Constraining the Age of the Universe with WMAP

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Throughout astronomy the key features for studying celestial objects are to measure how far away it is and how long it has existed. In cosmology, these two are coupled because the horizon of the visible universe is at a distance of ct_o, where t_o is the age of the universe. Though this consideration produces the horizon paradox, we can say that studying the age of the universe is equivalent to studying the distance to the horizon. There are many different ways to measure the age of the universe, we will discuss here how to determine it by measuring anisotropies in the cosmic microwave background (CMB) with the Wilkinson Microwave Anisotropy Probe (WMAP).

Historically, the age of the universe has always had a lower limit set by the age of celestial bodies, like globular cluster dating, Type Ia SN decay curves, white dwarf cooling curves and galactic evolution models. These measurements have yielded strongly varying results, from 9 - 20 Gyr with uncertainties of up to 10 Gyr (Spergel et al. 2003: Clayton 1988). But more recently, measurements have constrained the age of the oldest globular clusters to within 11.1 - 13.4 Gyr (Spergel et al. 2003). Similarly, white dwarf cooling curves have dated the age of galaxies to be 14.5 ± 1.5 Gyr (Spergel et al. 2003: Renzini et al. 1996) and 12.7 ± 0.35 Gyr (Richer et al. 2002; Hansen et al. 2002). For these reasons, we accept as a prior for the lower limit of 11 Gyr (95% confidence).

There are a number of methods for determining the age of the universe from WMAP's measurements of the CMB; most of which involve numerically optimizing using a Monte Carlo Markov Chain (MCMC) routine. The WMAP data constrains most cosmological parameters, like $\Omega_m, \Omega_b, h, n_s, \sigma_8$, etc and the age of the universe can be determined by a combination of these. We will discuss three methods for calculating the age of the universe from the CMB.

First, by integrating the Friedmann, and solving for t, reveals the integral (Spergel 2003),

$$t(z) = H_o^{-1} \int_0^z \frac{dz}{(1+z)[\Omega_\Lambda + \Omega_k(1+z)^2 + \Omega_m(1+z)^3 + \Omega_r(1+z)^4]^{1/2}}$$

After numerically determining Ω_m, Ω_k, Ω_Λ, Ω_r & h, by MCMC fitting the CMB power spectrum, and taking $z \to \infty$, the above integral determines the age of the universe.

Another method, from Knox et al. 2001, shows that,

$$t_o = 6.52(\Omega_\Lambda h^2)^{-1/2} \ln\left(\frac{1+(\Omega_\Lambda)^{1/2}}{(1-\Omega_\Lambda)^{1/2}}\right) \qquad \text{and} \qquad \frac{\Delta t_o}{t_o} = -0.12\left(3.0\frac{\Delta\omega_m}{\omega_m} + 1.2\frac{\Delta\omega_\Lambda}{\omega_\Lambda}\right)$$

Thus, by constraining Ω_Λ or ω_m, ω_Λ, $\Delta\omega_m$ and $\Delta\omega_\Lambda$ from the WMAP power spectrum, this equation determines the age of the universe today.

Similarly, we can measure the angular scale of the first peak, l_1, in the WMAP power spectrum and correlate that to the age of the universe (Page et al. 2003). The position of the first peak determines the curvature of the universe. By measuring the curvature of the universe we can understand the path that space-time has traveled since then. Combining this with $z_{dec} \approx 1000$ (Eisenstein & Hu 1998), we can determine the time since decouping. But $t_{universe} - t_{dec} \approx t_{universe}$. Thus this age is within the uncertainties of t_o.

Lastly, if we relax the assumption that the universe is flat ($k = 0$), several degeneracies form in parameters like Ω_{tot} & t_o , Ω_{tot} & h, Ω_Λ & Ω_m, Ω_m & h and Ω_b & n_s; as will be exhbited. This is a very visual method of determining the age of the universe, among other parameters. Using these plots, if we again take k = 0, we can constrain an equivalent range of t_0. This method does not rely on an independent measurement of H_o, but constrains it simultaneously.

Citations:

Bennet et al., ApJS 148:1-27, 2003 "First-Year WMAP Observations: Preliminary Maps and Basic Results"
Eisenstin & Hu, ApJ 496:605-614, 1998 "Baryonic Features in the Matter Transfer Function"
Hu et al., ApJ 549:669-680, 2001 "CMB Observables and Their Cosmological Implications"
Knox, Christensen & Skordis, ApJ 563:L95-L98, 2001 "The Age of the Universe . . . from CMB Anisotropy Measurements"
Komatsu et al., eprint arXiv:1001.4538, 2010 "Seven-Year WMAP Observatiosn: Cosmological Interpretation"
Page et al., ApJS 148: 233-241, 2003 "First Year WMAP Observations: Interpretation . . . Power Spectrum Peaks"
Spergel et al., ApJS 148: 175-194, 2003 "First-Year WMAP Observations: Determination of Cosmological Parameters"
Tegmark et al., PhRvD vol. 69, issue 10, id. 103501, 2004 "Cosmological Parameters from SDSS and WMAP"