

ASSIGNMENT No. 10

DUE: Thursday, May 8

Reading: Read Shu pp. 41-47

Atomic structure and spectrum from “old quantum theory”

From de Broglie’s proposal that matter has a wavelike character, the “wavelength”  $\lambda$  of an electron should be related to its momentum  $p$  by  $\lambda = h/p$ , where  $h$  is Planck’s constant. From Bohr’s principle that the “allowed” orbits in an atom should represent stationary states, the circumference  $2\pi r$  should be an integral number of wavelengths – so that system would be stationary and conceptually, an electron orbiting around the nucleus would “self-interfere” constructively. Together, these ideas imply that

$$\frac{2\pi r}{\lambda} = \frac{2\pi r p}{h} = n \quad \Rightarrow \quad r p = n \hbar, \quad (1)$$

where  $\hbar = h/(2\pi)$  and  $n$  is any positive integer. (*For those of you familiar with Hamiltonian dynamics, note that  $2\pi r p$  is the “action” of the orbit; Bohr’s original proposal was that the action should be quantized, equalling an integer times  $h$ .*)

(a) For a Hydrogen atom, if the electron’s orbit is also governed by force balance, then

$$\frac{m_e v^2}{r} = \frac{e^2}{r^2} \quad (2)$$

where  $v$  is the electron’s speed and  $\mp e$  is the electron/proton charge. Use  $p = m_e v$  and solve (1) and (2) simultaneously to find the spectrum (characterized by the quantum number  $n$ ) of: (i) radii, (ii) electron velocity, and (iii) total energy.

For each of (i)-(iii), give an expression in terms of fundamental constants for arbitrary  $n$ .

For (ii) with  $n = 1$ , evaluate  $v/c$  numerically. Comment on whether the non-relativistic treatment is validated.

For (i) and (iii), compute a value for  $n = 1$  in the following units:

*radius* – cm and Ångstroms ( $1 \text{ \AA} = 10^{-8} \text{ cm}$ )

*energy* – ergs and eV (electron-volts;  $1 \text{ eV} = 1.602 \times 10^{-12} \text{ ergs}$ )

(b) Repeat (a) with necessary modifications for an ion with nuclear charge  $Ze$  and a single electron. For specific ( $n = 1$ ) dimensional radius and energy, use an Iron ( $Z = 26$ ) ion.

(c) From Bohr's principle that transitions between levels  $n$  and  $m$  should involve emission and absorption of discrete photons, calculate the wavelengths (in  $\text{\AA}$ ) and energies (in eV) of the  $(n,m)=(2,3)$  and  $(n,m)=(1,3)$  transitions for Hydrogen, and state what part of the EM spectrum they would be observed in (X-ray,UV,Optical,Radio?). Repeat for an Iron ion with a single electron.