

ASSIGNMENT No. 8

DUE: Thursday, April 23

1. Galactic ISM vertical distribution

Consider a spiral galaxy in which the gravitational potential is dominated by a spherical dark matter halo which has a flat rotation curve  $\Omega = V_c/R$  for  $V_c = \text{const.}$

(a) Show, using a Taylor expansion about  $z = 0$ , that the vertical gravity at radius  $R$ , and height  $z$  above the midplane, is

$$g_z = -\frac{\partial\Phi}{\partial z} = -\frac{d\Phi}{dr} \frac{\partial r}{\partial z} \approx -\Omega^2 z.$$

(b) Review the derivation in class to show that hydrostatic equilibrium  $\rho^{-1}\partial P/\partial z = g_z$  for an isothermal layer and  $g_z = -\Omega^2 z$  results in a Gaussian density profile,

$$\rho = \rho_0 e^{-z^2/(2H^2)}$$

where  $H = c_s/\Omega$ ,  $\rho_0$  is the midplane density, and  $c_s^2 = kT/\mu$ .

(c) Suppose that the mean midplane number density is  $n \equiv \rho/\mu = 1 \text{ cm}^{-3}$  and the temperature is  $10^4 \text{ K}$ . Calculate the total surface density, in  $M_\odot \text{ pc}^{-2}$ , at radius  $R = 10 \text{ kpc}$ , assuming  $V_c = 200 \text{ km s}^{-1}$ . You may assume a Solar abundances so that  $\mu = 1.4m_p$ , where  $m_p$  is the mass of a proton. Make sure, in your integral, to include both sides of the disk ( $z > 0$  and  $z < 0$ ).

2. Cold interstellar cloud

Consider a cold molecular cloud with  $T = 10 \text{ K}$  and  $\mu = m_{H_2}$  embedded within the warm interstellar medium, with  $T_{ext} = 8000 \text{ K}$  and  $n_{ext} = 0.3 \text{ cm}^{-3}$ .

Treating the cloud as a truncated singular isothermal sphere (i.e. with  $\rho \propto r^{-2}$  everywhere between  $r = 0$  and  $r = R_{ext}$ ), remind yourself of why the outer radius obeys

$$R_{ext} = \frac{c_s^2}{(2\pi G)^{1/2} P_{ext}^{1/2}},$$

and the total mass is

$$M = \left(\frac{2}{\pi}\right)^{1/2} \frac{c_s^4}{G^{3/2} P_{ext}^{1/2}}.$$

Here,  $P_{ext}$  is the pressure of the external medium (warm ISM), and  $c_s^2 = v_{th}^2 = kT/\mu$  in the cloud.

(a) Evaluate the ratio of thermal to gravitational potential energy in the cloud, and related to the Virial Theorem. *Hint:* recall the expression we had before for the potential energy of a truncated power-law sphere,  $W = -(3/5)\beta GM^2/R$  for  $\beta = (1 - \alpha/3)/(1 - 2\alpha/5)$ .

(b) Find an expression for the average density in the cloud,  $\bar{\rho} \equiv M_{cloud}/V_{cloud}$ , in terms of  $P_{ext}$  and the cloud's temperature and mean molecular weight. Using the values from above, compare to the ambient ISM density.

(c) Using the values supplied above, compute the cloud mass in solar masses and the cloud size in parsecs. How would these change if the external pressure were larger by a factor 10? If stars originate as pressure-bounded cold molecular clouds that subsequently collapse, how do you think these results might be related to the observed distributions of stellar masses in the Galaxy?