

# Finding Solar System Analogs With SIM and HIPPARCOS

## A White Paper for the Exo Planet Task Force

March 2007

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arXiv:0704.3059v1 [astro-ph] 23 Apr 2007

## Abstract

The astrometric signature imposed by a planet on its primary increases substantially towards longer periods ( $\propto P^{2/3}$ ), so that long-period planets can be more easily detected, in principle. For example, a one Solar-mass ( $M_{\odot}$ ) star would be pulled by roughly 1 milli-arcsec (mas) by a one Jupiter-mass ( $M_J$ ) planet with a period of one-hundred years at a distance of 20 pc [cf. eqn. (3) below]. Such position accuracies can now be obtained with both ground-based and space-based telescopes. The difficulty was that it often takes many decades before a detectable position shift will occur. However, by the time the next generation of astrometric missions such as *SIM*<sup>1</sup> [e.g., (Edberg *et al.* 2005)] will be taking data, several decades will have past since the first astrometric mission, *HIPPARCOS* (ESA 1997).

Here we propose to use a new astrometric method that employs a future, highly accurate *SIM* Quick-Look (SQL) survey and *HIPPARCOS* data taken twenty years prior. Using a conservative position error for *SIM* of 10  $\mu$ as, this method enables the detection and characterization of “Solar-system analogs” (SOSAs) with periods up to 80 (165) years for 1 (10)  $M_J$  companions. Employing the standard *SIM* error of 4  $\mu$ as, this period range is extended by a factor of two to four. We might expect the PDF to turn over in this period regime.

Because many tens of thousands nearby stars can be surveyed this way for a modest expenditure of *SIM* time and SOSAs may be quite abundant, we expect to find many hundreds of extra-solar planets with long-period orbits. Such a data set would nicely complement the short-period systems found by the radial-velocity method. Brown dwarfs and low-mass stellar companions can be found and characterized if their periods are shorter than about 500 years. This data set will provide invaluable constraints on models of planet formation, as well as a database for systems where the location of the giant planets allow for the formation of low-mass planets in the habitable zone.

## 1. Introduction

This white paper is based on very recent work summarized in a review paper on *SIM*-science (Unwin *et al.* 2007), while we present more details elsewhere (Olling 2007).

Our current knowledge of the demography of extra-solar planetary systems is mostly a result of long-term radial velocity (RV) monitoring of nearby (mostly) FGK main-sequence (MS) stars. The results are spectacular with over 200 suspected planets in  $\gtrsim 171$  systems (Schneider 2006), with most of the planets in short-period orbits. Only 10% of the observed planets have periods exceeding 5 years, while just one (0.5%) has a period slightly longer than the period of Jupiter (11.9 yr). We use the PDF of extra-solar giant planets (ESGPs) of Tabachnik & Tremaine (2002, hereafter TT2002) but scaled-up by a factor 1.6 to account for current understanding the ESGP frequency [e.g., Sozzetti (2005)]. The updated PDF

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<sup>1</sup> [http://planetquest.jpl.nasa.gov/documents/WhitePaper05ver18\\_final.pdf](http://planetquest.jpl.nasa.gov/documents/WhitePaper05ver18_final.pdf)

indicates that the period range between 5 years and the maximum currently known period should account for 25% of the total number of ESGPs rather than the observed 10%. In fact, the PDF of ESGPs *increases* towards longer periods ( $P$ ) so that systems dominated by long-period planets such as the Solar system may be quite common, and we will use it to estimate the frequency of solar system analogs (SOSAs). However, the PDF<sub>ESGP</sub> *has* to turn over at some period to yield a finite integrated probability. If we *had to* guess where the planetary PDF might turn over, we might pick the period where the stellar PDF turns over, or about 170 years (Duquennoy & Mayor 1991).

We loosely define a SOSA as a system with a (single) planet in the mass range between Jupiter and Uranus/Neptune ( $\sim 0.05 M_J$ ) and with periods between 11.9 years ( $P_{Jupiter}$ ) and 165 years ( $P_{Neptune}$ ). Integrating the PDF<sub>ESGP</sub> over these ranges, we find that 12.6% of systems would be solar-system analogs. If we consider the group of long-period planets that can be detected astrometrically, we need to consider more massive systems with masses ( $M$ ) between 1 and 13  $M_J$ . We call such systems heavy SOSAs, or HOSAs. The PDF<sub>ESGP</sub> predicts that such systems make up about 20% of the total number of planetary systems with periods up to 165 years, and occur around 7.9% of apparently single stars<sup>2</sup>.

NASA’s *SIM* PlanetQuest can detect extra-solar planets weighing several times the mass of the Earth [e.g., Catanzarite et al. (2006)]. If multiple planets exist, their properties can also be determined with *SIM* [e.g., Sozzetti et al. (2003); Ford (2006)]. The mission-end astrometric accuracy of ESA’s *GAIA* astrometric mission is about twenty times worse than *SIM*’s, rendering it not very useful for the project described here.

## 2. Finding Solar-System Analogs

There are several astrometric methods that can be used to identify “long-period” companions of stars. These methods are based on the fact that the actual proper motion ( $\mu$ ) is non-linear if it contains a contribution from the reflex motion of the primary being orbited by a companion. In the “ $\Delta\mu$  method” a substantial difference between  $\mu$  values from a “short-term” catalog such as *HIPPARCOS* and those from a “long-term” proper motion catalog such as *TYCHO-2* is indicative of binarity (Wielen et al. 1999, 2000). Makarov & Kaplan (2005; hereafter referred to as MK2005) find that both the period and mass can be estimated from the acceleration ( $\dot{\mu}$ ) and the jerk ( $\ddot{\mu}$ ), but *only if* the “long-term” proper motion is known. Kaplan & Makarov (2003) developed a method that is appropriate for objects with periods up to twice the mission duration (i.e., up to 10 to 20 years for *SIM*).

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<sup>2</sup>Recent work [Cumming *et al.* (in preparation) as previewed by Butler et al. (2006)] indicates that the mass function declines more rapidly towards higher masses:  $dN/dM \propto M^{-1.9}$  rather than  $dN/dM \propto M^{-1.1}$  as derived by TT2002. As a result, the *number* and relative frequency of HOSAs would decrease, while their *detectability* is unchanged. Because we deal with detectability, we will use the re-scaled TT2002 PDF.

## 2.1. Past and Future Astrometry

Here we propose a new method to identify and quantify long-period systems comprising planetary, brown-dwarf (BD) and main-sequence (MS) companions. Our method uses astrometry of earlier epochs but uses the positions rather than the proper motions. The other component is a future, highly accurate astrometric mission such as *SIM*. The idea is to fit the *SIM* data for a given star with a simple astrometric models [e.g., linear, quadratic, etc.], and use this model to *predict* the position of the star at the *HIPPARCOS* epoch ( $\tau_H$ ). We assume that *SIM* data will be from 2013.5, leading to an epoch difference of 22 years. We assume that the *HIPPARCOS* data are accurate to  $\pm 1$  mas. However, this accuracy can be improved upon in the future by more careful modeling of the systematic effects, or by using the much improved *GAIA* reference frame to define the frame at the *HIPPARCOS* epoch. The former method has been applied recently by van Leeuwen & Fantino (2005) who reduced the *HIPPARCOS* errors by almost a factor of three, while the latter has been used in the construction of the *TYCHO-2* catalog (Høg et al. 2000).

## 3. Model Details

We assume circular, face-on orbits and neglect all the details associated with orbit fitting. The work of MK2005 indicates that the results will not be very sensitive to these simplification. We also assume that the secondary is “dark,” so that the photocenter tracks the motion of the primary. The position ( $z$ ) of the photocenter of the primary is thus a function of: 1) the position ( $z_0$ ) at time  $t = 0$ , 2) the proper motion of the barycenter ( $\mu_{z,B}$ ), 3) the semi-major axis of the orbit of the primary ( $a_o$ ), 4) the orbital period and phase  $\phi$ , and 5) the distance ( $d_{pc}$ ) in pc. (We use  $z$  as shorthand for either  $x$  or  $y$ ). With the period in years, the total mass ( $M_{tot}$ ) in  $M_\odot$  and the mass of the companion ( $M_{C,J}$ ) in  $M_J$ , we find:

$$x(t) = x_{t=0} + \mu_{x,B}t + X_o(t) \quad y(t) = y_{t=0} + \mu_{y,B}t + Y_o(t) \quad (1)$$

$$X_o(t) = a_o \cos(2\pi t/P + \phi) \quad Y_o(t) = a_o \sin(2\pi t/P + \phi) \quad (2)$$

$$a_o = \frac{0.9547}{d_{pc}} \left( \frac{P}{M_{tot}} \right)^{2/3} M_{C,J}, \quad [\text{mas}] \quad (3)$$

$$\tilde{X}_o(t) = a_{o,c\phi} - \left( \frac{2\pi}{P} \right) a_{o,s\phi} t - \frac{1}{2} \left( \frac{2\pi}{P} \right)^2 a_{o,c\phi} t^2 + \frac{1}{6} \left( \frac{2\pi}{P} \right)^3 a_{o,s\phi} t^3 \quad (4)$$

$$\tilde{Y}_o(t) = a_{o,s\phi} + \left( \frac{2\pi}{P} \right) a_{o,c\phi} t - \frac{1}{2} \left( \frac{2\pi}{P} \right)^2 a_{o,s\phi} t^2 - \frac{1}{6} \left( \frac{2\pi}{P} \right)^3 a_{o,c\phi} t^3 \quad (5)$$

with  $a_{o,c\phi} \equiv a_o \cos(\phi)$  and  $a_{o,s\phi} \equiv a_o \sin(\phi)$ , and where we expand the position change  $Z_o(t)$  due to the orbit to third order to arrive at eqns. (4) and (5). We identify the orbit-induced position ( $z_o$ ), the proper motion, acceleration and jerk as the coefficients of the  $t$ -terms with powers 0,1,2 and 3, respectively.  $\tilde{X}$  and  $\tilde{Y}$  are a 3<sup>rd</sup>-order, orbit-based astrometric model. On the other hand, the observed trajectory can be fit by a polynomial up to  $n^{\text{th}}$  order:

$$z_{F,SIM}(t) \approx z_{0,F,SIM} + z_{1,F,SIM} t + z_{2,F,SIM} t^2 + z_{3,F,SIM} t^3 + \mathcal{O}(t^4), \quad (6)$$

where the subscript “ $F, SIM$ ” indicates that the fit is performed to the  $SIM$  data only.  $z_{F, SIM}$  can be evaluated at any previous epoch and compared with the the observed position at that epoch. The position error  $[\delta_z(t)]$  on  $z_{F, SIM}$  depends on the accuracy of the fit and strongly on the epoch difference. The difference between the true position at the  $HIPPARCOS$  epoch and the  $SIM$  prediction is given by  $\Delta_z(\tau_H) = z_H - z_{F, SIM}(\tau_H)$ , while the significance of  $\Delta_z(\tau_H)$  is readily computed. In order to make a significant detection,  $\Delta_z(\tau_H)$  has to be smaller than the errors on both the  $SIM$  prediction and the  $HIPPARCOS$  position.

While  $z_{F, SIM}$  can fit the space motion during the  $SIM$  observing span extremely well, the *extrapolation* of the model to the  $HIPPARCOS$  epoch can lead to large  $\Delta_z$  values when the primary has a companion. In general, large  $\Delta_z(\tau_H)$  values indicate heavy companions, while small values indicate either no companions at all or a low-mass companion.

### 3.1. Period and Mass Estimates from $\Delta_z(\tau_H)$

Ideally, one would like to know the motion of the barycenter so that it could be subtracted from  $z_{F, SIM}(t)$  to yield the orbital contribution. In that case, the period follows from the ratio of the coefficients of eqns. (4) and (5), and would be independent of orbital phase and inclination. Unfortunately, because we do not know  $\mu_B$ , this method can not be used.

Alternatively, it is possible to eliminate the phase effects is by combining the  $x$  and  $y$  positions differences, at least for face-on circular orbits. Initial investigations indicate that the effects of inclination ( $i$ ) are not all that large, as long as  $i \lesssim 45^\circ$ . Keeping in mind that the results are indicative rather than definite, we proceed with circular, face-on orbits. The position differences can be found analytically (Olling 2007), and read:

$$\Delta_x = x(\tau_H) - \tilde{X}(\tau_H) \quad \Delta_y = y(\tau_H) - \tilde{Y}(\tau_H) \quad \Delta_{xy} = \sqrt{\Delta_x^2 + \Delta_y^2} \quad (7)$$

$$\Delta_{xy, \mu}^2 = \frac{a_o^2}{p^2} [1 - 2s_\alpha p + 2(1 - c_\alpha)p^2] \quad \& \quad \Delta_{xy, \mu+\dot{\mu}}^2 = \frac{a_o^2}{p^4} \left[ \frac{1}{4} + c_\alpha p^2 - 2s_\alpha p^3 + 2(1 - c_\alpha)p^4 \right] \quad (8)$$

with  $\alpha \equiv 2\pi\tau_H$ ,  $s_\alpha \equiv \sin(\alpha/P)$ ,  $c_\alpha \equiv \cos(\alpha/P)$ , and  $p = P/(2\pi\tau_H)$ , and where the “ $\mu + \dots$ ” subscripts indicate that an expansion of the orbital motion is used that includes all listed components. The position differences can be ratioed to yield a period estimator:

$$\tilde{P}_{\mu, \dot{\mu}} = \pi \tau_H \frac{\Delta_{xy, \mu}}{\Delta_{xy, \mu+\dot{\mu}}} \approx \begin{cases} P & P \ll \tau_H \\ \frac{3}{2}P & P \gtrsim 2\tau_H \end{cases} \quad (9)$$

which is accurate for either short or long periods. In the intermediate regime,  $\tilde{P}$  oscillates due to the trigonometric terms in eqns. (8). Once the period has been estimated, the companion mass follows from solving either of the  $\Delta_{xy}$  relations for  $a_o$  [and hence mass via eqn. (3)].

The proper motion of the barycenter is not important for this method because it does not matter how the observed proper motion is divided between the center-of-mass- and orbital components. The  $SIM$  model is good because it predicts the observed *positions* at the  $SIM$  epoch, while any position difference at the  $HIPPARCOS$  epoch depends *only* on the orbital parameters and  $\tau_H$ , so that  $\Delta_{xy}$  can be calculated employing the orbital parameters only.

We have performed extensive numerical simulations to test analytical relations for the position differences derived above (Olling 2007). Our modeling of the system comprises an implementation of equations (1) with an arbitrary barycentric motion and a periodic signal in both coordinates (with random phases). We use this model to predict the position at the *HIPPARCOS* epoch. We then generate, in Monte-Carlo fashion, many random realizations of the model which are fitted by a polynomial *to the SIM positions only*. The so-determined *SIM* astrometric model is extrapolated to the *HIPPARCOS* epoch to yield  $\Delta_{xy}$ . We perform: 1) a first order fit to compute  $\Delta_{xy,\mu}$ , and 2) a second-order fit for  $\Delta_{xy,\mu+\dot{\mu}}$ . The numerical results are virtually identical to our analytical predictions [eqn. (8)].

#### 4. Results

We ran simulations that might be relevant for the *SIM* quick-look survey (§5 below): 5 observations per coordinate per star during a period of 18 months. Conservatively, we assume *SIM* position errors of 10  $\mu\text{as}$  per observation. The results are presented in Figure 1 where we plot two  $\Delta_{xy}$  metrics as determined from the first-order fit (abscissa) and the second-order fit (ordinate). In this figure, systems with a given period but with varying mass fall along diagonal lines from the lower-left to the upper-right (drawn lines)<sup>3</sup>. In Figure 1, constant-mass systems are indicated by the dashed lines. The “bunching up” of the lines around  $\Delta_{xy,\mu+\dot{\mu}} = 5$  mas is due to the limited accuracy of the *SIM*-based determination of the acceleration, and this limit is used to generate the thick horizontal line (at 3 times this level). Thus the *SIM* measurements are the limiting factor for the determination of  $\Delta_{xy,\mu+\dot{\mu}}$  rather than the *HIPPARCOS* error. This is the reason why the much less accurate *GAIA* data would not be very useful for this application. Figure 1 indicates that the companion’s mass and period can be determined in a large region of parameter space. The residuals suggest that the orbits of a 1 (10)  $M_J$  ESGPs can be characterized up to periods of 10 (80) years, while this is possible for stellar companions up to 500 years<sup>4</sup>. If we use the expected *SIM* accuracy of 4  $\mu\text{as}$ , this period range is extended by a factor 2 – 4.

#### 5. A SIM Quick-Look Survey for HOSAs

Figure 1 also indicates that a few highly accurate observations suffice to identify planetary, MS and BD companions. Such data could be generated by a *SIM* quick-look (SQL) survey. The targets are bright *HIPPARCOS* stars, so we assume that an SQL observation can be achieved in one minute per position per baseline. Thus, one-thousand stars can be done in 10,000 minutes (6.9 days), so that several thousand stars can be included in an SQL survey without impacting the overall *SIM* mission significantly. Given that the PDF for

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<sup>3</sup> The well-known period-mass degeneracy would result if we were to plot the fitted  $\dot{\mu}$  instead of  $\Delta_{xy,\mu+\dot{\mu}}$ .

<sup>4</sup> The 1<sup>st</sup>-order  $\Delta_{xy,\mu}$  values are significant up to 1,000 years at the Hydrogen burning limit and 4,000 years for a double star with solar-mass components.

Position Difference Predictions  
HIPPARCOS Position – SQL Prediction

$[N, \delta(\mu\text{as})] = [5, 10.0]$

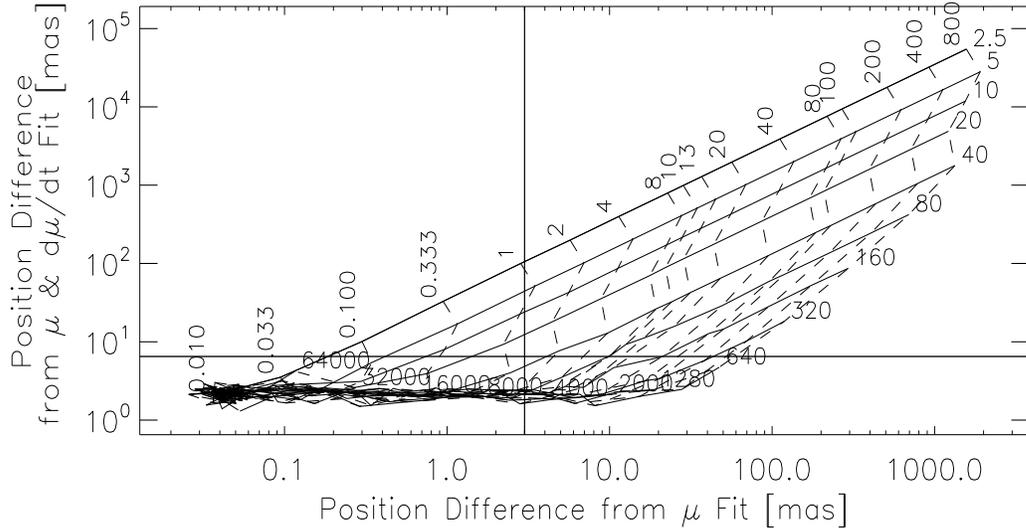


Fig. 1.— For a series of models with various companion masses (vertically oriented numbers in the plot in units of  $M_J$ ) and orbital periods (in years; horizontal numbers) we plot two positions difference computed from our model data. The abscissa is the position difference ( $\Delta_{xy,\mu}$ ), while ordinate is  $\Delta_{xy,\mu+\dot{\mu}}$ . The 3-sigma observational limits are indicated by the thick vertical and horizontal lines.

ESGPs predicts heavy solar-system analogs around 7.9% of stars, a survey of  $\sim 5,000$  stars may find 400 HOSAs. Such a sample would firmly establish the PDF in the long-period regime and indicate how unique the Solar system really is.

In order to maximize the yield of HOSAs (and some SOSAs), an SQL program needs to avoid MS and BD multiples. The subset of 73,000 *ARIHIP* stars (Wielen et al. 2001) that show no signs of binarity is a good starting point for the target selection of an SQL survey.

Those systems that do not show signs of binarity in the SQL+*HIPPARCOS* survey are likely to have either sub-stellar companions or stellar companions with very long-periods. Those systems warrant further SIM observations. The SQL follow-up survey of those stars with suspected sub-stellar companions would be significantly more sensitive than the SQL survey. Figures similar to figure 1 but with employing the SQL follow-up data indicate (not shown) that the ESGPs can be detected with masses as low as 0.1  $M_J$  in 10 year orbits. Period estimation for 1 [10]  $M_J$  is extended by a factor four [two] (to 40 [160] years).

## 6. Conclusions

A judicial combination of *HIPPARCOS* data, a SIM quick-look survey and follow-up SIM observations at full accuracy can uncover several hundred extra-solar planetary systems with periods comparable to the gas giants of the Solar system. Such a program is only possible with *SIM*-like accuracies, and the results would nicely complement radial velocity

and imaging surveys.

Given the importance of accurate pre-*SIM* astrometry, it is sad to realize that the canceled *FAME* mission (Johnston 2003) would have provided an excellent reference catalog for detection and characterization of solar-system analogs. Likewise, it is of pre-eminent importance to continue all-sky astrometric programs at intervals of ten to twenty years to probe the long-period regime. The required accuracy depends on the desired period- and mass ranges, but a survey with an accuracy at the *HIPPARCOS* level (one-half to one mas) would already be very valuable.

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