Masses of Neutron Stars – Longair pg 417

- Observed masses of non-accreting NS in binaries strongly clusters around 1.4 $M_\odot$ the Chandrasekar mass

Midterm

- Mean=77
- A>85
- 70<B<84
- 60<C<69

- The original scheme was
  - 90%+ A
  - 80-90% B
  - 70-80% C
  - 60-70% D
  - <60% F
Basics of Accretion – Longair 14.2

- If accretion takes place at a rate \( \frac{dM}{dt} = \mathcal{M} \) then the potential energy gained by the material is

\[
E = \frac{GMm_x}{R} \quad \text{(where } m_x \text{ is the mass of the accreting object)} - \text{ if this energy is released as radiation it also is the luminosity } L_{\text{acc}}.
\]

- Alternatively (Longair 443-444) one can calculate the free-fall velocity from infinity an

\[
\frac{1}{2}m_p v^2_f = \frac{GMm_p}{r}.
\]

We can write this as

\[
L = \xi mc^2
\]

and show that \( \xi \) depends on \( r \).

Normalizing the observed luminosity to a typical value of \( 1.3 \times 10^{37} \text{ erg/sec} \) gives accretion rates of

- \( L_{\text{acc}} = 1.3 \times 10^{37} \mathcal{M}_{17} m_x R_6 \)

- \( \mathcal{M}_{17} \) is \( \mathcal{M} \) in units of \( 10^{17} \text{ gm/sec} = 1.5 \times 10^{-9} M_{\odot}/\text{yr} \)
- \( m_x \) is the mass in solar units of the accretor
- \( R_6 \) is the radius in units of \( 10^6 \text{ cm} \)
- \( m_x \) is the mass in solar units of the accretor

Basics of Accretion – Longair 14.2

- For a white dwarf star with \( M = M \) and \( R \approx 5 \times 10^6 \text{ m}, \xi \approx 3 \times 10^{-4} \).
- Neutron star with mass \( M = M \) and \( R = 10 \text{ km}, \xi \approx 0.15 \).
- For nuclear energy generation the conversion of hydrogen into helium has \( \xi \approx 7 \times 10^{-3} \).
Basics of Accretion Longair 14.2.2

Is there a limit on accretion?

If the accreting material is exposed to the radiation it is producing it receives a force due to radiation pressure

The minimum radiation pressure is

\[(\text{Flux}/c)\times \mathcal{A} \]  

(\mathcal{A} is the relevant cross section)

assume that the infalling matter is fully ionised and that the radiation pressure force is provided by Thomson scattering of the radiation by the electrons in the plasma.

the smallest cross-section

Or

\[L_\sigma T/4\pi^2m_pc \]  

(\sigma_T is the Thompson cross section \(6.6x10^{-25} \text{ cm}^2\) \(m_p\) is the mass of the proton)

The gravitational force on the proton is

\[GM/m_r \]

Equating the two gives the Eddington limit

\[L_{\text{Edd}}=4\pi M_x Gm_pc/\sigma_T=1.3x10^{38}M_{\text{sun}} \text{ erg/sec} \]

Eddington Limit- More Detail Longair pg 446

• \(f_{\text{grav}} \approx GMm_pr^2\) force due to gravity acting on the protons

The radiation pressure acts upon the electron-

• Each photon gives up a momentum \(p = h\nu/c\) to the electron in each collision

• force acting on the electron is the momentum communicated to it per second by the incident flux density of photons \(N_{ph}\).

• Thus, \(f_{\text{rad}} = \sigma N_{ph} p\) \((p\text{ is momentum, } \sigma\text{ is the relevant cross section }, \text{ the smallest is the Thompson cross section } \sigma_T \approx 6.6x10^{-29} \text{ m}^2\)

• As we go away from the source of photons the flux of photons is

\[N_{ph}/4\pi^2; N_{ph}=L/h<\nu>; L \text{ is the luminosity of the source.}\]

• so the outward force on the electron is \(f = \sigma_T L/4\pi cr^2\).

• Equate this to gravity (e.g. radiation pressure and gravity balance)

Gives \(L_{E} = 4\pi GMm_pc/\sigma_T\)

• maximum luminosity a spherically symmetric source of mass \(M\) can emit in a steady state. The limiting luminosity is independent of the radius \(r\) and depends only upon the mass \(M\) of the emitting region
Simplistic Check

- If a NS is accreting at the Eddington limit and radiating via a black body what is its temperature?
- \(4\pi r_{\text{NS}}^2 a T^4 = L_{\text{edd}}\)

So put in 10km for \(r_{\text{ns}}\) and 1.3x10\(^{31}\) W for \(L_{\text{edd}}\) for 1 solar mass and get
  - \((a=5.67x10^{-8}\text{Wm}^{-2}\text{K}^{-4})\)
- \(T\approx2\times10^7\text{K} ; \) 'natural' for NS to radiate in the x-ray band.

Accretion -Basic idea

- Viscosity/friction moves angular momentum outward
  - allowing matter to spiral inward
  - Accreting onto the compact object at center
- Gravitational potential energy is converted by friction to heat. Some fraction is radiated as light – physical origin of friction is complex.

Very efficient process: Energy \(\sim GM/R=1.7\times10^{16}\text{ (R/10km)}^{-1}\text{J/kg} \sim 1/2mc^2\)
Nuclear burning releases \(\sim7\times10^{14}\text{J/kg}\) (0.4% of \(mc^2\))
- \(L = 1/2m\cdot\dot{c}^2\left(r_{\phi}/R\right)\) (14.3)
- This expression for the luminosity can be written \(L = \xi m\cdot\dot{c}^2\), where \(\xi\) is the efficiency of conversion of the rest-mass energy of the accreted matter into heat.
- The efficiency is roughly \(\xi = (r_{\phi}/2R)\) and so depends upon how compact the star is. For a white dwarf star with \(M = M_\odot\) and \(R = 5\times10^6\) m, \(\xi \approx 3\times10^{-4}\).
- For a neutron star with mass \(M = M_\odot\) and \(R = 10\) km, \(\xi \sim 0.15\).
- In the case of nuclear energy generation, the greatest release of nuclear binding energy occurs in the conversion of hydrogen into helium for which \(\xi \approx 7\times10^{-3}\).
- Thus, accretion onto neutron stars is an order of magnitude more efficient as an energy source than nuclear energy generation.
Gravitational potential of spherically symmetric mass $M$ of radius $R$

$$\Phi = -\frac{GM}{r} \quad (r > R)$$

Acceleration of gravity

$$g = -\nabla \Phi = -\frac{GM}{r^2} \hat{r}$$

Particles freely falling from $r \to \infty$ to $r$:

$$E_K = \frac{1}{2} v^2 \quad \text{(kinetic energy per unit mass)}$$

Energy conservation: $E_K + \Phi = E = \text{cst.}$

At $r$:

$$v^2 = \frac{2GM}{r} \quad \text{(free-fall or escape speed)}$$

Viral temperature $T_{\text{viral}} = GM/kr$; for a NS $M \approx 1.4 M_{\text{sun}}$, $R \approx 10$ km

$T \approx 10^{12}$K

(H. Spruit)

Accretion from a Dwarf Companion

http://physics.technion.ac.il/~astrogr/research/animation_cv_disc.gif
Geometry of heated accretion disk + coronal in LMXB

Jimenez-Garate et al. 2002

binary system's Roche lobes

The blue area is the Roche lobe for the companion. The green area is the Roche lobe for the compact object (shown as a black dot).

Anything in the blue area is bound to the companion; anything in the green area is bound to the compact object. However, material found where the two lobes meet could find itself moving from one lobe to the other!
Pulsars

- The rate of change of the pulse period can
  - measure the orbital period of the source
  - The accreted angular momentum (e.g. the amount of material accreted)
  \[
  \frac{dP}{dt}/P \sim (L/10^{37})^{6/7} \quad \text{(Ghosh and Lamb 1978) Longair 14.62,14.63}
  \]

\[
\log_{10} \left( \frac{dP}{dt}/P \right) = -4.4 + \log_{10} L_{37}^{6/7}
\]

$L_{37}$ is the luminosity in units of $10^{37}$ ergs/sec
How much energy is released by accretion onto a compact object?

- Consider matter in an accretion disk assume that...
  - The matter orbits in circular paths (will always be approximately true)
  - Centripetal acceleration is mainly due to gravity of central object (i.e., radial pressure forces are negligible... will be true if the disk is thin)

- Energy is...

\[ \frac{v^2}{r} = \frac{GM}{r^2} \]

\[ E = \frac{1}{2} v^2 - \frac{GM}{r} = -\frac{GM}{2r} \]

Total luminosity liberated by accreting a flow of matter is

- Longair 14.48

\[ L = \left[ 0 - \left( -\frac{GM}{2r_{\text{in}}} \right) \right] \dot{M} = \frac{GM \dot{M}}{2r_{\text{in}}} \]

- Initial energy (at infinity)
- Final energy
- Mass flow rate

- Total luminosity of disk depends on inner radius of dissipative part of accretion disk
• The matter falling in from infinity passes through a series of bound Keplerian orbits for which the kinetic energy is equal to half the gravitational potential energy.
• The matter dissipates half its potential energy in falling from infinity to radius $r$ and this is the source of the luminosity of the disc.
• When the matter reaches the boundary layer, it has only liberated half its gravitational potential energy. If the matter is then brought to rest on the surface of the star, the rest of the gravitational potential energy can be dissipated.
• Thus, the boundary layer can be just as important a source of luminosity as the disc itself. Of course black holes do not have a surface so the energy 'disappears'.

**Thin accretion disks**

Accretion disks form due to angular-momentum of incoming gas

Once in circular orbit, specific angular momentum (i.e., per unit mass) is

$$J = vr = \sqrt{GMr}$$

So, gas must shed its angular momentum for it to actually accrete...

Releases gravitational potential energy in the process!

Matter goes in, angular momentum goes out!
A Simple Disk- see Longair 14.48

C. Done IAC winter school

- The underlying physics of a Shakura-Sunyaev accretion disc (a very simple derivation just conserving energy - rather than the proper derivation which conserves energy and angular momentum).
- A mass accretion rate $\dot{\mathcal{M}}$ spiraling inwards from $R$ to $R-dR$ liberates potential energy at a rate $dE/dt = L_{\text{pot}} = (GM\dot{\mathcal{M}}/R^2)xRdR$.
- The virial theorem says that only half of this can be radiated, so $dL_{\text{rad}} = GM\dot{\mathcal{M}} dR/(2R^2)$.
- If this thermalises to a blackbody then $dL = (dA)xkT^4$ where $k$ is the Stephan-Boltzman constant and area of the annulus $dA = 2\piRxdR$ (where the factor 2 comes from the fact that there is a top and bottom face of the ring).
- Then the luminosity from the annulus $dL_{\text{rad}} = GM\dot{\mathcal{M}} dR/(2R^2) = 4\piRkdT^4$ or $-kT^4(R) = (GM\dot{\mathcal{M}}/8\piR^3)$.
- This is only out by a factor $3(1-(R_\text{in}/R)^{1/2})$ which comes from a full analysis including angular momentum.
- Thus the spectrum from a disc is a sum of blackbody components, with increasing temperature and luminosity emitted from a decreasing area as the radius decreases.

The First Physical Disk Model- Longair 14.3.3

- The first physical model of a disk was developed by Shakura and Sunyaev in 1973.
- They made a set of reasonable assumptions which have proved to be reasonable.
- The disk is optically thick.
- The local emission should consist of a sum of quasi-blackbody spectra.
Temperature Structure of Accretion disk
Longair 14.3.5

• Energy released by an element of mass in going from \( r + dr \) to \( r \)
Gravitational potential energy is
\[
E_p = -\frac{GMm}{2r}
\]
so energy released is
\[
E_g = -\frac{GMmdr}{r^2}.
\]
the luminosity of this annulus, for an accretion rate \( \dot{M} \), is
\[
dL \sim \frac{G\dot{M}M}{r^2}.
\]
assuming the annulus radiates its energy as a blackbody
For a
• blackbody, \( L = \sigma A T^4 \). The area of the annulus is \( 2\pi r dr \), and since
• \( L = \frac{\dot{M} M}{r^2} \) dr we have
• \( T^4 \sim \frac{\dot{M}}{M r} \), or
• \( T \sim \left( \frac{\dot{M} M}{r^3} \right)^{1/4} \); e.g. \( T \sim r^{-3/4}; T \sim \dot{M}^{1/4} \)

Total Spectrum- see Longair eqs 14.54-14.57

• If each annulus radiates like a black body and the temperature scales as
\( T \sim r^{-3/4} \)
• The emissivity scales over a wide range of energies as \( I(\nu) \sim \nu^{1/3} \)
• At lower frequencies the spectrum has a Raleigh-Jeans \( \nu^2 \) shape and at higher energies has an exponential cutoff corresponding to the maximum temperature \( (e^{\nu h/\nu kT_{inner}}) \)
• Thus the spectrum from a disc is a sum of blackbody components, with increasing temperature and luminosity emitted from a decreasing area as the radius decreases.
Total Spectrum

• If each annulus radiates like a black body and the temperature scales as \( T \sim r^{-3/4} \) (Longair 14.54)
• The emissivity scales over a wide range of energies as \( I(\nu) \sim \nu^{1/3} \)
• At lower frequencies the spectrum has a Raleigh–Jeans \( \nu^2 \) shape and at higher energies has an exponential cutoff corresponding to the maximum temperature \( (e^{h\nu/kT_{\text{inner}}}) \)
• Thus the spectrum from a disc is a sum of blackbody components, with increasing temperature and luminosity emitted from a decreasing area as the radius decreases.

If the disk 'cuts off' at some radius \( r_{\text{inner}} \) then the temperature profile is
\[
T(r) = \frac{3GM\dot{m}}{8\pi\sigma} \left[ 1 - \left( \frac{r_{\text{inner}}}{r} \right)^{1/2} \right]^{1/4}
\]
eq in 14.7.1.

X-ray spectrum of accreting Neutron star at various intensity levels

Do They Really Look Like That

• Right panel is \( T(r_{\text{in}}) \) vs flux - follows the \( T^4 \) law
Fit to Real Data

The data is of very high signal to noise
Simple spectral form fits well over a factor of 20 in energy
Emitted energy peaks over broad range from 2-6 kev

Actual Neutron Star X-ray Spectra

- Low Mass x-ray binaries (NS with a 'weak' magnetic field) have a 2 component spectrum
  - The low energy component is well described by a multi-color disk black body spectrum
  - And the hotter temperature black body is related to the boundary layer
- However the observed temperatures disagree with simple theory due to three effects
  - General relativity
  - The 'non-black body' nature of the radiation
  - Reprocessing of the radiation of the central regions by the outer regions and then re-emission
Where is the Accretion Occurring

- In a weak field neutron star (LMXB) has the accretion disk and the place where the material hits the star surface (boundary layer) produce the radiation

How is the Potential Energy Released

- Suppose that there is some kind of “viscosity” in the disk
  - Different annuli of the disk rub against each other and exchange angular momentum
  - Results in most of the matter moving inwards and eventually accreting
  - Angular momentum carried outwards by a small amount of material

- Process producing this “viscosity” might also be dissipative... could turn gravitational potential energy into heat (and eventually radiation)

- Physics of the 'viscosity' is very complex- it turns out that it is due to magnetic fields and an instability magnetorotational instability (MRI), by which weak magnetic fields are amplified by differential rotation, gives the required viscosity
General Considerations

• The luminosity that results from accretion is roughly
  \[ L \sim \varepsilon c^2 \rho (\frac{dM}{dt}) \text{ Where } \varepsilon = \frac{GM}{RC^2} \text{ (the depth of the potential)} \]
  \[ \varepsilon \sim 3 \times 10^{-4} \text{ for a white dwarf and 0.1 for a neutron star} \]

• If the gas flow is spherically symmetric and steady state the luminosity should not exceed the Eddington limit (outward force from Compton scattering balances gravity)

• The Compton optical depth in a spherical accretor is \( \tau = \frac{2}{\varepsilon \lambda} \frac{L}{L_{edd}} \)

• Two natural temperatures
  
  – Free fall \( kT = \frac{3}{16} \varepsilon m_n c^2 = 210^5 \varepsilon \text{ kev} \)
  
  – Black body temperature: minimum temperature for the object to radiate the observed luminosity
    \[ T_{BB} \sim \frac{L}{A\sigma}^{1/4} ; A \text{ is the area and } \sigma \text{ is the Stefan-Boltzman constant} \]
    \[ \text{about 0.2 keV for a white dwarf and 2 keV for a neutron star} \]

General Considerations

• Time scales:
  \[ \tau_{dyn} = (r^3/G M)^{1/2} \text{ This is about 0.1 ms for matter at } r = 10 \text{ km, and 2 ms at } r = 100 \text{ km.} \]
  
  The typical orbital period of circulating matter,
  \[ P_{orb} = 2\pi \tau_{dyn} \sim 1 \text{ ms} \]
  
  • Characteristic velocity is \( \sim (GM/R)^{1/2} \sim 0.5c. \)

• The two main accretion mechanisms are
  
  – Roche lobe overflow, which most often occurs in low-mass binaries (LMXB, low B field, accretion disk and boundary layer dominated)
  
  – and stellar wind capture, which is common for high-mass binaries with super-giant companions (high B fields, pulsars)
Basics of Accretion

- Because of angular momentum considerations an accretion disk, **almost** always forms.
- Matter is thought to form a physically thin (but optically thick* disk) which has Keplerian rotation.
- Matter falls into by losing angular momentum via viscosity, the angular velocity is $\Omega_k = \sqrt{GM/r^3}$.

The binding energy of a parcel of the disk is $E = GM\dot{M}/2R = 1/2 L_{acc}$.

The other half of $L_{acc}$ is released very close to the star surface (the boundary layer) as matter in the disk tries to co-rotate with the NS (what happens for a black hole??)

* Optically thick means that a photon emitted inside the disk always interacts with matter at least once before 'escaping'.

Total Spectrum

- If each annulus radiates like a black body and the temperature scales as $T \sim r^{-3/4}$ (Longair 14.54)
- The emissivity scales over a wide range of energies as $I(\nu) \sim \nu^{1/3}$
- At lower frequencies the spectrum has a Raleigh-Jeans $\nu^2$ shape and at higher energies has an exponential cutoff corresponding to the maximum temperature $(e^{h\nu/kT_{inner}})$
- Thus the spectrum from a disc is a sum of blackbody components, with increasing temperature and luminosity emitted from a decreasing area as the radius decreases.

If the star spins more slowly than the innermost part of the accretion disk (angular speed $\omega_k$), the BL must release a large amount of energy as the accreting matter comes to rest at the stellar surface. Some of this is used to spin up the star, but there remains an amount $GM\dot{M}/2R(1 - \omega_k/\Omega_k)^2$ which is radiated.

If the disk 'cuts off' at some radius $r_{inner}$ then the temperature profile is $T(r) = \frac{3GMm}{8\pi\sigma r^2}\left[1 - (r_{inner}/r)^{1/2}\right]^{1/4}$ eq in 14.7.1.

the emission spectrum of an optically thick accretion disc.

The exponential cut-off at high energies occurs at frequency $\nu = kT_i/h$, where $T_i$ is the temperature of the innermost layers of the thin accretion disc. At lower frequencies the spectrum tends towards a Rayleigh–Jeans spectrum $I \propto \nu^2$. 

Effects of Geometry on Observed Properties can be Huge (P. Charles)