

ASSIGNMENT No. 5

DUE: Tuesday, March 24

Dynamical Friction

In class, we showed that the dynamical friction drag force on a mass M orbiting with speed V through a distribution of (smaller) masses m , with $n =$ the number density of masses m per unit volume, is given by

$$F_{DF} = -\frac{4\pi n(GM)^2 m \ln(\Lambda)}{V^2}, \quad (1)$$

where $\ln(\Lambda) = \ln(b_{max}/b_{min})$. The results are insensitive to the exact value of Λ ; $b_{max} = R$ and $b_{min} = GM/V^2$ or $= n^{-1/3} \sim R/N^{1/3}$ are adequate for the current purposes.

Consider the case where M is on a near-circular orbit of radius r , and the total density of the background masses is $mn = \rho(r) = \rho_R(r/R)^{-2}$.

(a) Show that this density distribution yields a mass within r of $\mathcal{M}(r) = 4\pi\rho_R r^3 (r/R)^{-2}$, and a circular velocity V_c that is independent of radius, $V_c^2 = G\mathcal{M}(r)/r = G\mathcal{M}(R)/R = 4\pi G\rho_R R^2$. The angular momentum of M on its circular orbit at r is $L_M = V_M r M = V_c r M$, so that if the mass M remains on a near-circular orbit and spirals in to the center of the distribution, the rate of change of angular momentum is related to the rate of change of radius by $dL_M/dt = V_c M dr/dt$.

(b) Using the above expression (1) for F_{DF} , compute the torque $\tau_M = rF_{DF}$ on M , and equate to dL_M/dt in order to find a relation between \dot{r} , r , and constant factors. Integrate this relation (it should have the form $r\dot{r} = K$) to find r as a function of t , assuming that M is initiated at r_i at time $t = 0$. In particular, write down an expression for the time t_{DF} when M reaches $r = 0$.

(c) Apply the above considerations to compute t_{DF} for a $10M_\odot$ star in a globular cluster, taking the total cluster mass as $10^5 M_\odot$ within 1 pc radius, a density profile $\propto r^{-2}$, and an initial orbit for M at the edge of the cluster. Compare t_{DF} to the crossing time, and age of clusters, as discussed in class. Also compare to the physical collision time of stars, given by $t_{collis} = (n\sigma V)^{-1}$, where the physical crosssection $\sigma = \pi R_*^2$ (you may use $R_* = R_\odot$).

(d) Similarly to (c), compute t_{DF} for a globular cluster within a galaxy. For orbits away from the disk, most of the dynamical friction comes from the dark matter halo, which has a density law $\rho \propto r^{-2}$. You may use $V_c = 250 \text{ km/s}$, and an initial cluster distance from the center of the galaxy of 20 kpc . Compare to the crossing time and age of the galaxy.