Astronomy 622 Cosmology **Homework Assignment #3 – Due 2:00 pm April 8th**
100 points (later arbitrarily renormalized to match Dr. McGaugh's earlier homeworks)
Please explain your answers in complete sentences where called for. Although collaboration is encouraged, remember to answer in your own words. Where math is involved, please be sure to show all your work, and answer in no more than a reasonable number of significant digits.

1. (25) Read over section 15.2 in Peacock, and/or look at your *excellent* notes from class. Starting from the three equations (15.24) in Peacock, derive eq. (15.25) by linearizing the variables, dropping 2nd order terms, assuming the 0th order equations are satisfied, etc., etc. Please be sure to show all the gory detail of why you drop or alter any terms. You will notice that you have an extra term $-\left(\frac{kc^2}{3a^2}\delta_k\right)$ (and you will also notice that Peacock is sloppy and doesn't use the *k* subscript; however, you *should* use it!). Why do you have this extra term?! Do not panic: read the paragraph below eq. (15.25) beginning, "In this simplified exercise..." and all will be clear (maybe).

2. (30) You know how $\rho_0 = \bar{\rho} = \rho_{background}$ evolves with time in different eras (yes, you do, it's the Friedmann equation). Below, you will drop the pressure terms in both (15.21) and your version of (15.25) and solve for $\delta_k(t)$ in the radiation (RD) and matter domination (MD) eras by presuming, like a good astrophysicist that the answer is a power law in time. **But first:**

    a. (5) Explain in English at what scales, or for what reasons are you justified in dropping those terms during radiation domination and during matter domination. At which scales would you get nervous about dropping them (be specific)?

    b. (15) Now, drop those pressure-related terms and plug and grind (show all your work): solve for $\delta_k(t)$ during MD and then solve it during RD (or vice versa). Note how much "faster" the growth rate seems to be during RD compared to MD. BUT, see part d.

    c. (5) Should I care about the power laws with negative exponents? Explain.

    d. (5) What is $\delta_k(a)$ in both RD and MD? (Ha! *Now* RD doesn't seem quite so fast compared to MD, does it? That explains why "*t*" is not the typical dependent variable of choice for cosmologists: either *a* or *z* works better. In fact, diehard theorists prefer η, the conformal time.)

3. (10) There's something funny going on with extremely large scales: imagine a perturbation δ_k on a **superhorizon** scale. It's "frozen" in the sense that causality won't let us do *anything* with it (let alone pressure). Why can I presume it will simply expand along with the background? In fact, GR folks would be very squeamish about this if we don't state that this approach is using the Newtonian gauge...

4. (15) As you may have heard, the CMB is remarkably "smooth." In particular, deviations from 2.73K are on the order of 10^{-5}: $\frac{\delta T}{\bar{T}} \sim 10^{-5}$. (I'll drop the "bar" over the *T* from here on.) But, this statement is angular-scale dependent. In much the way that $\delta(\mathbf{x})$ for a point is meaningless, $\frac{\delta T}{T}(\theta, \phi)$ for a particular position on the sky is meaningless.

(4. continued)

a. At what scale is $\frac{\delta T}{T} \sim 10^{-5}$? (Hint: at what value of l is most of the angular power? What angle does that correspond to?)

b. The deviation due to the CMB **dipole** is (one) considerably larger than this, (two) presumed to be entirely due to our relative motion and (three) is (of course) on the angular scale of half the sky! The dipole is in the mK range. Given that the overall temperature is in the $O(1)$ K range, what **order of magnitude** (please do not overthink this!) speeds are required to get a temperature shift of mK?

c. Does the temperature in that direction mean that the CMB isn't a blackbody in that direction? If not, why not? You may find section 9.4 in Peacock to be helpful with this part.

5. (10) Consider $\frac{\delta T}{T}(\theta,\phi) = \sum_{l,m} a_l^m Y_l^m(\theta,\phi)$. Now think about correlating the temperatures across all pairs of points (θ,ϕ) and (θ',ϕ') separated by the same angle on the sky. By plugging the above into "angle brackets" (i.e., averaging), show that: $\left\langle \frac{\delta T}{T}(\theta,\phi) \frac{\delta T}{T}(\theta',\phi') \right\rangle = \frac{1}{4\pi} \sum_l (2l+1) C_l P_l(\cos\chi)$ and thereby identify the connection between C_l's and the a_l^m's. This will also force you to look up the Spherical Harmonic Addition Theorem if you've forgotten it.

6. (10) The Planck mission is going to probe l in the 1000's.

   a. What are some of the problems facing any CMB experiment trying to look at extremely low $l < 10$? Consider statistics and real issues on the sky.

   b. Pick any of your favorite mathematical software packages and see if you can plot $P_l(\cos\theta)$ for $l = 1000$. What's are some of the issues involved?