity + “orbital” + random
    \[ \Rightarrow 3+1+3+3 = 10 \text{ unknowns} \]
  - **Coordinate system:**
    - Origin of coordinate system \( \Rightarrow 2 \text{ unknowns} \)
    - Position angle of major axis \( (\phi) \) \( \Rightarrow 1 \text{ unknown} \)
    - Distance and Inclination \( \Rightarrow 2 \text{ unknowns} \)
    - Star position in galaxy \( \Rightarrow 3 \text{ unknowns} \)
  - **TOTAL:** \( \Rightarrow 18 \text{ unknowns} \)

- **OBSERVABLES** (per star):
  - 2 positions, 2 proper motions, \( V_{\text{rad}} \)
  \[ = 5 \text{ knowns} \]
• **However:**

  - Many unknowns are shared between test particles:
    - Center of galaxy + PA: 3 shared vars.
    - Systemic velocity: 3 shared vars.
    - Rotation Speed: 1 shared var.
    - Distance: 1 shared var.
    - Inclination: 1 shared var.
    - Velocity dispersion: 3 shared vars.
    - **TOTAL** 12 shared variables

  - Left with: 3 $V_p$'s & x,y,z: 6 star-dependent unknowns

  - No solution because we have 5 observables per star

  - No solution $\Rightarrow$ Eliminate 2 more variables
    - e.g., assume $<V_{p;z}> = 0$ and $<z>=0$
General Rotational Parallaxes (cntd)

- **Then:** \((4 \, N_\ast + N_{sv})\) unknowns
  5 \(N_\ast\) observables

- \(\implies \text{Solution if:}\) \(5 \, N_\ast \geq (4 \, N_\ast + N_{sv})\)
  \(N_\ast \geq N_{sv}\)

- **In our example, if** \(N_\ast \geq N_{sv} = 12\)

- **Alternatively,** allow for corrugations
  
  - \(z(\theta) = z_0 + \sum_{n=1}^{nz} A_n \cos(2n\pi\theta) + B_n \cos(2n\pi\theta)\)
  
  - \(V_{p;z}(\theta) = V_{p;z;0} + \sum_{n=1}^{nvpz} C_n \cos(2n\pi\theta) + D_n \cos(2n\pi\theta)\)

  - \(\implies \text{Increase } N_{sv} \& N_\ast \text{ by: } 2 \ast (n_z + n_{vpz} + 1)\)

  - \(\implies \text{Measure/determine } V_c, D, i \text{ and:}\)
    
    Could add: \(V_c(R) = V_c(R_0) + \frac{dV}{dR} \ast (R-R_0)\) \([N_{sv} = N_{sv} + 2]\)
    
    \(i(R) = i(R_0) + \frac{di}{dR} \ast (R-R_0)\) \([N_{sv} = N_{sv} + 2]\)

  - \(\implies 4 \text{ measured } \& 2 \text{ modeled phase-space cmpnts per star}\)
• **Alternatively:**

  - **Model in-plane peculiar motions,**
    - Either physically from observed light distribution & “M/L”
      - Including: \(i(R), V_c(R), \phi(R),\) Bar, Spiral Arms, Tides
        - Adds more variables [\(i > 1; V_c > 1; \phi > 1;\) Bar: >3; Spiral: >3]
    - Or as Fourier series to check the z-assumptions
      - Iterate between \((z; V_z)\) and \((V_p;x; V_p;y)\)

• **Similar Procedures are/will-be employed for:**

  - Distance determination with maser-regions in galaxies
    - ~17 \(H_2O\) Masers in M31 & M33 at SKA sensitivity
    - Barely exceeds the minimum number of shared variables
  - Velocity-field/Rotation Curve determination of Milky Way
The equations to solve

\[ \kappa D\mu_x = V_{\text{sys},x} + V_{\sigma,x} + V_{c,x} + V_{p,x} \]
\[ \kappa D\mu_{y'} = V_{\text{sys},y'} \sin i_s - (V_{p,z} + V_{\sigma,z}) \cos(i) + (V_{c,y} + V_{p,y} + V_{\sigma,y}) \sin(i) \]
\[ V_r = V_{\text{sys},y'} \cos i_s + (V_{p,z} + V_{\sigma,z}) \sin(i) + (V_{c,y} + V_{p,y} + V_{\sigma,y}) \cos(i) \]

are mildly non-linear with reasonably well-known initial conditions

**Good solutions expected**

*Problem investigated by Olling & Peterson* [2000, aph/0005484]

- Solve \( V_r \) relation for \( (V_{p,z} + V_{\sigma,z}) \) and substitute in \( \mu_{y'} \)
- Or solve \( V_r \) relation for \( (V_{c,y} + V_{p,y} + V_{\sigma,y}) \) and substitute in \( \mu_{y'} \)
- Or solve \( \mu_{y'} \) relation for \( V_{c,y} = V_{c,x} * x/y \) and substitute in \( \mu_x \)
Rewrite equations employing observables \( x, y' \)

- \( \mu_{y'}(V_R) = \alpha_{y'r} \cdot V_R + \gamma_{y'r} \)
- \( \mu_x(V_R; y'/x) = \alpha_{xr} \cdot V_R \cdot y'/x + \beta_{xr} \cdot y'/x + \gamma_x \)
- \( \mu_x(\mu_y; y'/x) = \alpha_{xy'} \cdot \mu_{y'} \cdot y'/x + \beta_{xy'} \cdot y'/x + \gamma_x \)
- Solve for unknown \( \alpha, \beta \) and \( \gamma \) coefficients

- The \( \alpha \) and \( \gamma \) coefficients yield the desired parameters
  - \( \cos^2(i) = -1 / \alpha_{xy'} \)
  - \( D = 1 / [ \alpha_{y'r} \cdot \kappa \cdot \tan(i) ] \)
- Non-circular motions and \( V_{SYS} \) appear only in \( \gamma_{y'r} \) and \( \beta_s \)
- Accuracy of fitted parameters follows from back-substitution and Fourier analysis of velocity field
Rotational Parallax: Observability

• **Need bright sources:**
  • Minimize confusion & Maximize observing speed
    • All stars share (almost) the same proper motion
  • More than enough available”
    • M 33: 1,000 ($\pm 200$) 2MASS stars ($K_s \leq 15$)
    • M 31: 2,000 ($\pm 270$) 2MASS stars ($K_s \leq 15$)
    • LMC: 23,000 UCAC stars ($V \leq 16$)

• **Need least disturbed galaxy**
  M33, M31, LMC

• **My Preference for SIM: M33, M31, LMC**
Probing the Hubble Flow:

- Need to go to $>100$ Mpc
  \[ \epsilon(H_0) \sim \frac{V_{pec}}{V_{Hubble}} \sim 200 \text{ km/s} / (100 \text{ Mpc} \times 75 \text{ km/s/Mpc}) \sim 2.6\% \]
- The only known geometric method that probes that far:
  - Extra-galactic $H_2O$ Masers
    Thin, edge-on disks
    - NGC 4258: $D \sim 7.3$ Mpc
      \[ \Delta D/D \sim 5\% \]
    - NGC 1068 $D \sim 14$ Mpc
    - .... $D \sim 200$ Mpc
      [e.g., Argon et al., 2007, ApJ, 659, 1040]
Mega Maser Distance Uncertainties:

- **N 4258 Distance:**
  \[ 7.2 \pm 0.3 \text{ (random)} \pm 0.4 \text{ (systematic)} \]

- mostly due to orbital eccentricity \cite{Argon2007},

- Up to \( e \sim 0.3 \) due to, e.g., binary black holes \cite{Eracleousetal1995}

- But ruled out by monitoring \cite{Gezarietal2007}

- Not clear that elliptical orbits exist, if not >60% has emissivity variations \cite{StorchiBergmannetal2003}

**Distance error in case of unmodeled eccentricity:**

- \( D_{\text{CIRC}} = D_{\text{TRUE}} \left[ \frac{(1 \mp e)^3}{(1 \pm e)} \right]^{1/2} \sim D_{\text{TRUE}} \left[ 1 \mp 2e \right] \)
  \cite{Olling2007}
Conclusions

- $H_0$ is important for Cosmology
- **1% Galaxy Distances will be possible**
  - SIM should do M33, M31
  - GAIA will do LMC, SMC
- 1% Distance to LG galaxies will calibrate secondary calibrators (Cepheids, TRGB, EBs, ...) to determine $H_0$
- Other methods will also become available for cross-checks: very important
Backup Slides
The Future of Astrometry-enabled Astrophysics in the US

- **Is GAIA going to go before SIM for sure?**
  - If so, then there will be many (10,000's) interesting objects too look at with SIM to get better data

- **Is there going to be dedicated US funding to work with GAIA data?**
  - This would be required to prepare GAIA-follow-up SIM programs

- **Would it not be useful to upgrade SIM,**
  - Make a larger difference with GAIA
  - Deal with the extra-source “burden” (faster of faint sources)
  - Make *definite detections* of Earth-mass planets
  - Improve extra-galactic capabilities

- **How about DARWIN & TPF-I?**
Calibrating the Extragalactic Distance Scale

- I review several methods in my 2007 (MNRAS) paper
- "Standard Candle Methods:"
  - No great fan, but will be calibrated with GAIA
  - Extinction may be greatest difficulty. For known Galactic Cepheids: $\langle A_V \rangle \sim 1.7$ mag
  - GAIA expects: $\epsilon (A_V) \sim 0.1$ mag [Jordi et al, 2006MNRAS.367..290J]
Achievable distance errors as a function of proper motion errors:
LEFT: random errors, RIGHT: systematic component
Symbols: accuracy of radial velocity data (2.5 – 10 km/s)
400 Stars used from: Olling & Peterson (2000)
Distance accuracy (LEFT panel) and Systematic Effects (3 RIGHT panels) as a function of proper motion accuracy.
- We do not understand convection
- We do not consider the variation in micro-turbulent velocity

- We do not have good spectra of the Sun or any star
- We do not have energy distributions for the Sun or any star
- We do not know how to determine abundances:
  - we do not know the abundances of the Sun or any star
- We do not have good atomic and molecular data:
  - 50% of the lines in the solar spectrum are not identified
"A Few Things We do not Know About Stars"

R.L. Kurucz (2002nqsa.conf....3K), cntd

- Cepheids have convective pulsation but the models do not:
  - we do not have high quality spectra over phase for any Cepheid

- We do not understand abundance evolution in early type stars

- Many early type stars are oblate fast rotators
  - e.g., Vega  [Peterson etal,2006Natur.440..896P]
“People sometimes complain that I am too pessimistic and that I criticize too much. In fact I am the most optimistic person. I believe that the human race is tremendously improvable and that humans can solve any problem. **But the most important step in solving a problem is to realize that the problem exists.** When I identify a problem I tell, or try to tell, the people who are capable of doing something about it. I also work on correcting the problem myself, if I am capable. A pessimist does not believe that problems can be solved so does not question the present and does not search for errors. A pessimist acts so “optimistically” about the present that a pessimist prevents progress. **Why worry about basic physics when everything is fine as it is?”**

See also: kurucz.harvard.edu.