

ASSIGNMENT No. 2

DUE: Tuesday, Feb. 17

READING: Read Maoz pp. 22-27 (§2.2.4) on binary orbits and mass measurements.

1. Moon-planet system

A new planet-moon system is discovered in our Solar system, and your job is to figure out the orbital parameters a and e of the moon, based on a series of observed line-of-sight velocities v_{los} of the moon (supposing that it has a weak atmosphere with emission lines). You know that if the moon's mass is small compared to the planet's mass M , the moon's orbit will obey

$$R = \frac{a(1 - e^2)}{1 + e \cos \varphi},$$

and also that the specific angular momentum

$$\Lambda = R^2 \dot{\varphi} = [GMa(1 - e^2)]^{1/2}.$$

First, use the above relations to show that

$$v_R = \left(\frac{GM}{a}\right)^{1/2} \frac{e \sin \varphi}{(1 - e^2)^{1/2}}$$

and

$$v_\varphi = \left(\frac{GM}{a}\right)^{1/2} \frac{1 + e \cos \varphi}{(1 - e^2)^{1/2}}.$$

Then use these to evaluate v_x and v_y , applying the usual cylindrical to cartesian coordinate transformation.

The plane of the moon-planet system and its semimajor axis are not conveniently aligned with respect to Earth. Suppose that the direction of the observer on Earth with respect to the planet-moon system is $\hat{z} \sin i + (\hat{x} \cos \psi + \hat{y} \sin \psi) \sin i$. Use this to write down an expression for $v_{los} = (v_x \cos \psi + v_y \sin \psi) \sin i$, in terms of i , ψ , φ , M , and the unknowns a and e . Then describe how you would use the observed variation of v_{los} over an orbit of period P to deduce the values of a and e . What constraints could your observations provide on the planet's mass M ?

2. Planetary orbit properties

The orbit with the minimum energy for a given angular momentum is circular. Show this, based on the formula relating orbital eccentricity, energy, and angular momentum:

$$e = \left(1 + \frac{\Lambda^2 2\epsilon}{(GM)^2} \right)^{1/2} .$$

Then discuss what this principle might have to do with the fact that the major planets in our solar system have low eccentricity. Interestingly, many of the recently-discovered extrasolar planets have *high* eccentricity. Can you think of possible reasons for the differences? *Note: there is no “right answer” to this question yet! The differences between other planetary systems and our own came as a surprise, and theorists are very busy right now trying to explain why. Lots of ideas have been proposed, but there is not yet any generally-accepted explanation.*

3. Epicycles for Kepler potential

Consider a near-circular orbit with epicyclic perturbations $x_1 = -A \sin \Omega t$ and $y_1 = 2A \cos \Omega t$, where $R = R_0 + x_1$ and $\varphi = -\Omega t + y_1/R_0$. Draw the orbit of the guiding center and the epicycle motion about the guiding center (note that the signs are different from what we did in class!). Based on your sketch, show that the guiding center is offset by approximately a distance A from the central mass.