

ASSIGNMENT No. 5

DUE: Thursday, March 13

**Reading:** Read Shu pp. 175-178 and 265.

**Dynamical Friction**

In class, we showed that the dynamical friction drag force on a mass  $M$  orbiting with speed  $V$  through a distribution of (smaller) masses  $m$ , with  $n =$  the number density of masses  $m$  per unit volume, is given by

$$F_{DF} = -\frac{4\pi n(GM)^2 m \ln(\Lambda)}{V^2}, \quad (1)$$

where  $\ln(\Lambda) = \ln(b_{max}/b_{min})$ . The results are insensitive to the exact value of  $\Lambda$ ;  $b_{min} = GM/V^2$  and  $b_{max} = \text{radius of system}$  are adequate for the current purposes.

Consider the case where  $M$  is on a near-circular orbit of radius  $r$ , and the total density of the background masses is  $mn = \rho(r) = \rho_R(r/R)^{-2}$ . From HW #4, you found that the total mass within  $r$  for this mass distribution is  $\mathcal{M}(r) = 4\pi\rho_R r^3 (r/R)^{-2}$ .

**(a)** Show that the circular velocity  $V_c$  is independent of radius,  $V_c^2 = GM(r)/r = GM(R)/R = 4\pi G\rho_R R^2$ . The angular momentum of  $M$  on its circular orbit at  $r$  is  $J = V_c r M$ , so that if the mass  $M$  remains on a near-circular orbit and spirals in to the center of the distribution, the rate of change of angular momentum is related to the rate of change of radius by  $dJ/dt = V_c M dr/dt$ .

**(b)** Using the above expression (1) for  $F_{DF}$ , compute the torque on  $M$ , and equate to  $dJ/dt$  in order to find a relation between  $\dot{r}$ ,  $r$ , and constant factors. Integrate this relation (it should have the form  $r\dot{r} = K$ ) to find  $r$  as a function of  $t$ , assuming that  $M$  is initiated at  $r_i$  at time  $t = 0$ . In particular, write down an expression for the time  $t_{DF}$  when  $M$  reaches  $r = 0$ .

**(c)** Apply the above considerations to compute  $t_{DF}$  for a  $10M_\odot$  star in a globular cluster, taking the total cluster mass as  $10^5 M_\odot$  within 1 pc radius, a density profile  $\propto r^{-2}$ , and an initial orbit for  $M$  at the edge of the cluster. Compare  $t_{DF}$  to the crossing time, and age of clusters, as discussed in class. Also compare to the physical collision time of stars, given by  $t_{collis} = (n\sigma V)^{-1}$ , where the physical crosssection  $\sigma = \pi R_*^2$  (you may use  $R_* = R_\odot$ ).

**(d)** Similarly to (c), compute  $t_{DF}$  for a globular cluster within a galaxy. For orbits away from the disk, most of the dynamical friction comes from the dark matter halo, which has a density law  $\rho \propto r^{-2}$ . You may use  $V_c = 250 \text{ km/s}$ , and an initial cluster distance from the center of the galaxy of  $20 \text{ kpc}$ . Compare to the crossing time and age of the galaxy.