

ASSIGNMENT No. 7

DUE: Thursday, April 17

Reading: Read Shu pp. 358-362, 386-388

1. Cosmic evolution

In class, we found that the evolution of the Universe in the matter-dominated era is accordance with the Newtonian theory. Thus, if M is the total mass contained within a “comoving” radius R (where R is a function of time), the equation of motion obtained by setting $\ddot{R} = -GM/R^2$ leads to the integral

$$\frac{1}{2}\dot{R}^2 - \frac{GM}{R} = \text{const.} \equiv E.$$

This can be rearranged to yield $dR = \pm [2(E + \frac{GM}{R})]^{1/2} dt$. In class, we solved this to obtain $R \propto t^{2/3}$ when $E = 0$.

(a) Show that a parametric solution to this equation in the case $E > 0$ is

$$R = \frac{GM}{E} \sinh^2(\theta) \quad \text{and} \quad t = \frac{GM}{\sqrt{2}E^{3/2}} (\frac{1}{2} \sinh(2\theta) - \theta),$$

and a parametric solution to this equation in the case $E < 0$ is

$$R = \frac{GM}{E} \sin^2(\theta) \quad \text{and} \quad t = \frac{GM}{\sqrt{2}|E|^{3/2}} (\theta - \frac{1}{2} \sin(2\theta)).$$

(b) Show that in the small- t limit, both of the above solutions yield $R \propto t^{2/3}$. Interpret this result in terms of the early evolution of the Universe.

2. Cosmic energy densities

Adopting current values of $\Omega_{matter} = 0.3$, $T_{rad} = 2.73\text{K}$, and using $H_0 = 100h \times \text{km s}^{-1} \text{Mpc}^{-1}$ with $h = 0.7$, compute the cosmic contraction factor relative to the present, $R_0/R \equiv (1 + z)$, at which the energy density of matter and energy density of radiation were equal. How does the redshift compare to the redshift of matter-radiation decoupling, $z \sim 1400$?