ASTR 320 – Spring 2009
Prof. Ostriker

ASSIGNMENT No. 4

DUE: Tuesday, March 10

Reading: Read Maoz pp. 140-145 on MW galaxy, pp. 149-151 on dark matter, pp. 171-173 on clusters.

1. Elliptical galaxy mass

From the Virial Theorem, kinetic energy (K) and potential energy (W) are related by $2K + W = 0$. For a spherical system with an isotropic distribution of velocities, $K = 3\sigma_1^2 M/2$ where $\sigma = \langle (v_{los} - \bar{v}_{los})^2 \rangle^{1/2}$ is the velocity dispersion in any one direction, such as the observer's line-of-sight. Use these relations together with the relationship

$$W = -\frac{GM^2(3-\alpha)}{R(5-2\alpha)}$$

for a power-law density distribution $\rho \propto r^{-\alpha}$ to estimate the mass (in $\mathrm{M}_\odot$) of a giant elliptical galaxy with $R = 40\,\mathrm{kpc}$ and dispersion in the observed velocities of member stars $\sigma_{los} = 300\,\mathrm{km\,s^{-1}}$. You may assume that $\alpha = 2$. Make sure to be careful with units!

2. Dark matter in galaxy clusters

Consider a cluster of galaxies of radius 1Mpc, with line-of-sight dispersion in velocities of member galaxies $800\,\mathrm{km\,s^{-1}}$, containing 1000 galaxies each of typical luminous mass $10^{11}\,\mathrm{M}_\odot$. What is the ratio of dark matter to visible matter (i.e. the total mass of the visible galaxies) in the cluster? You may assume $\alpha = 2$.

3. Open and bound clusters

Consider an evolving star cluster that obeys the Virial Theorem. Initially, the cluster begins with a mixture of stars and gas, and a radius R. Let the initial mass fraction of stars in the cluster be $f_* = M_*/M_{init}$, so that the initial mass of gas is $M_g = (1 - f_*)M_{init} = M_*(1 - f_*)/f_*$. You may assume that the gas molecules initially have the same velocity dispersion as the stars, as well as the same spatial distribution. Suppose that the gas is expelled very rapidly from the cluster by the action of supernovae when the cluster's most massive stars explode. You may assume that the change to the cluster's *stellar* mass is small (since the massive stars are a small fraction of the total), so that $M_{final} = M_*$. You may also assume that the distribution of the positions and velocities of the rest of the stars are not altered when the supernovae drive out the gas. Thus, the kinetic energy of the stars does not change, when the gas is removed, but the potential energy of the stars changes because the potential well changes when the gas is blown away. For the following, you may assume uniform density ($\alpha = 0$); think about how your answer would change if the cluster were centrally concentrated.

(a) What is the minimum value of f_*, f_{min}, such that the cluster would remain bound after the gas is expelled?

(b) If $f_* > f_{min}$, the cluster remains bound. If the cluster re-virializes while conserving its total energy, what would the final cluster radius be (in terms of R)? You may assume when the cluster re-virializes, it will have the same functional form for the density distribution as it did initially (e.g. same power-law index). Check that your answer has the correct behavior in the limits $f_* \to f_{min}$ and $f_* \to 1$.