

ASSIGNMENT No. 10

DUE: Tuesday, May 12

Reading: Read Maoz §4.2 (pp. 69-81) on white dwarfs, and §4.3.2 (pp. 84-85) on neutron stars.

1. White dwarf properties

In class, we showed that the pressure and density for a non-relativistic, degenerate electron gas, are locally related by

$$P = \frac{h^2}{20m_e m_p^{5/3}} \left(\frac{3}{\pi}\right)^{2/3} \left(\frac{Z}{A}\right)^{5/3} \rho^{5/3}.$$

In particular, this will be true when $P = P_c$ and $\rho = \rho_c$, the central pressure and density of a white dwarf. We also argued in class that hydrostatic equilibrium (or the Virial Theorem) will require $P_c = CGM^2/R^4$ for some numerical constant C . The exact solution of hydrostatic equilibrium, evaluated numerically, yields $C = 0.770$, and the ratio of the central density to the average density $\rho_c/\bar{\rho} = 5.99$. The mass is related to the mean density and radius by $M = (4\pi/3)R^3\bar{\rho}$.

(a) Combine the above relations to find the mass-radius relationship for a white dwarf. This should have the form $R = \text{const.}M^{-1/3}$. You must evaluate the constant in terms of m_e , m_p , h ; you may use $Z/A = 1/2$ (explain why).

(b) Use your result in (a) to evaluate the radius, in km, of white dwarfs with masses $0.5 M_\odot$ and $1 M_\odot$.

(c) For the $0.5 M_\odot$ case, evaluate ρ_c and $\bar{\rho}$ in g/cm^3 , and compare to the density of the Sun. Evaluate n_e at the center of the white dwarf, for this case.

(d) Using your result from (c) for n_e at the center of the white dwarf, calculate the Fermi momentum p_F and corresponding Fermi speed $v_F = p_F/m_e$, in km/s . What is the corresponding electron temperature, using $kT \sim p_F^2/(2m_e)$?