

ASSIGNMENT No. 2

DUE: Tuesday, Feb. 19

READING: Read Shu pp. 265-267 (epicyclic theory) and 182-183 (binary orbits).

1. Two-body orbits: direct solution

In class, we showed that for masses M_1 and M_2 in a two-body orbit with vector separation $\mathbf{R} = \mathbf{R}_1 - \mathbf{R}_2$, the equation of motion for \mathbf{R} is

$$\ddot{\mathbf{R}} = -\frac{G(M_1 + M_2)\mathbf{R}}{R^3},$$

and the bodies' positions with respect to the center of mass are $\mathbf{R}_1 = (\mu/M_1)\mathbf{R}$ and $\mathbf{R}_2 = -(\mu/M_2)\mathbf{R}$, where $\mu = M_1M_2/(M_1 + M_2)$ is the reduced mass. Show that the equation of motion for M_1 is therefore

$$\ddot{\mathbf{R}}_1 = -\left(\frac{\mu}{M_1}\right)^3 \frac{G(M_1 + M_2)\mathbf{R}_1}{R_1^3}$$

with an analogous expression for M_2 . Thus, M_1 obeys a “one-body” equation of motion with an *equivalent* central mass of $M_{ctr,1} = (\mu/M_1)^3(M_1 + M_2)$. A similar equation holds for M_2 with an equivalent central mass $M_{ctr,2} = (\mu/M_2)^3(M_1 + M_2)$; i.e. simply switching indices 1 and 2. Use the “equivalent one-body” equation of motion to derive the specific energy and specific angular momentum of M_1 , ϵ_1 and Λ_1 , by following the same steps we used for our original one-body derivation. Show that $\Lambda_1 = (\mu/M_1)^2\Lambda$ and $\epsilon_1 = (\mu/M_1)^2\epsilon$, where Λ and ϵ are the specific angular momentum of the “reduced-mass” particle derived in class, $\Lambda = R^2\dot{\varphi}$ and $\epsilon = (1/2)(\dot{R}^2 + (\Lambda/R)^2) - G(M_1 + M_2)/R$. Then use the expressions relating specific energy and specific angular momentum to semimajor axis, semiminor axis, and eccentricity, to derive expressions for a_1 , b_1 , and e_1 , showing that $a_1 = (\mu/M_1)a$, $b_1 = (\mu/M_1)b$, and $e_1 = e$, where a , b , and e are the semimajor axis, semiminor axis, and eccentricity of the “reduced mass” particle. Since the same would hold for Mass 2 by substituting $M_1 \rightarrow M_2$, this shows directly that the individual masses' orbits are just rescaled versions of each other. [35 points]

2. Epicyclic orbits

The solution we found for the first-order perturbations involved in epicyclic motion was

$$x_1 = A \cos \Omega_0 t, \quad y_1 = -2A \sin \Omega_0 t,$$

where $A/R_0 \ll 1$ is required for self-consistency in the perturbation approach. Show that to first order in the small parameter A/R_0 , the specific energy and angular momentum of the epicyclic orbit are the same as for the circular orbit, and hence the semimajor and semiminor axes are also unchanged to order A/R_0 . Show, however, that the center of the epicyclic orbit is displaced from the central mass by a distance A (draw a picture). Explain these results in terms of the number of powers of e (the eccentricity) that appear in the expressions for a , ϵ , and f in terms of M , Λ , and e .

[35 points]

3. Extrasolar planet

You have obtained a time-series set of radial-velocity observations of a solar-type star (you may assume the mass = $1 M_\odot$), and find a sinusoidal variation in v_{obs} with a period of 8 days, and amplitude $v_{max} = 220 \text{ m s}^{-1}$. You interpret these variations as reflex motion caused by a planet in a circular orbit around the star. With sensitive photometric observations from HST, you conclude that there is a periodic eclipse of the star with the same period as that of the planet; you interpret this as a transit of the planet between you and the star. From this information, you add a new extrasolar planet to your list. What are the mass of the new planet and its distance from its parent star? If you had not observed the eclipsing behavior, how would your conclusions have changed?

[30 points]