

ASSIGNMENT No. 7

DUE: Thursday, April 16

Reading: Read Maoz Ch. 8. Additional (optional) material on observational probes of cosmic parameters is in Ch. 7 and 9.

1. Cosmic evolution

In class, we solved for the evolution of the Universe in the matter-dominated era for the case when the curvature is zero, corresponding to zero energy for our Newtonian analogue. Here, we will consider the non-zero curvature cases.

The Friedmann equation for a matter-dominated, zero- Λ Universe yields

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G\rho}{3} - k\frac{c^2}{R^2}$$

where k is the curvature. In our Newtonian derivation, the term $-kc^2$ represents $2E$, so that $k = 1$ is bound and $k = -1$ is unbound. This can be rearranged to yield $dR = \pm [-kc^2 + \frac{2GM}{R}]^{1/2} dt$, where $M = 4\pi R^3\rho/3$ is the mass inside R . In class, we solved this to obtain $R \propto t^{2/3}$ when $k = 0$.

(a) Show that a parametric solution to this equation in the case $k = -1$ is

$$R = \frac{2GM}{c^2} \sinh^2(\theta) \quad \text{and} \quad t = \frac{GM}{c^3} (\sinh(2\theta) - 2\theta),$$

and a parametric solution to this equation in the case $k = +1$ is

$$R = \frac{2GM}{c^2} \sin^2(\theta) \quad \text{and} \quad t = \frac{GM}{c^3} (2\theta - \sin(2\theta)).$$

(b) By Taylor expanding for θ small, show that in the small- t limit, both of the above solutions yield $R \propto t^{2/3}$. Interpret your result in terms of the early evolution of the Universe with differing k .

2. Cosmic energy densities

Adopting current values of $\Omega_{matter} = 0.27$, $T_{rad} = 2.73\text{K}$, and using $H_0 = 100h \times \text{kms}^{-1} \text{Mpc}^{-1}$ with $h = 0.73$, compute the cosmic contraction factor relative to the present, $R_0/R \equiv (1+z)$, at which the energy density of matter and energy density of radiation were equal. How does this redshift compare to the redshift of matter-radiation decoupling due to recombination, $z \sim 1100$?