ASTR 320 - Theoretical Astrophysics – Spring 2004 March 18, 2004
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MIDTERM EXAM

1. Virial theorem

Use the virial theorem to estimate the mass of a galaxy cluster with outer radius 1 Mpc and line-of-sight velocity dispersion 1000 $\mathrm{km\,s^{-1}}$. Give your answer in $M_\odot$. *Note*: you may set the dimensionless constant factor in the potential energy equal to unity. **[25 points]**

2. Dynamical relaxation and dynamical friction

(a) Choose *one* of the following and show (by comparing the relaxation time with the age) whether or not it can have relaxed by 2-body effects: (i) a globular cluster, (ii) a galaxy, (iii) a galaxy cluster. From class, the ratio of relaxation time to crossing time is $\approx 0.1N/\ln(N)$ where N is the total number of bodies. **[25 points]**

(b) Consider a supermassive black hole (BH), of mass $M = 10^6 M_\odot$, which is orbiting in the bulge of a galaxy containing a total stellar mass of $M_{tot} = 10^{10} M_\odot$ within 1kpc of the center. From class, the dynamical friction drag force (with total stellar background mass/volume ρ) was

$$F_{DF} = \frac{-4\pi\rho(GM)^2}{V^2}\ln(\Lambda),$$

where $\ln(\Lambda)$ is $\ln(M_{tot}/M)$. *Estimate* the dynamical friction time t_{DF} for the black hole to sink to the center of the galaxy by taking the ratio of the initial angular momentum of the black hole (for a circular radius at distance 1 kpc) to the initial torque due to dynamical friction at that distance. *Hint:* You will need $V^2/R = GM_{tot}/R^2$, the force balance equation for the initial circular orbit. **[25 points]**

3. One-body orbit

Prove that specific angular momentum $\Lambda = R^2\dot{\varphi}$ and energy $E = (1/2)[\dot{R}^2 + R^2\dot{\varphi}^2 - GM/R]$ are conserved for the orbit of a mass m about another (fixed) mass M. You may find it convenient to use the equation of motion for m in cylindrical coordinates,

$$(\ddot{R} - R\dot{\varphi}^2)\hat{R} + (R\ddot{\varphi} + 2\dot{R}\dot{\varphi})\hat{\varphi} = -\frac{GM}{R^2}\hat{R}.$$

[25 points]