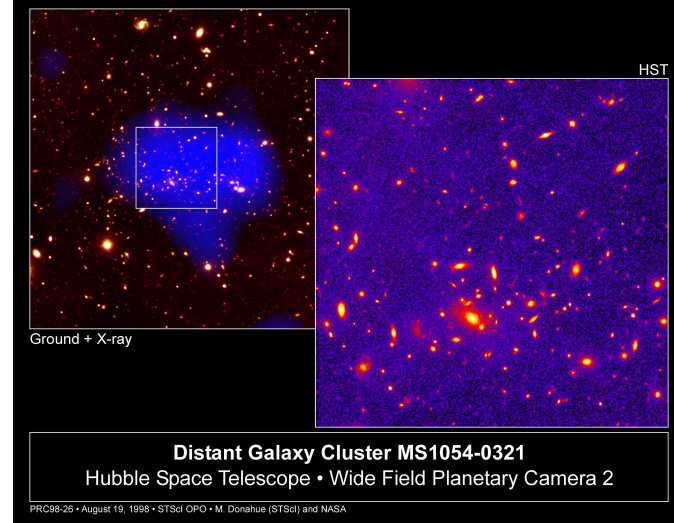
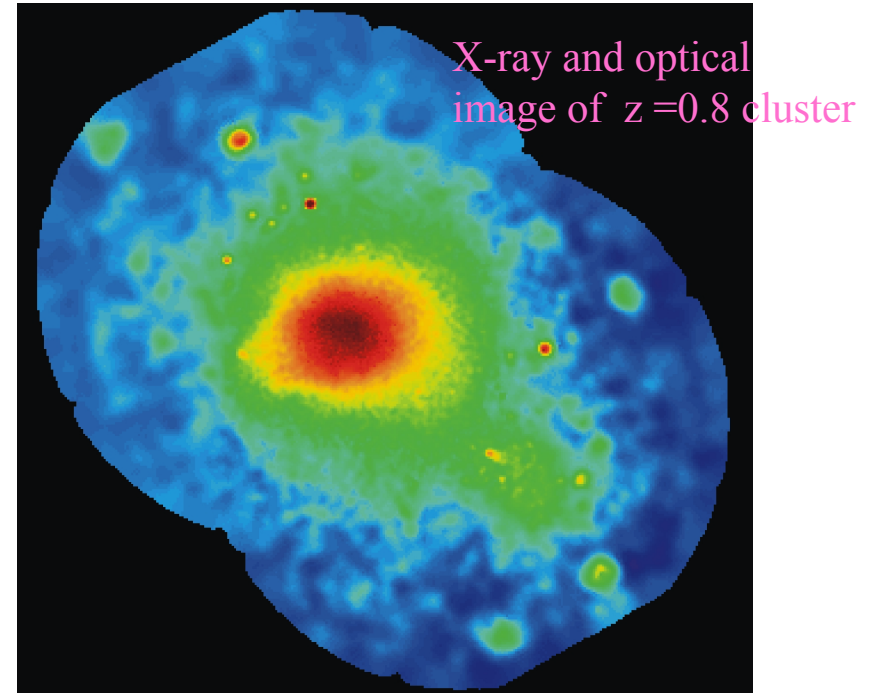


Virialized systems- Clusters, Groups and Big galaxies

X-ray Image of Coma Cluster of Galaxies

- The virial temperature of most bound systems corresponds to $kT \sim 2-100 \times 10^6 \text{K}$ (velocity dispersions of 180-1200 km/sec)
- Most of the baryons lie in the hot x-ray emitting gas which is in virial equilibrium with the dark matter potential well (the ratio of gas to stellar mass is $\sim 2-10:1$)
- This gas is enriched in heavy elements and is thus the reservoir of stellar evolution in these systems.
- There is a strong relation between the hot gas temperature and the total mass.



X-ray Emission from Clusters of Galaxies: C.Sarazin
<http://nedwww.ipac.caltech.edu/level5/March02/Sarazin/frames.html>

Chemical Evolution of the Universe

- A major area of astrophysical research is understanding when stars and galaxies formed and how the elements are produced
 - With the exception of H and He (which are produced in the big bang) all the other elements (called metals in astrophysical jargon) are "cooked" in the centers of massive stars and supernova and then "ejected" by explosions or winds
 - The gas in these explosions is moving very fast (1000 km/sec) and can easily escape a galaxy.
 - Clusters are essentially giant "boxes" which can hold onto all their material
- Measurement of the amount and change of metals with time in clusters directly measures their production
 - In the hot gas elements such as silicon and iron have only 1 or 2 electrons
These ions produce strong H,He like x-ray emission lines.
The strengths of these lines is 'simply' related to the amount of silicon or iron in the cluster

Why are Clusters Interesting or Important

- Laboratory to study
 - Dark matter
 - Can study in detail the distribution and amount of dark matter and baryons
 - Chemical evolution
 - Most of the 'heavy' elements are in the hot x-ray emitting gas
 - Formation and evolution of cosmic structure
 - Feedback
 - Galaxy formation and evolution
 - Mergers
 - Cosmological constraints
 - Evolution of clusters is a strong function of cosmological parameters
 - Plasma physics on the largest scales
 - Numerical simulations
 - Particle acceleration

**Each one of these issues
Leads to a host of topics**

Dark matter:

How to study it
lensing

Velocity and density distribution
of galaxies

Temperature and density distribution
of the hot gas

Chemical Evolution

Hot and when where the elements
created

Why are most of the baryons in the hot
gas

Does the chemical composition of the
hot gas and stars differ?

X-rays from Clusters of Galaxies

- The baryons thermalize to $> 10^6$ K making clusters strong X-ray sources- the potential energy of infall is converted into kinetic energy of the gas.
- Most of the baryons in a cluster are in the X-ray emitting plasma - only 10-20% are in the galaxies.
- Clusters of galaxies are self-gravitating accumulations of dark matter which have trapped hot plasma (intracluster medium - ICM) and galaxies. (the galaxies are the least important constituent)

Today's Material

- How do we know that clusters are massive
 - Virial theorem
 - Lensing
 - X-ray Hydrostatic equilibrium (but first we will discuss x-ray spectra) Equation of hydrostatic equilibrium (*)
- What do x-ray spectra of clusters look like
- * $\nabla P = -\rho_g \nabla \phi(\mathbf{r})$ where $\phi(\mathbf{r})$ is the gravitational potential of the cluster (which is set by the distribution of matter) P is gas pressure and ρ_g is the gas density ($\nabla f = (\partial f / \partial x_1, \partial f / \partial x_2, \dots, \partial f / \partial x_n)$)

The First Detailed Analysis

- Rood et al used the King (1969) analytic models of potentials (developed for globular clusters) and the velocity data and surface density of galaxies to infer a very high mass to light ratio of ~ 230 .
- Since "no" stellar system had $M/L > 12$ dark matter was necessary

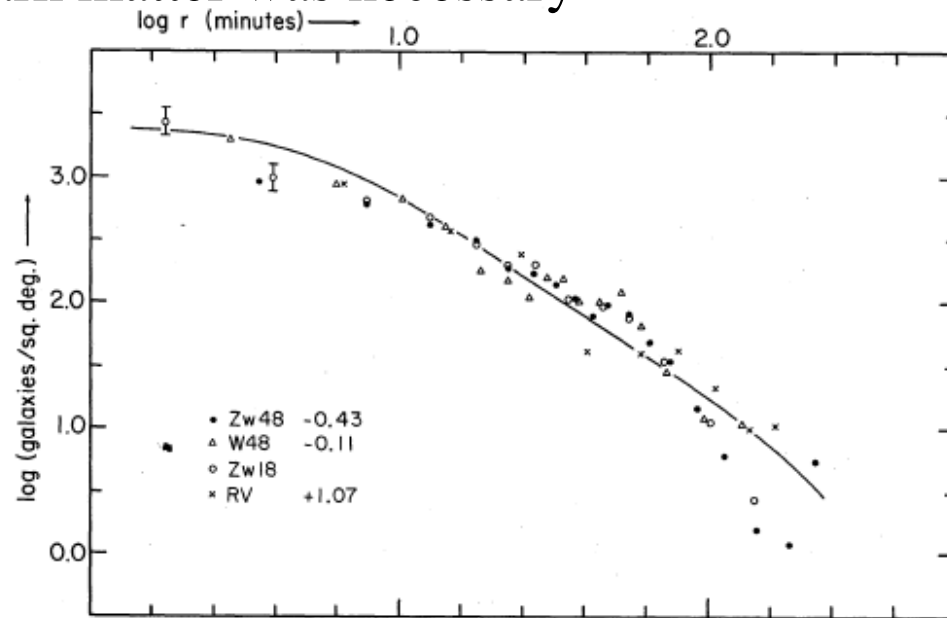
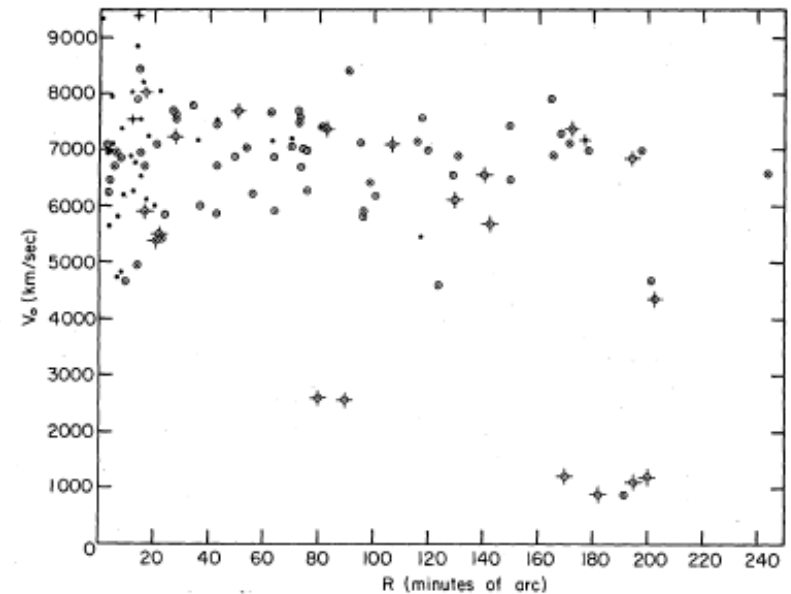


FIG. 5.—Surface densities, corrected for backgrounds given in table 2. For this fitting, logarithms of



**Rood 1972- velocity vs
position of galaxies in
Coma
Surface density of
galaxies**

**Paper is worth reading
ApJ 175,627**

Virial Theorem (Kaiser sec 26.3)

- The virial theorem states that, for a stable, self-gravitating, spherical distribution of equal mass objects (stars, galaxies, etc), the total kinetic energy of the objects is equal to -1/2 times the total gravitational potential energy.

$$2\langle T \rangle = -\langle W_{\text{TOT}} \rangle$$

T is the time average of the Kinetic energy and W is the time overage of the potential energy

- In other words, the potential energy must equal the kinetic energy, within a factor of two. Consider a system of N particles with mass m and velocity v.
- kinetic energy of the total system is
$$\text{K.E.}(\text{system}) = \frac{1}{2} m N v^2 = \frac{1}{2} M_{\text{tot}} v^2$$

$$\bullet \text{PE} \sim \frac{1}{2} G N^2 m^2 / R_{\text{tot}} = \frac{1}{2} G M_{\text{tot}}^2 / R_{\text{tot}}$$

(dimensional analysis)

If the orbits are random

KE = 1/2 PE (virial theorem)

$$M_{\text{tot}} \sim 2 R_{\text{tot}} v_{\text{tot}}^2 / G$$

Binney, J. and Tremaine, S.
"The Virial Equations."
§4.3 in Galactic Dynamics.
Princeton, NJ: Princeton
University Press, pp. 211-
219, 1987

Virial Theorem Actual Use (Kaiser 26.4.2)

- Photometric observations provide the surface brightness Σ_{light} of a cluster. On the other hand, measurements of the velocity dispersion σ_v^2 together with the virial theorem give $\sigma_v^2 \sim W/M \sim GM/R \sim G\Sigma_{\text{mass}} R$

Σ_{mass} is the projected mass density.

At a distance D the mass to light ratio (M/L) can be estimated as

$$M/L = \Sigma_{\text{mass}} / \Sigma_{\text{light}} = \sigma_v^2 / GD\Theta \Sigma_{\text{light}}$$

where Θ is the angular size of the cluster.

Applying this technique, Zwicky found that clusters have $M/L \sim 300$ in solar units

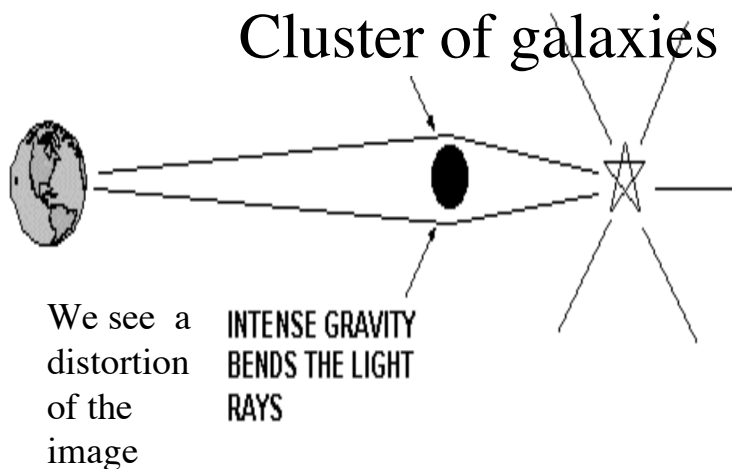
- The virial theorem is exact, but requires that the light traces the mass-but will fail if the dark matter has a different profile from the luminous particles.

Mass Estimates

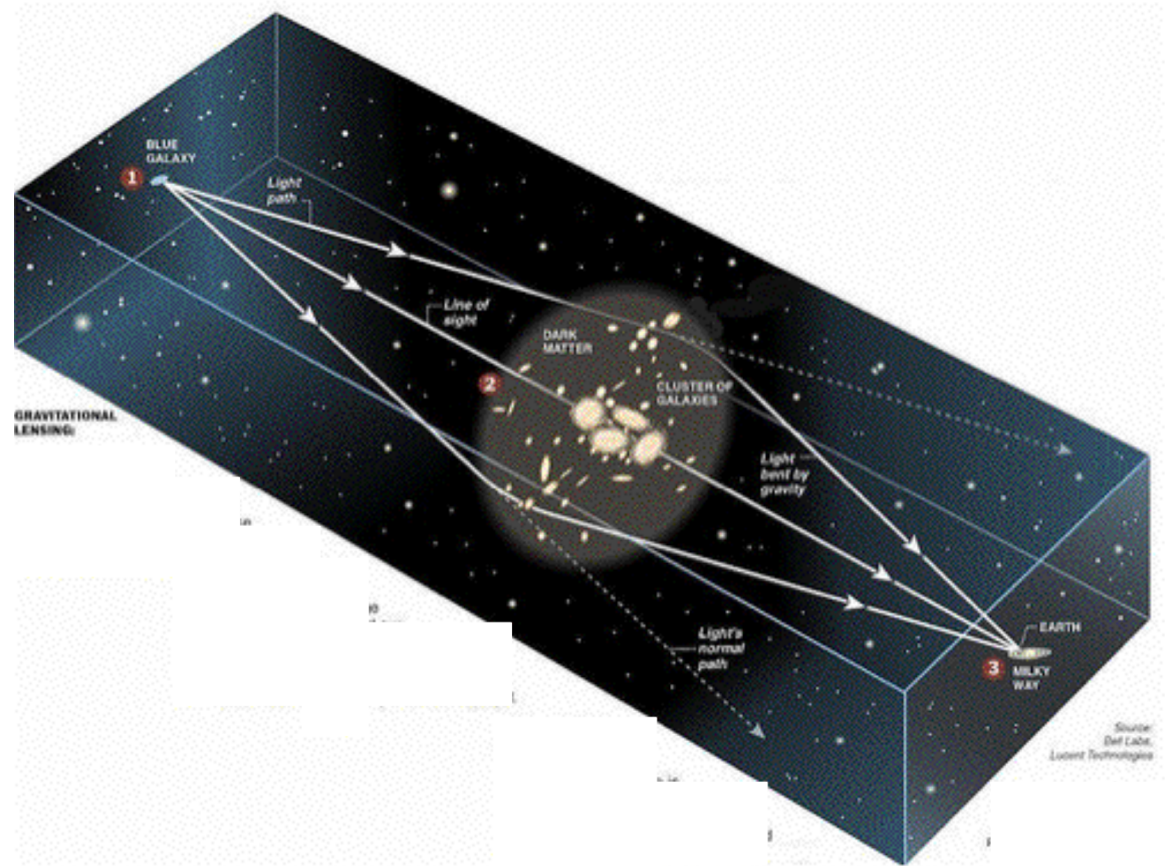
- While the virial theorem is fine it depends on knowing the time averaged orbits, the distribution of particles etc etc- a fair amount of systematic errors
- Would like better techniques
 - Gravitational lensing
 - Use of spatially resolved x-ray spectra

Light Can Be Bent by Gravity

The more mass-
the more the
light is bent



Gravitational Lensing.



faculty.lsmsa.edu

Amount and type of distortion is related to
amount and distribution of mass in gravitational lens

Evidence for Dark Matter in Galaxy Clusters



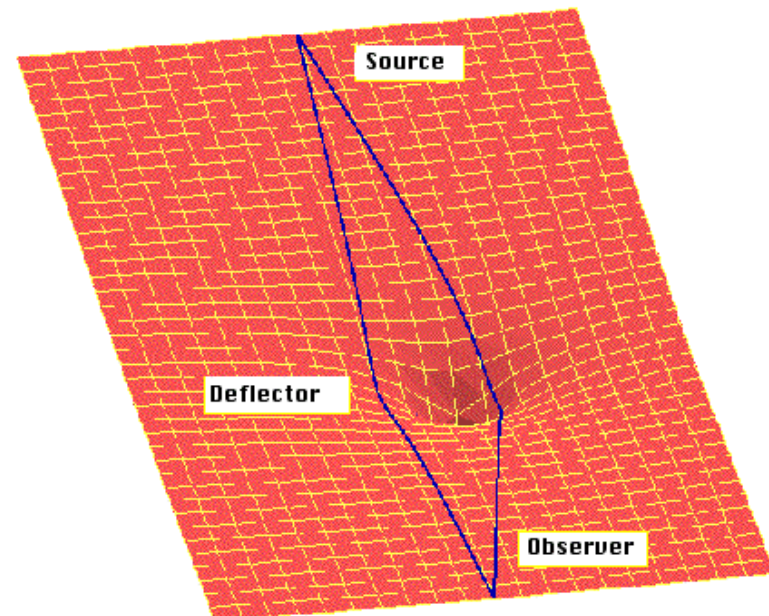
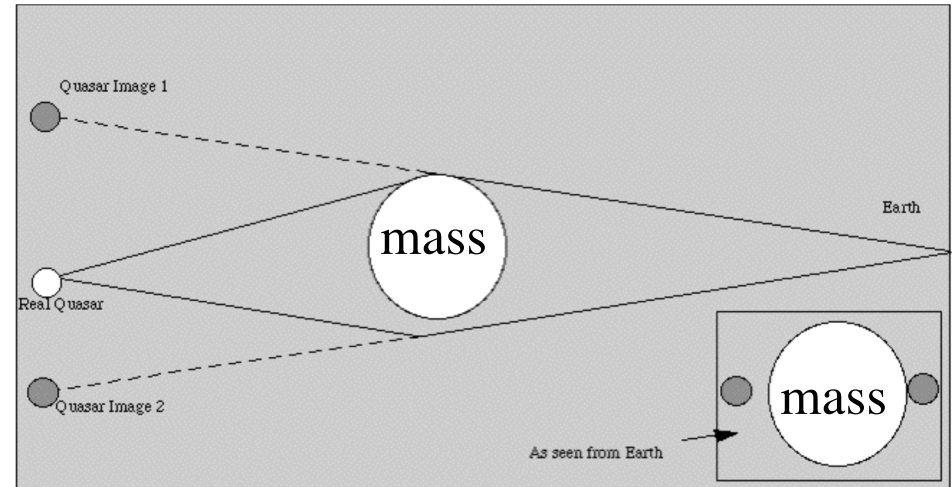
Basics of Gravitational Lensing

- See Lectures on Gravitational Lensing by Ramesh Narayan Matthias Bartelmann or <http://www.pgss.mcs.cmu.edu/1997/Volume16/physics/GL/GL-II.html>

For a detailed discussion of the problem

- Rich centrally condensed clusters occasionally produce giant arcs when a background galaxy happens to be aligned with one of the cluster caustics.
- Every cluster produces weakly distorted images of large numbers of background galaxies.
 - These images are called arclets and the phenomenon is referred to as weak lensing.
- The deflection of a light ray that passes a point mass M at impact parameter b is

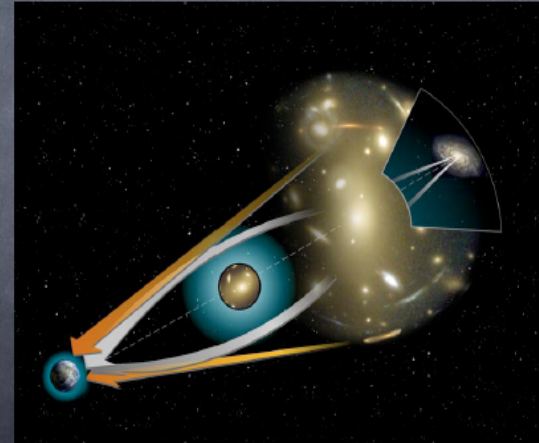
$$\Theta_{\text{def}} = 4GM/c^2 b$$



Ways of Thinking About Lensing (Kaiser sec 33.5)

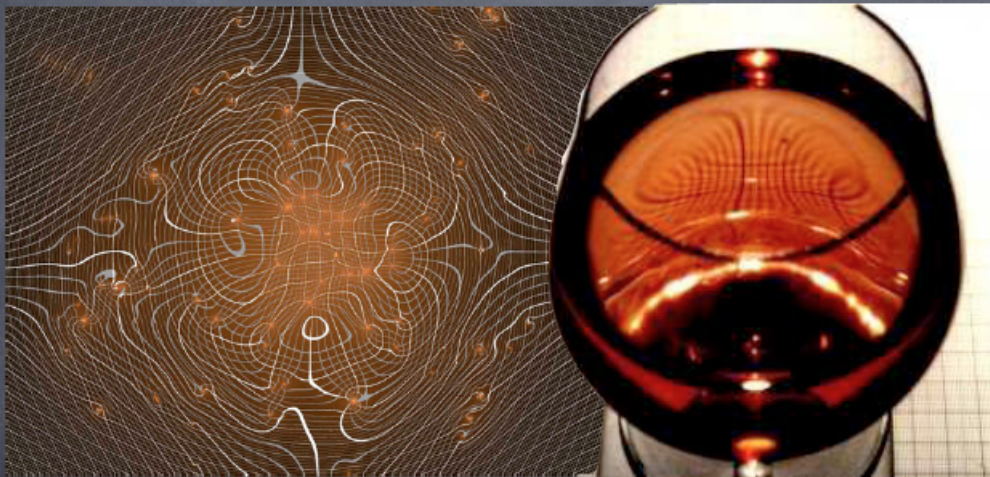
- This deflection is just twice what Newtonian theory would give for the deflection of a test particle moving at $v = c$ where we can imagine the radiation to be test particles being pulled by a gravitational acceleration.
- another way to look at this using wave-optics; the inhomogeneity of the mass distribution causes space-time to become curved. The space in an over-dense region is positively curved.
light rays propagating through the over-density have to propagate a slightly greater distance than they would in the absence of a the density perturbation. Consequently the wave-fronts get retarded slightly in passing through the over-density and this results in focusing of rays.
- Another way : The optical properties of a lumpy universe are, in fact, essentially identical to that of a block of glass of inhomogeneous density where the refractive index is $n(r) = (1 - 2\phi(r)/c^2)$ with $\phi(r)$ the Newtonian gravitational potential. In an over-dense region, ϕ is negative, so n is slightly greater than unity. In this picture we think of space as being flat, but that the speed of light is slightly retarded in the over-dense region.
- All three of the above pictures give identical results

Hoekstra 2008 Texas Conference



The angle of deflection is a direct measure of mass!

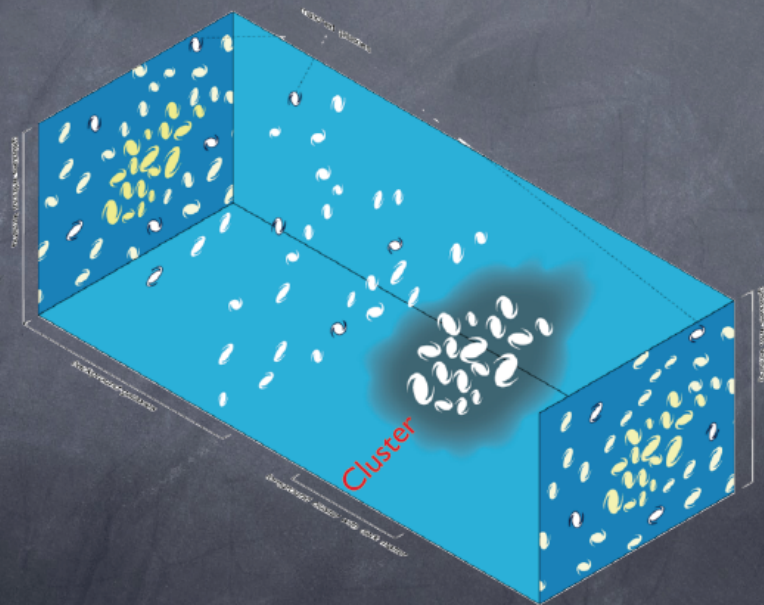
Gravitational lensing



Inhomogeneities in the mass distribution distort the paths of light rays, resulting in a remapping of the sky. This can lead to spectacular lensing examples...

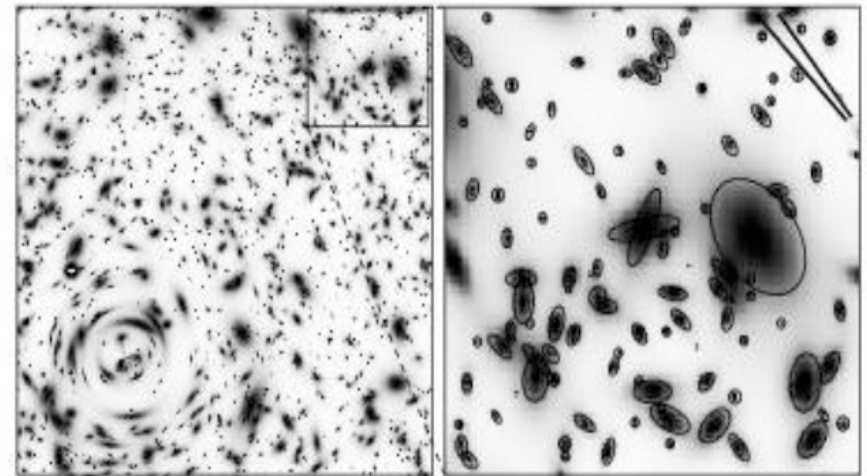
Weak gravitational lensing

Credit: Michael Sachs



In the absence of noise we would be able to map the matter distribution in the universe (even “dark” clusters).

Weak gravitational lensing



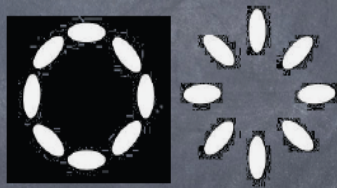
A measurement of the ellipticity of a galaxy provides an unbiased but noisy measurement of the shear

Hockstra 2008 Texas Conference

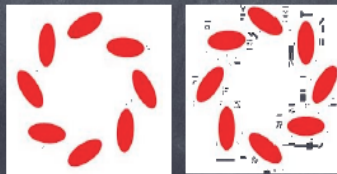
Diagnostics

The lensing signal should be curl-free. We can project the correlation functions into one that measures the divergence and one that measures the curl: *E-B mode decomposition*. We can also look for correlations between the corrected galaxy shapes and the PSF anisotropy.

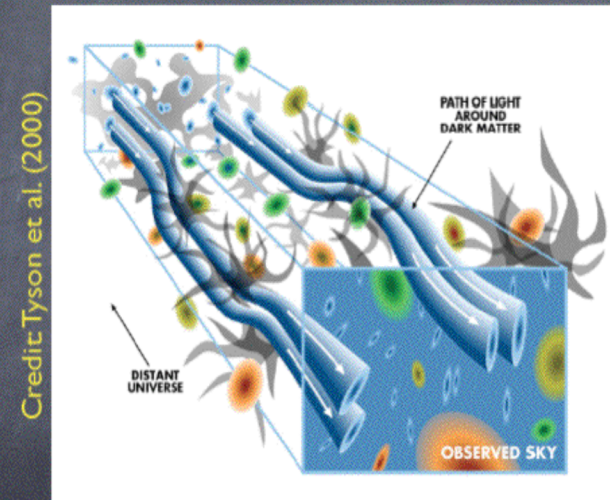
E-mode (curl-free)



B-mode (curl)



Cosmic shear



Cosmic shear is the lensing of distant galaxies by the overall distribution of matter in the universe: it is the most “common” lensing phenomenon.

What we try to measure with X-ray Spectra

- From the x-ray spectrum of the gas we can measure a mean temperature, a redshift, and abundances of the most common elements (heavier than He).
- With good S/N we can determine whether the spectrum is consistent with a single temperature or is a sum of emission from plasma at different temperatures.
- Using symmetry assumptions the X-ray surface brightness can be converted to a measure of the ICM density.

What we try to measure II

If we can measure the temperature and density at different positions in the cluster then assuming the plasma is in hydrostatic equilibrium we can derive the gravitational potential and hence the amount and distribution of the dark matter.

There are two other ways to get the gravitational potential :

- The galaxies act as test particles moving in the potential so their redshift distribution provides a measure of total mass.
- The gravitational potential acts as a lens on light from background galaxies.

Why do we care ?

Cosmological simulations predict distributions of masses.

If we want to use X-ray selected samples of clusters of galaxies to measure cosmological parameters then we must be able to relate the observables (X-ray luminosity and temperature) to the theoretical masses.

Theoretical Tools

- Physics of hot plasmas
 - Bremsstrahlung
 - Collisional equilibrium
 - Atomic physics
- How to use lensing

Physical Processes

- Continuum emission
 - Thermal bremsstrahlung, $\sim \exp(-h\nu/kT)$
 - Bound-free (recombination)
 - Two Photon
- Line Emission
(line emission)

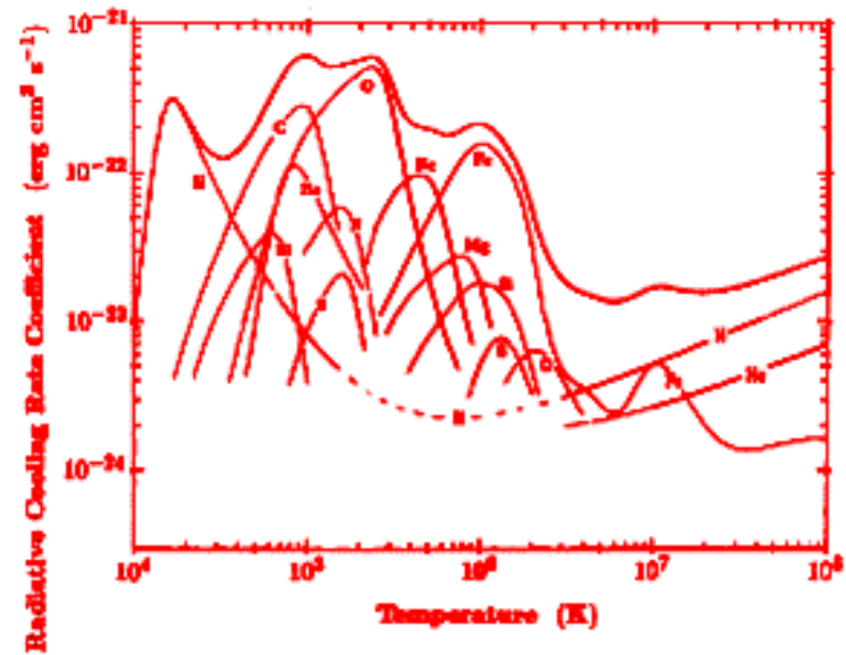
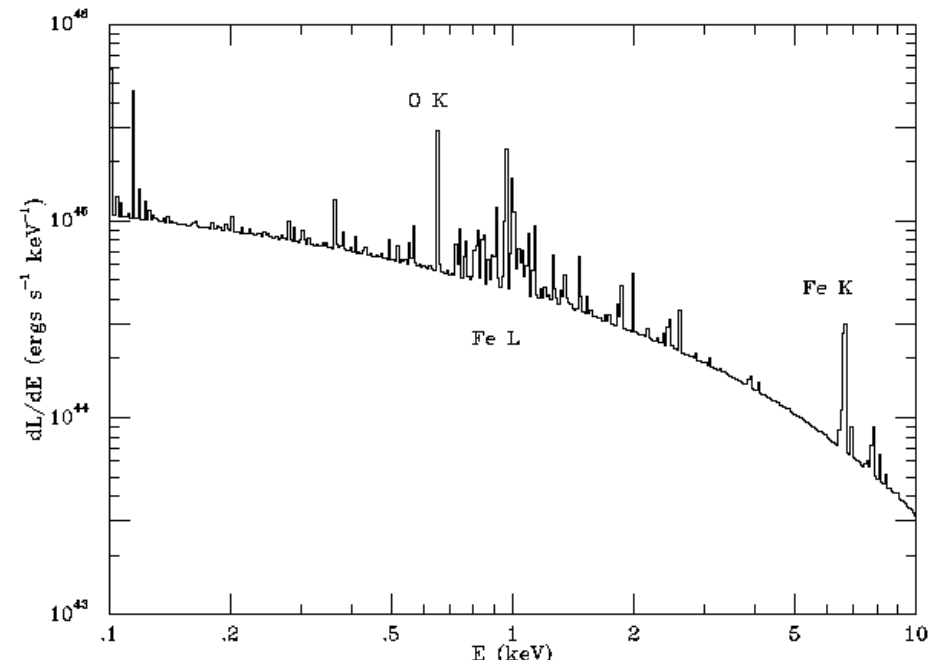
$$L_\nu \sim \epsilon_\nu(T, \text{abund}) (n_e^2 V)$$

$$I_\nu \sim \epsilon_\nu(T, \text{abund}) (n_e^2 l)$$

Line emission dominates cooling
at $T < 10^7$ K

Bremsstrahlung at higher
temperatures

$$\epsilon(\nu) = \frac{16 e^6}{3 m_e c^2} \left(\frac{2\pi}{3 m_e k_B T_X} \right)^{1/2} n_e n_i Z^2 g_{ff}(Z, T_X, \nu) \exp\left(\frac{-h\nu}{k_B T_X}\right),$$



cooling rate of hot plasma as a function of the plasma temperature. The contribution to the cooling is of different important abundant elements is indicated (Böhinger and Hensler 1989). Most of

Cooling Function

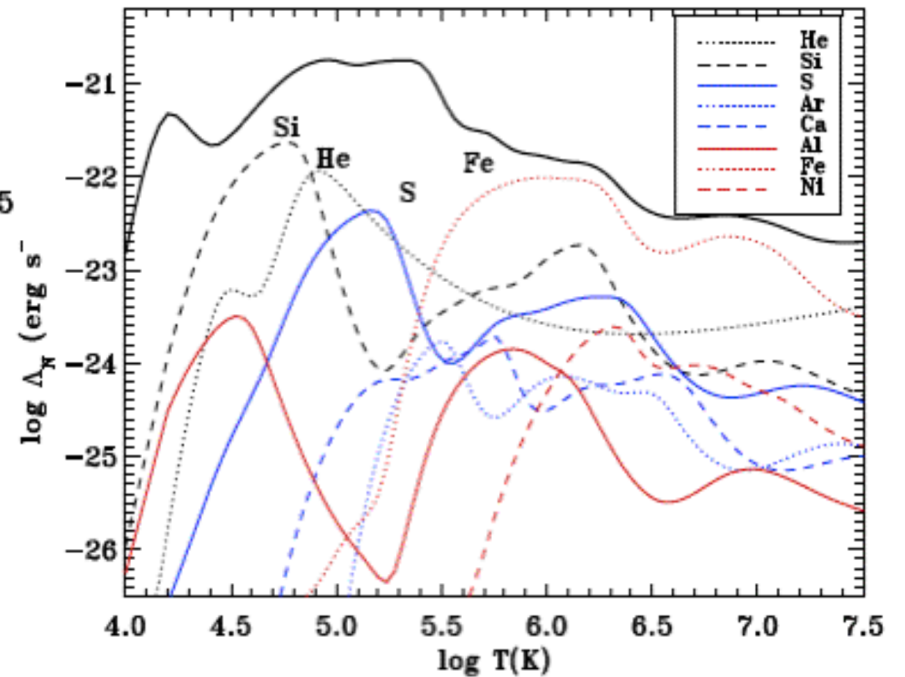
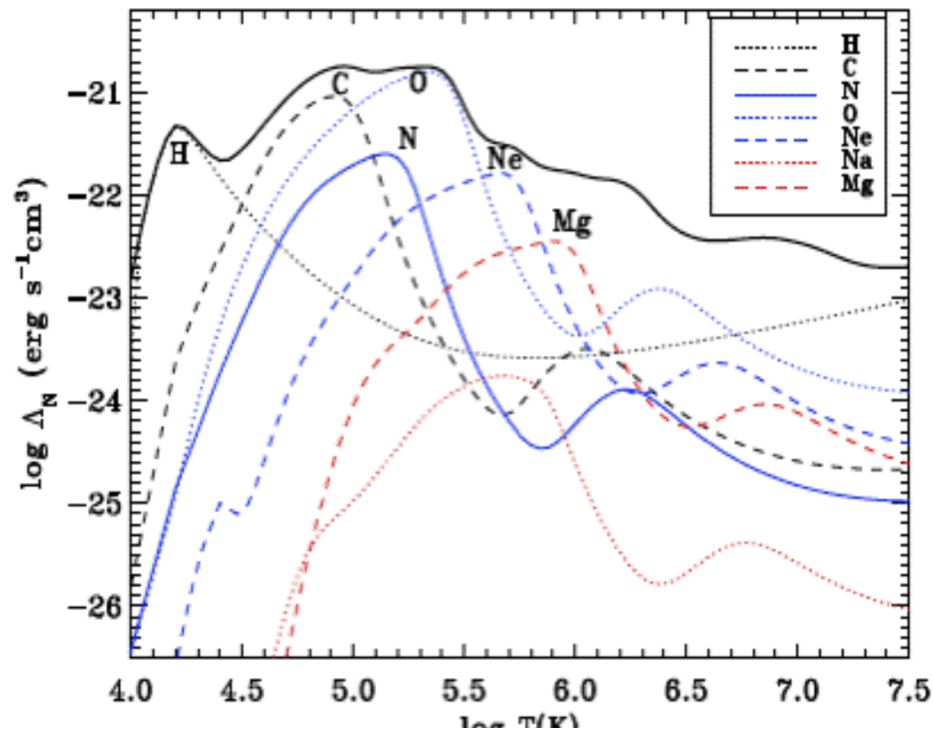
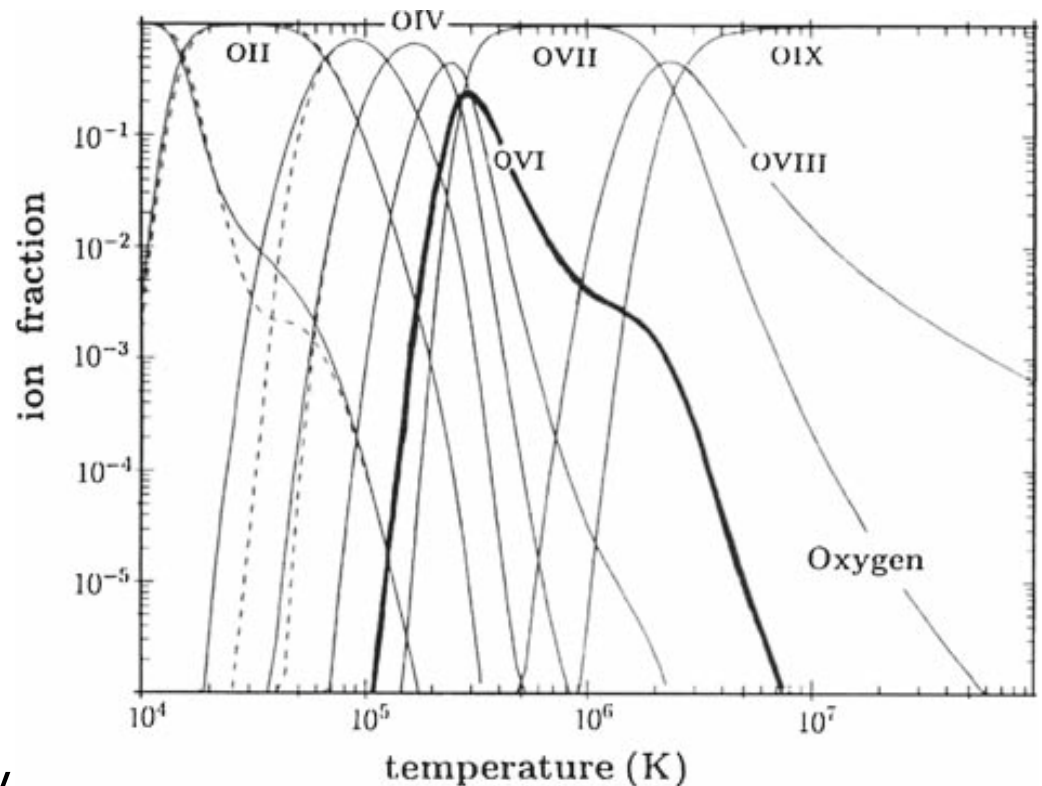


Fig. 2. Contributions of different elements to the cooling curve are given. Each of the plots shows a different set of elements. Important peaks are labelled with the name of the element. The total cooling curve (black solid line) is an addition of the individual elemental contributions.

Plasma Parameters

- Electron number density $n_e \sim 10^{-3} \text{ cm}^{-3}$ in the center with density decreasing as $n_e \sim r^{-2}$
- $10^6 < T < 10^8 \text{ K}$
- Mainly H, He, but with heavy elements (O, Fe, ..)
- Mainly emits X-rays
- $10^{42} L_X < 10^{45.3} \text{ erg/s}$, most luminous extended X-ray sources in Universe
- Age $\sim 2\text{-}10 \text{ Gyr}$
- Mainly ionized, but not completely
e.g. He and H-like ions of the abundant elements (O...Fe) exist in thermal equilibrium

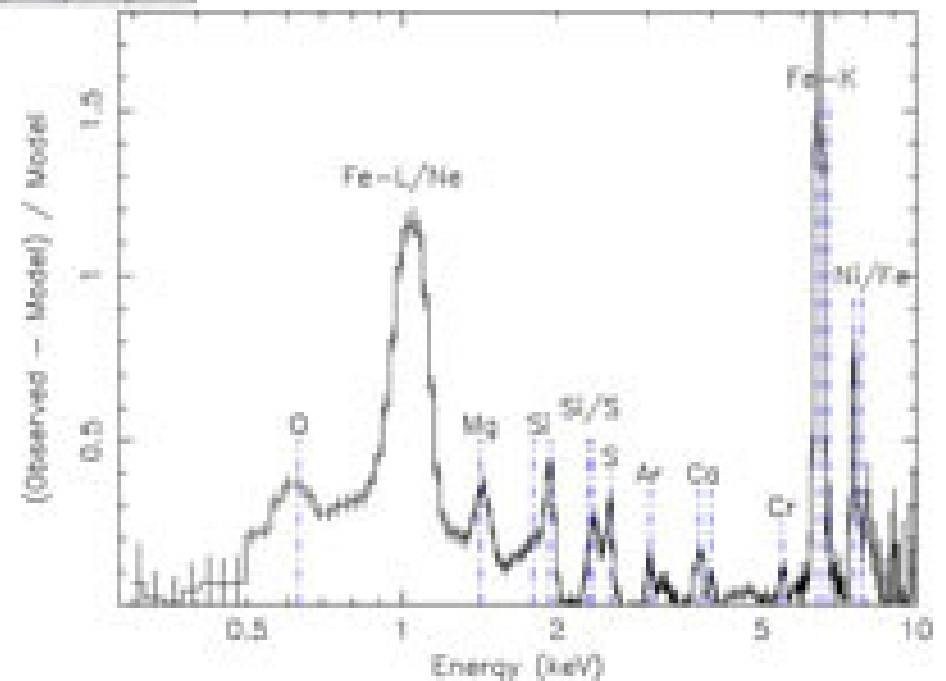
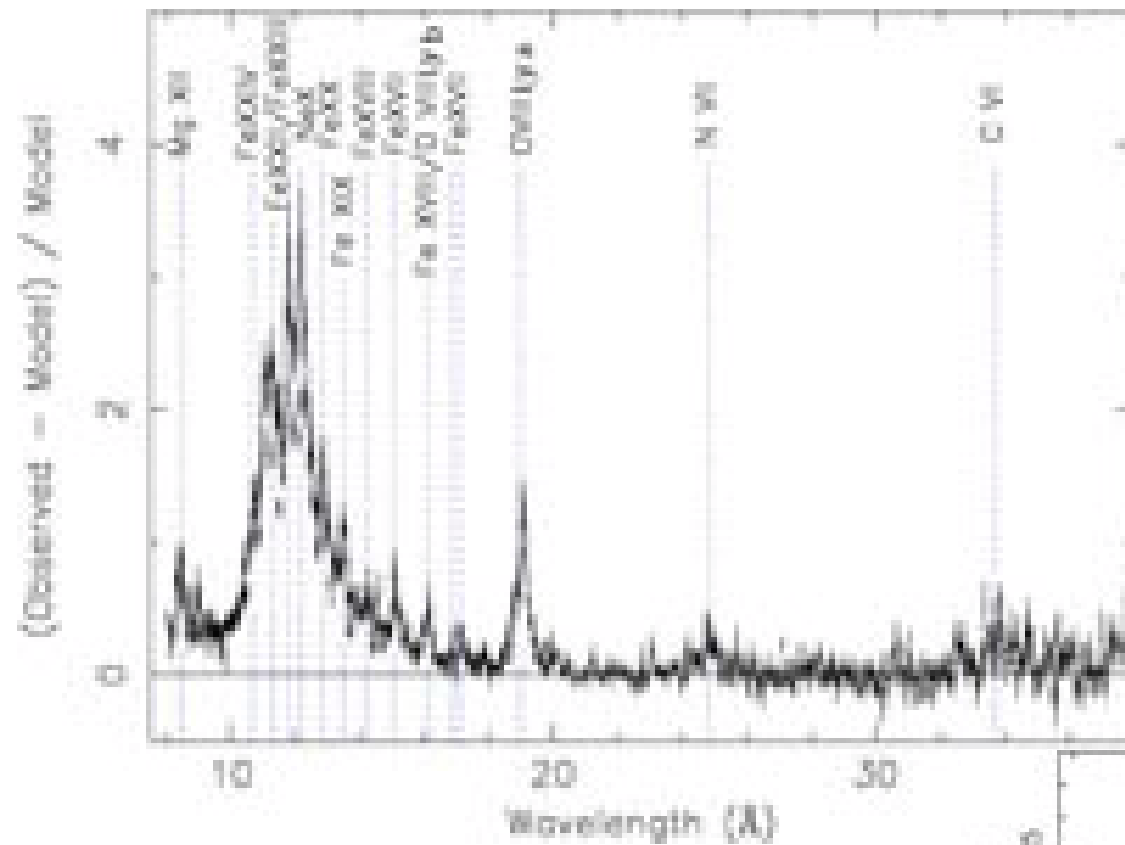


Ion fraction for oxygen vs electron temperature

How Did I Know This??

- Why do we think that the emission is thermal bremsstrahlung?
 - X-ray spectra are consistent with model
 - X-ray 'image' is also consistent
 - Derived physical parameters 'make sense'
 - Other mechanisms 'do not work' (e.g. spectral form not consistent with black body, synchrotron from a power law: presence of x-ray spectral lines of identifiable energy argues for collisional process; ratio of line strengths (e.g. He to H-like) is a measure of temperature which agrees with the fit to the continuum)

X-ray Spectra of Clusters



X-ray Spectra Data

- For hot plasmas the spectra are continuum dominated

Simulated X-ray Spectrum of a $kT=4$ keV Plasma with Solar Abundance
Observed with the XMM CCD Camera

