ASTR622 Class Project - Kari Helgason

The Shape Parameter

The cold dark matter (CDM) power spectrum is commonly used to compare with cosmological observations. It can be conveniently descibed as $P(k) = Ak^nT_k^2$ where the n is usually taken as $n \simeq 1$ and T_k is the *transfer function*. The transfer function characterizes any modifications of the primordial power spectrum. These include gravitational growth, effects of pressure and dissipative forces, all of which tend to reduce power on small scales. An accurate calculation of the transfer function is a challenging numerical task since one has to consider a mixture of baryons, dark matter and radiation, requiring the solution of the Boltzmann equation and particle interactions. Therefore it is useful to find simple analytic formulae to fit the transfer function, T_k. The BBKS approximation assumes pure adiabatic CDM (Bardeen et al. 1986):

$$T_k = \frac{\ln(1+2.34q)}{2.34q}\left[1+3.89q+(16.1q)^2+(5.46q)^3+(6.71q)^4\right]^{-0.25}$$

where $q = k(\Omega_m h^2)$Mpc. This is assuming $\Omega_m \gg \Omega_b$. A non-zero baryonic energy density will lower the dark matter contribution to Ω_{tot}. A **shape parameter** Γ for the transfer function can be defined as (Efstathiou, Bond & White, 1992):

$$q \equiv \frac{k/h\mathrm{Mpc}^{-1}}{\Gamma}.$$

The shape parameter becomes $\Gamma = \Omega_m h$ for zero baryon content. For a non-zero baryon contribution, it can approximated as (Sugiyama 1995):

$$\Gamma = \Omega_m h \cdot \exp\left[-\Omega_b(1+\sqrt{2h}/\Omega_m\right]$$

If we accept the CDM picture at face value then the best fit of Γ can be used to constrain Ω_m, Ω_b and h. Large redshift surveys, such as CfA, LCRS, 2df and SDSS, have mapped the distribution of galaxies and given us the linear power spectrum to a reasonable accuracy. The shape parameter currently favoured is

$$\Gamma \simeq 0.25 + 0.31(n^{-1} - 1)$$

which reduces to $\Gamma \sim 0.25$ for the spectral index $n \simeq 1$. For a reasonable baryon content of $\Omega_b = 0.04$ we end up with $\Gamma \simeq 0.9\Omega_m h$. To match the observations, this strongly favours a universe with $\Omega_m < 1$ but should be closer to $\Omega \simeq 0.3$. To salvage the $\Omega_m = 1$ universe, scientists played with different models of dark matter mixtures (CDM, WDM, HDM) which can lead to lower apparent values of Γ. In light of existing data that supports the ΛCDM universe with $\Omega_m \simeq 0.3$, these models now seem ill-motivated.