ASTR 320 – Spring 2008
Prof. Ostriker

ASSIGNMENT No. 9

DUE: Thursday, May 1

Reading: Read Shu pp 95-96; and pp. 68-70, 395

1. Convection in a disk

For the disk around a star (or black hole) of mass M, recall that the vertical equilibrium profile at radius R from the center is given by

$$\frac{1}{\rho}\frac{dP}{dz} = -z\Omega^2$$

where $\Omega^2 = GM/R^3$.

Recall also that the entropy per particle is given by

$$S = \frac{k}{\gamma - 1}\ln(P\rho^{-\gamma}) + const.$$

where $\gamma = C_p/C_V \to 5/3$ for an ideal, monatomic gas.

Show that the condition for convective instability to occur is

$$\frac{dT}{dz} < -\frac{(\gamma-1)}{\gamma}\frac{\mu}{k}z\Omega^2,$$

where $T(z)$ is the vertical temperature profile.

Show therefore that if the temperature varied $\propto e^{-z/H}$, a region near the midplane would become convectively unstable.

2. Jeans masses and sizes

For the following ISM conditions, compute the Jeans length

$$L_J \equiv c_s\left(\frac{\pi}{G\rho}\right)^{1/2}$$

and Jeans mass

$$M_J \equiv \rho L_J^3 :$$

(a) the warm ISM, with $T = 8000\,\mathrm{K}$ and $n = 0.3\,\mathrm{cm}^{-3}$; (b) diffuse atomic hydrogen clouds with $T = 60\,\mathrm{K}$, $n = 40\,\mathrm{cm}^{-3}$; (c) cold molecular clouds with $T = 10\,\mathrm{K}$, $n = 100\,\mathrm{cm}^{-3}$; (d) dense molecular cores with $T = 10\,\mathrm{K}$, $n = 10^4\,\mathrm{cm}^{-3}$; Comment on what you would expect for the location of star formation. *Note: recall* $c_s^2 = v_{th}^2 = kT/\mu$.