

ASSIGNMENT No. 6

DUE: Thursday, April 10

Reading: Read Shu pp. 62-68, 77-79

1. Thermal radiation distribution

As discussed in class, the number of photons as a function of momentum in a thermal distribution can be written as $dn(\mathbf{p}) = f_{Rad}(p, T)d^3p$, where dn is the number of photons/volume with momenta within the element d^3p , and $p = |\mathbf{p}|$. We will show later on (using quantum mechanics) that

$$f_{Rad} = \frac{2}{h^3(e^{\frac{cp}{kT}} - 1)},$$

where h is Planck's constant.

(a) Integrate over momenta to evaluate $n = \int f_{Rad}(p)d^3p$ in terms of fundamental constants and the temperature. *Hint:* spherical-polar coordinates are useful for this; also, there is an integral you will have to look up or compute using Mathematica, Matlab, etc.

Compute the mean number per volume (in cm^{-3}) of Cosmic Background Radiation (CBR) photons now (when $T = 2.725 \text{ K}$), and at the radiation-matter decoupling epoch when the radiation temperature was larger by a factor ~ 1400 .

(b) As discussed in class, the energy density (energy/volume) in radiation is given as

$$u_{rad} = \int cpf_{Rad}(p)d^3p.$$

We showed in class (using principles of thermodynamics and the fact that energy density and pressure for radiation are related by $u_{rad} = 3P_{rad}$), that u must be given by $u = a_{rad}T^4$ for some constant factor a_{rad} . Evaluate the integral for u_{rad} to find an expression for a_{rad} in terms of fundamental constants. (Compare to tabulated values to check your work.)

Compute the CBR energy density now and at decoupling, in erg/cm^3 .

(c) Using $S = (4/3)a_{rad}T^3V$ from class and your result from (a), compute the mean entropy/ k /photon. Interpret in terms of the statistical meaning of entropy discussed in class: $S = Nk \ln(\# \text{ of states per particle})$. *Note:* since the total number of states is multiplicative for independent particles, $\ln(\# \text{ of states for N-particle system}) = \ln(\# \text{ states per particle})^N = N \ln(\# \text{ states per particle})$.