

FINAL EXAM

1. Cosmological structure formation

(a) Show that a collisionless system (stars or dark matter particles) formed via gravitational collapse starting from rest at near-uniform density ρ_0 will have final density $\rho \approx 8\rho_0$. [8 points]

(b) Explain qualitatively how the average density of a galaxy relates to the cosmological time at which it formed. [6 points]

2. Convective Instability

(a) State and derive the general criterion for convective instability. [16 points]

(b) Describe an astronomical situation in which convective instability is important, and what consequences it has. [8 points]

3. Degeneracy pressure and white dwarfs

(a) Argue from quantum principles (be specific) that a degenerate electron gas has typical particle momentum $p \sim n_e^{1/3} \hbar$, where n_e is the number/volume of electrons. [8 points]

(b) Show, based on the definition of pressure P , why a relativistic, degenerate electron gas has $P \sim n_e^{4/3} c \hbar$. [8 points]

(c) Explain why the average pressure, mass, and radius of a star are related by $GM^2 \sim R^4 P$. [8 points]

(d) Using the results in (a)-(c), find an estimate for the Chandrasekhar mass M_{Ch} in terms of \hbar , c , G , and m_p . Plug in values of these fundamental constants and compare to the known value of M_{Ch} . [12 points]

4. Entropy

(a) The energy density of thermal radiation is $u = aT^4$, and the pressure for any relativistic fluid is $P = u/3$. Starting with the thermodynamic identity $dE = -PdV + TdS$, prove that $S = (4/3)aT^3V + const.$ for thermal radiation. [10 points]

(b) Based on (a), how does T vary with the scale factor R of the universe during the radiation-dominated epoch? [4 points]

5. Self-gravitating instability

Use scaling relations to show that for a near-uniform medium of density ρ and temperature T , the masses of fragments that form via gravitational instability should exceed $M_J \sim [kT/(G\mu)]^{(3/2)}\rho^{-(1/2)}$. What does this imply about the regions within the ISM where star formation can occur? [**12 points**]