

How is the Potential Energy Released

- Suppose that there is some kind of “viscosity” in the disk
 - Different annuli of the disk rub against each other and exchange angular momentum
 - Results in most of the matter moving inwards and eventually accreting
 - Angular momentum carried outwards by a small amount of material
- Process producing this “viscosity” might also be dissipative... could turn gravitational potential energy into heat (and eventually radiation)
- Physics of the 'viscosity' is very complex- it turns out that it is due to magnetic fields and an instability magnetorotational instability (MRI) , by which weak magnetic fields are amplified by differential rotation, gives the required viscosity

General Considerations

- The luminosity that results from accretion is roughly
 - $L \sim \epsilon c^2 \dot{M}$ ($dM/dt = \dot{M}$) Where $\epsilon = GM/Rc^2$ (the depth of the potential)
 - $\epsilon \sim 3 \times 10^{-4}$ for a white dwarf and 0.1 for a neutron star
- If the gas flow is spherically symmetric and steady state the luminosity should not exceed the Eddington limit (outward force from Compton scattering balances gravity)
- Two natural temperatures
 - Free fall $kT = 3/16 \epsilon m_p c^2 = 2 \times 10^5 \epsilon$ keV
 - Black body temperature: minimum temperature for the object to radiate the observed luminosity
 - $T_{BB} \sim (L/A\sigma)^{1/4}$; A is the area and σ is the Stefan-Boltzman constant
 - about 0.2 keV for a white dwarf and 2 keV for a neutron star

General Considerations

- Time scales:

$\tau_{\text{dyn}} = (r^3/G M)^{1/2}$ This is about 0.1 ms for matter at $r = 10$ km, and 2 ms at $r = 100$ km.

The typical orbital period of circulating matter,

$$P_{\text{orb}} = 2\pi \tau_{\text{dyn}} \sim 1 \text{ ms:}$$

- Characteristic velocity is $\sim(GM/R)^{1/2} \sim 0.5c$.
- The two main accretion mechanisms are
 - Roche lobe over flow, which most often occurs in low-mass binaries (LMXB, low B field, accretion disk and boundary layer dominated)
 - and stellar wind capture, which is common for high-mass binaries with super-giant companions (high B fields, pulsars)

Basics of Accretion

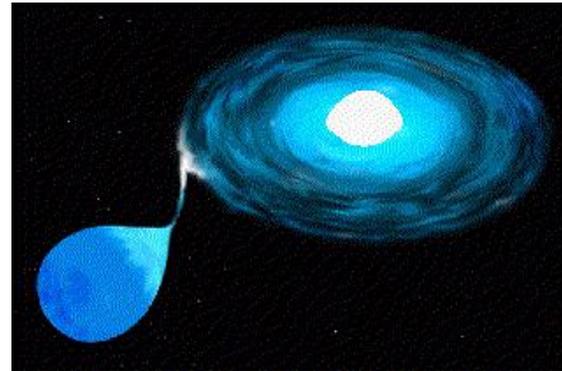
- Because of angular momentum considerations an **accretion disk**, **almost** always forms
- Matter is thought to form a physically thin (but optically thick* disk) which has Keplerian rotation
- Matter falls into by losing angular momentum via viscosity

the angular velocity is $\Omega_k = \sqrt{GM/r^3}$

The binding energy of a parcel of the disk is $E = GM_{\text{disk}} M_x / 2R = 1/2 L_{\text{acc}}$

The other half of L_{acc} is released very close to the star surface (the boundary layer) as matter in the disk tries to co-rotate with the NS (what happens for a black hole??)

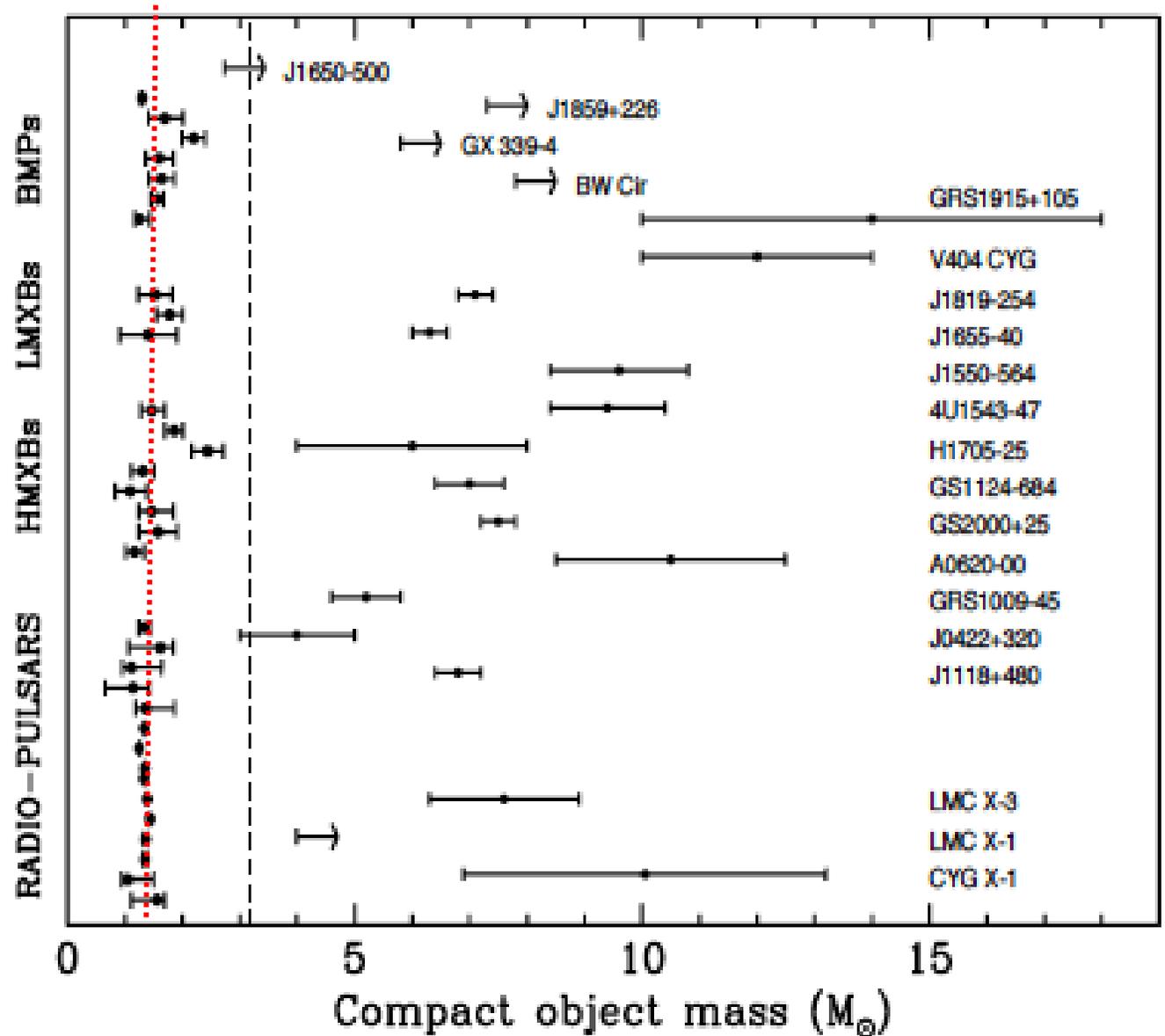
If the star spins more slowly than the innermost part of the accretion disk (angular speed ω_k), the BL must release a large amount of energy as the accreting matter comes to rest at the stellar surface. Some of this is used to spin up the star, but there remains an amount $GM_x/2R(1 - \omega_k/\Omega_k)^2$ which is radiated



* Optically thick means that a photon emitted inside the disk always interacts with matter at least once before 'escaping'

Masses of Compact Objects

- Masses of neutron stars cluster around 1.4M (1 is near 2 M)
- other sources are galactic black holes

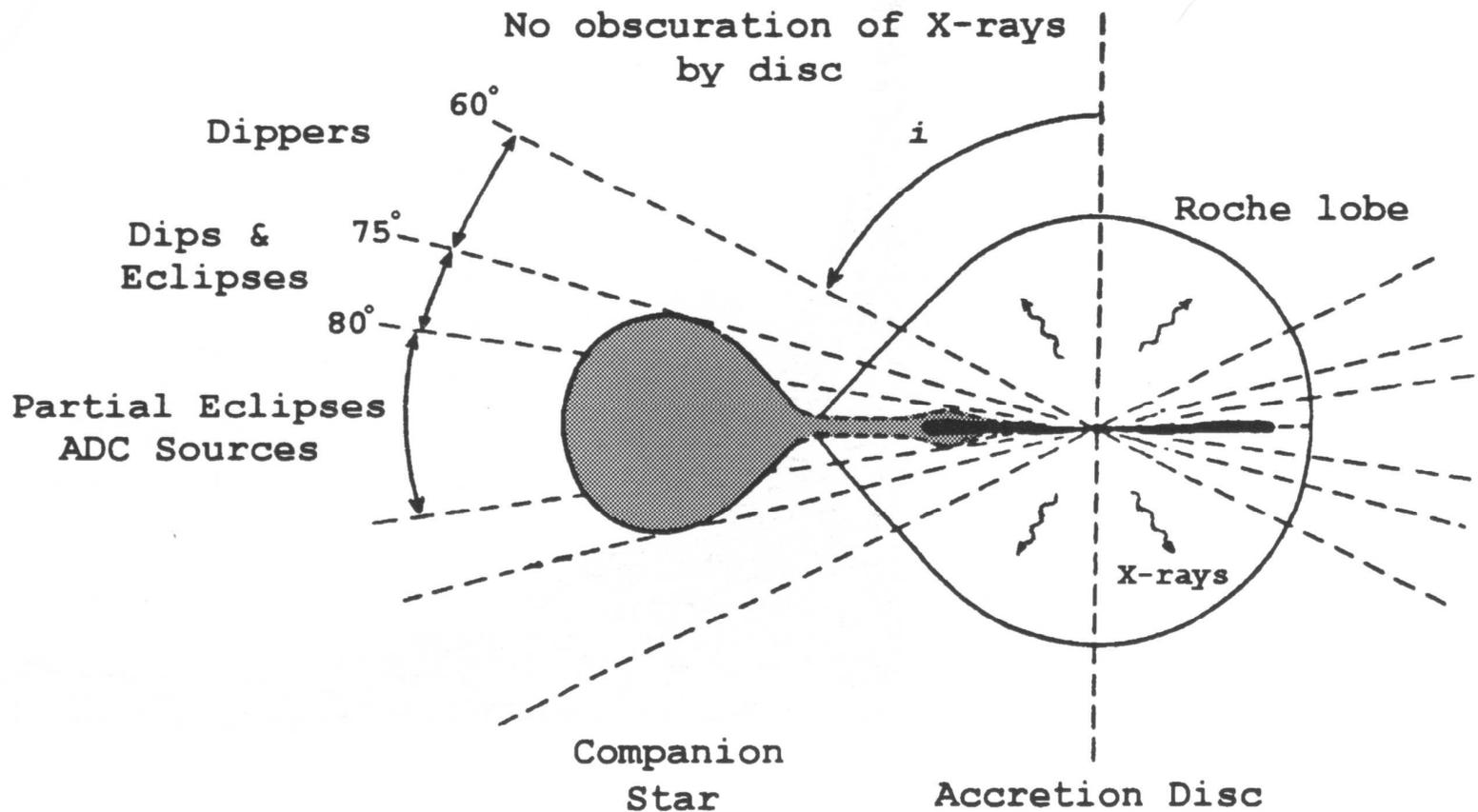


Accreting Neutron Stars-Mass transfer Fuels Accretion, Creating X-rays

- Two types- based on mass of companions
 - Low mass x-ray binaries-NS star tends to have *low* magnetic field ($B \sim 10^{8-10}$ G) and accrete via Roche lobe overflow
 - High mass-NS tends to have *high* magnetic field ($B \sim 10^{11-13}$ G), accrete from stellar wind
- A bit about observations
 - LMXBs tend to be rather luminous and not show pulsations- spectra are 'quasi-thermal' due to radiation from accretion disk and surface of NS
 - HMXBs are often 'pulsars', spectra are non-thermal dominated by effects of energy generation and transfer in a high B field. Can measure B field via detection of cyclotron emission/absorption features.

$$h\omega_c/2\pi = \hbar e B / 2\pi m_e c = 11.6 B_{12} \text{ keV}$$

Effects of Geometry on Observed Properties can be Huge (P.Charles)



Two Modes of Accretion

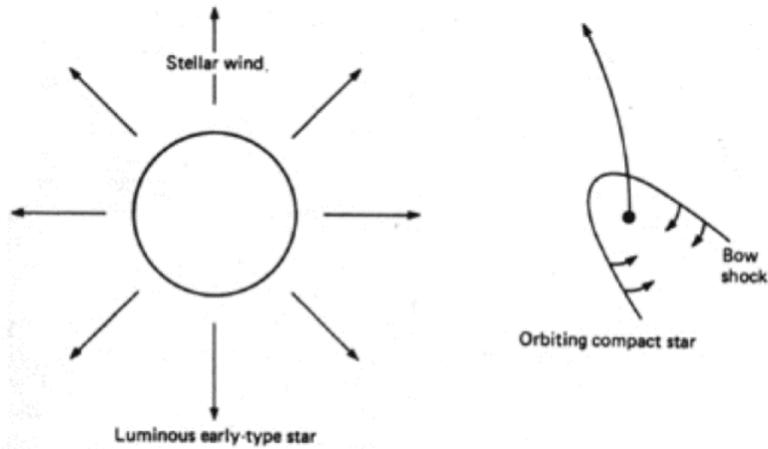


Figure 9: Accretion from a stellar wind.²³

Accretion from a stellar wind

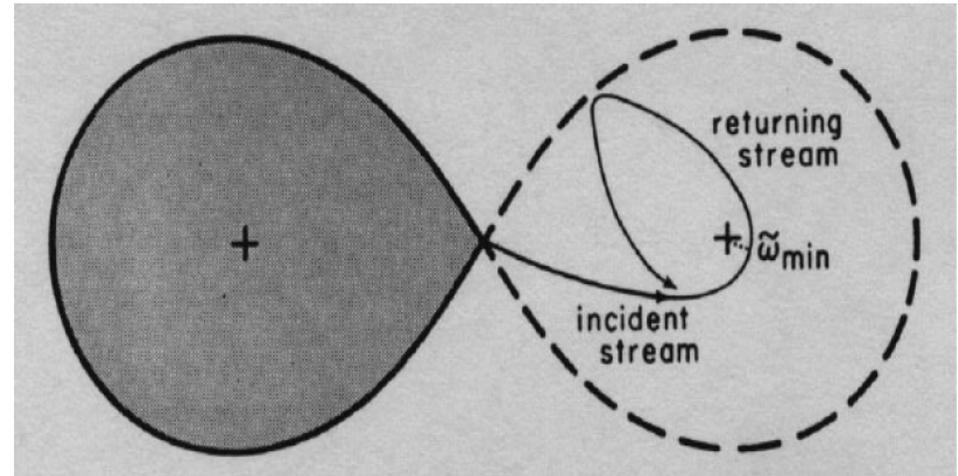


Figure 6: Roche lobe geometry, showing the incident mass stream through the inner Lagrangian point and the intersection of the returning stream with the newly arriving material. The low mass companion on the left hand side completely fills its Roche lobe, while the position of the compact object is denoted by a plus sign (+) inside the right hand lobe.²⁴

Accretion from Roche Lobe Overflow

Cominsky (2002)

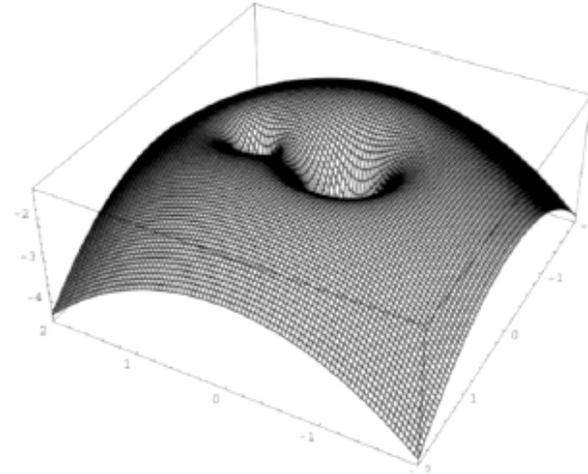
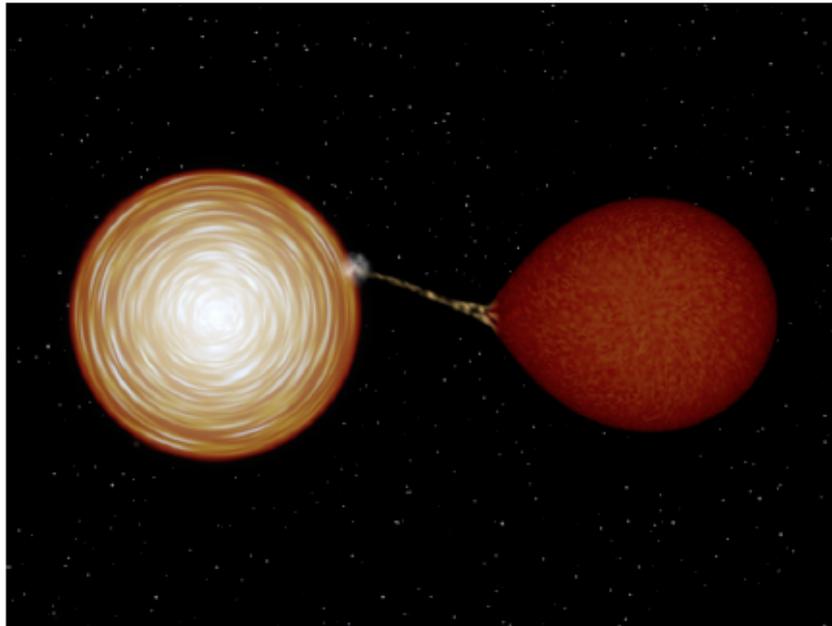
HMXBs T. Kallman

- Among the first discovered extra-solar sources (eg. Vela X-1, Cyg X-1, Cyg X-3, Her X-1)
- Often contain pulsar
- Often eclipsing
- Pulse timing + stellar radial velocity + eclipses = mass, orbital separation, inclination determination
- Accretion can occur from wind from primary, or from Roche-lobe overflow
- Two different subtypes:
 - Be binaries
 - Supergiant binaries
- Statistics: ~50 known in galaxy
- Young population, lifetime $\sim 10^5$ yrs: mass transfer is unstable - wide range in luminosities

Roche Lobe Overflow Systems

Sample

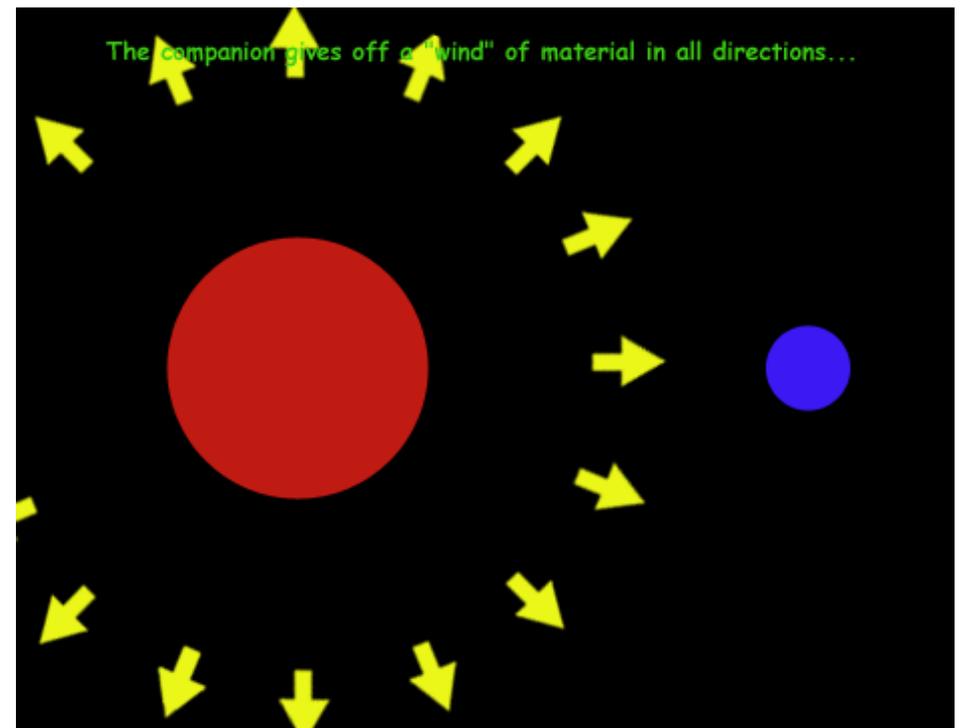
- Almost all LMXBs and IMXBs
- Small fraction of HMXBs



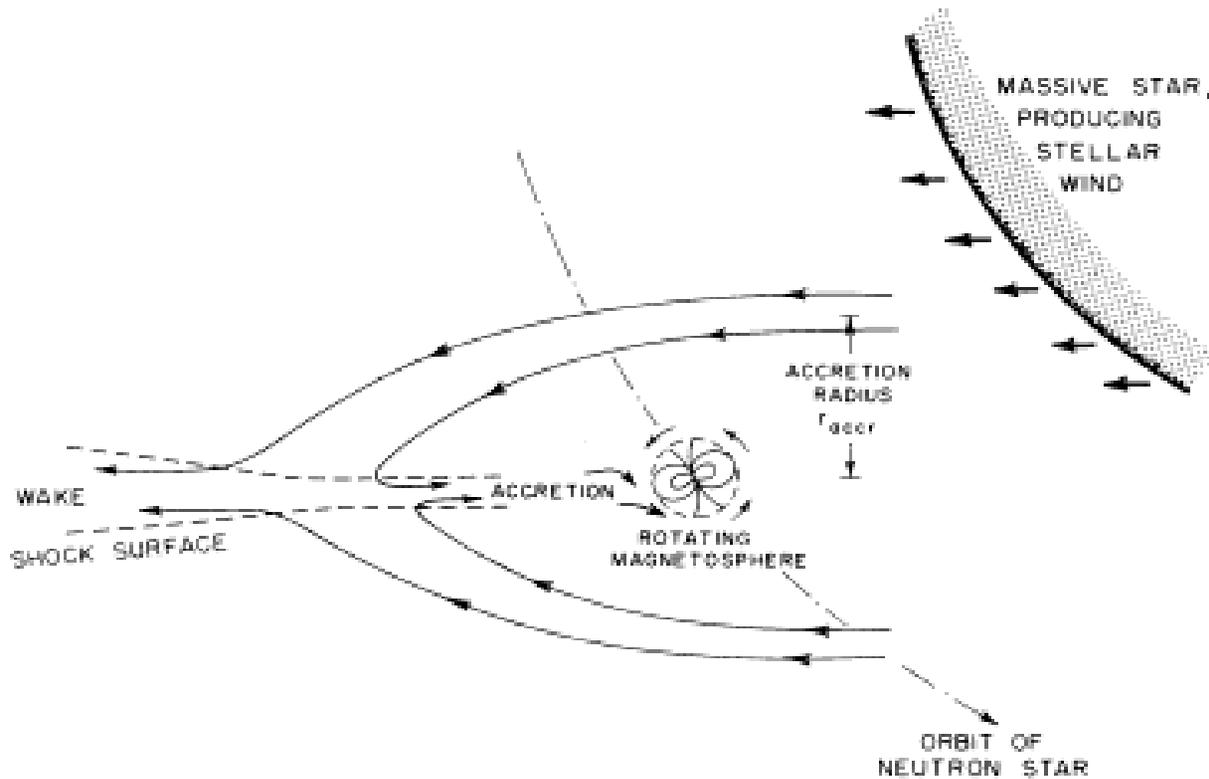
From Frank et al., 2002, *Accretion Power in Astrophysics*

High Mass X-ray Binary

- The high mass companion sheds mass through a wind.
- Wind flows isotropically from the companion, so a portion of it 'hits' the compact object.
- This material is accreted and releases its potential energy as X-rays.



HMXB- wind accretion



Observed X-ray luminosity $L_x \sim \dot{M}_{\text{wind}} c^2 G^2 M^2 \eta / (a^2 v_w^4)$

From similar stars

Efficiency Guess: ~ 0.1

separation

Wind speed

Theory of wind accretion:
(Davidson and Ostriker 1973):

$$\rho v_w^2 / 2 = GM\rho / r$$

Accretion From a Wind

- The process is called Bondi accretion
- Consider a star of mass m , traveling through a gas of density ρ at relative velocity v_{rel} .
- Material inside a cylinder of radius $r_{acc} = 2GM/v_{rel}^2$ can lose enough energy to fall onto the star at an accretion rate of $S = \pi r_{acc}^2 v_{rel} \rho f$ (where f is a fudge factor due to things not properly modeled due to radiation pressure effects and gas dynamics)

giant stars stellar wind speeds are
~700km/sec giving

$$r_{acc} \sim 5 \times 10^{10} \text{cm}$$

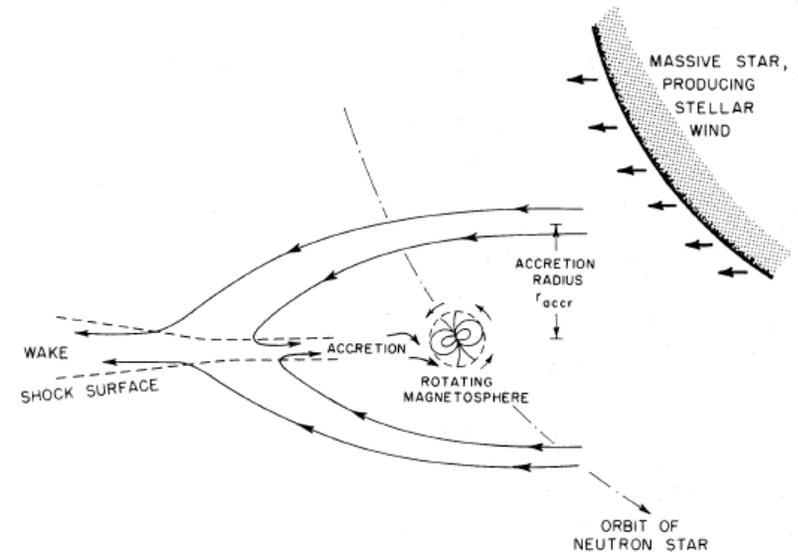
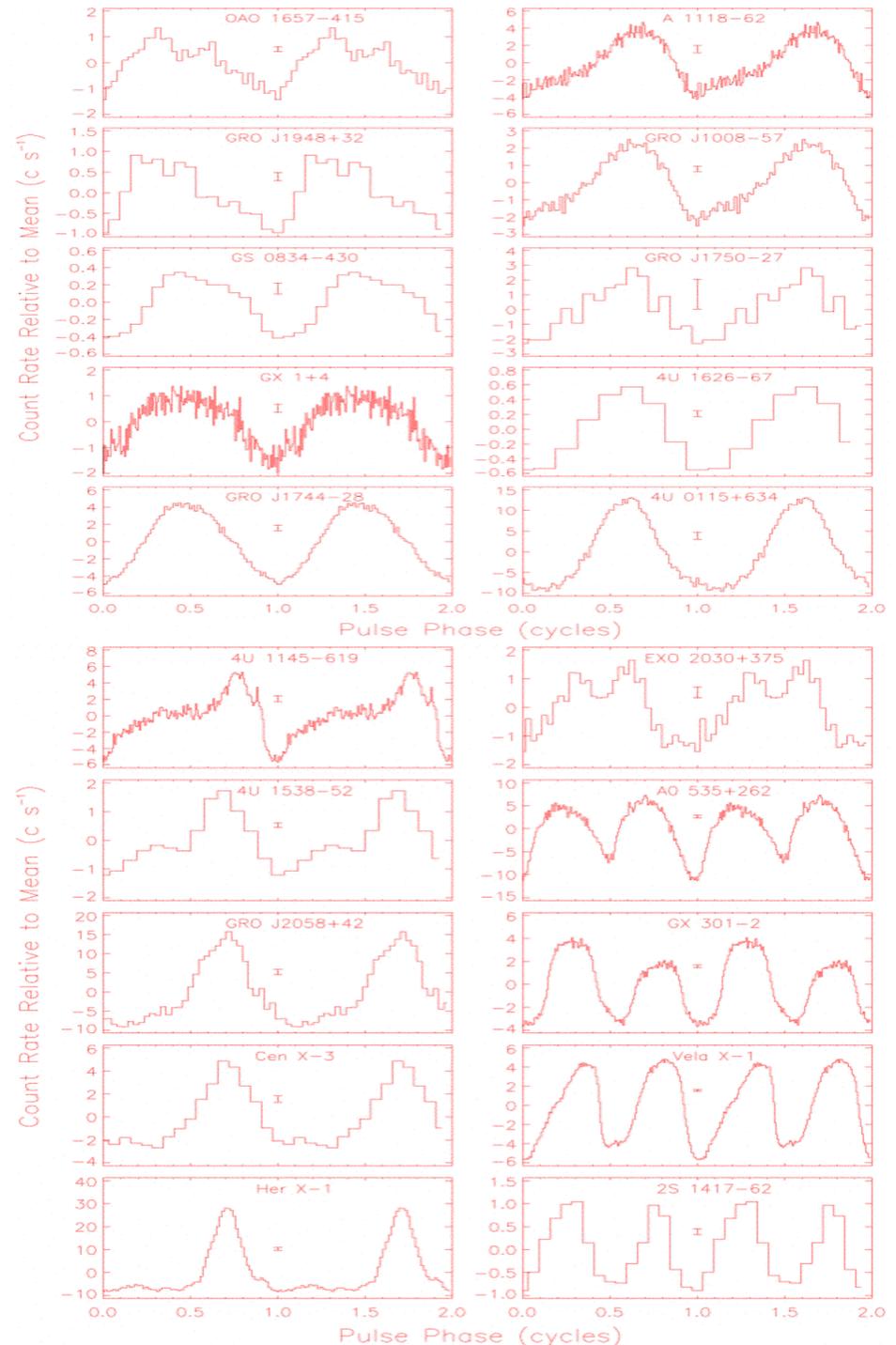


FIG. 3.—Streamlines of stellar-wind material, relative to an accreting neutron star. Dimensions are not to scale.

X-ray Pulsars

- Accrete matter through wind or via disk from a high mass companion. Because of a large magnetic field strength (typically 10^{12} G) the material is channeled onto small spots at the magnetic poles.
- the relativistically moving plasma is decelerated in a radiative shock near the surface and this subsonically settling plasma radiates in the X-ray band
- Pulsations are observed if the magnetic field is inclined relative to the rotation axis.
- Studies of the pulse profiles of individual pulsars allow one to constrain the emission pattern of the hotspots (or accretion columns) at the NS surface as well as the geometry of the magnetic field



If the Magnetic Field is Strong (As In NS Pulsars)

- If magnetic pressure dominates over thermal pressure the magnetic field channels the accretion flow and matter flows along field lines that connect to the magnetic polar regions:
- As a result, almost all of the accretion energy is released in a “hot spot” near the two magnetic poles. If the magnetic axis is not aligned with the rotational axis, then as the star rotates we see more or less of the hot spot, and hence see pulsations in the X-rays.

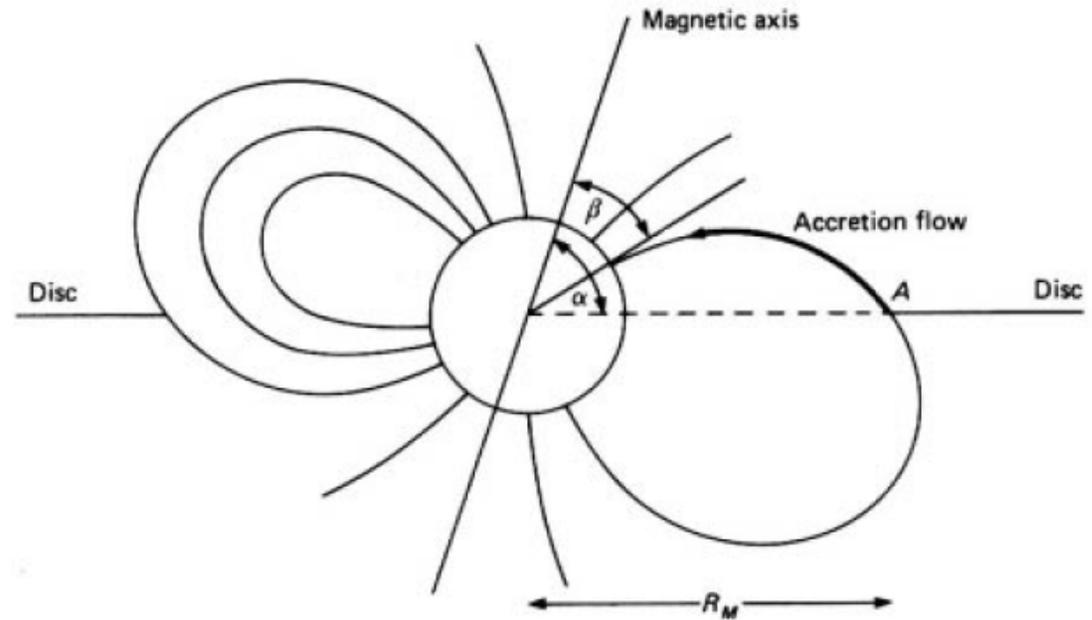


Figure 8: Accretion in a strong ($\sim 10^{12}$ Gauss) magnetic field. Note that the accretion disk is held off the neutron star surface by the centrifugal barrier formed by the rotating magnetosphere.²³

Cominsky (2002)

Accreting Magnetic Neutron Stars

- Effect of magnetic field
 - flow of ionized gas is channeled by the field
 - Photon production in a strong field is different (cyclotron radiation)
- When/where does the magnetic field dominate the accretion flow? (C. Miller)

The magnetic energy density is $B^2/8\pi$, and the kinetic energy density of the accreting matter is $1/2 \rho v^2$, where ρ is the density and v is the typical velocity.

Accreting Magnetic Neutron Stars

By continuity, $\rho v_{\text{ff}} = dM/dt / (4 \pi r^2)$ (gas flow) ($dM/dt = \dot{M}$)

Magnetic energy density = $B^2 / 8 \pi$

Notice the radial dependences

magnetic energy density goes as r^{-6}

material energy density goes as $r^{-5/2}$.

Close to the star, **magnetic stresses will dominate** if the field is strong enough;

A magnetic moment of $\mu_{30} = 10^{30} \text{ G cm}^3$ which gives a surface field of $\sim 10^{12} \text{ G}$ is typical of neutron stars in high-mass X-ray binaries.

Since the radius of a neutron star is $R \approx 10^6 \text{ cm}$, the accretion flow onto a strongly magnetized neutron star is dominated by the magnetic field.

Where Does the Magnetic Field Start to Dominate?

This is called the Alfvén radius

The radius at which the pressure due to the pulsar's magnetic field equals the ram pressure of infalling material.

If the magnetic field is dipolar, $B = \mu / r^3$, (μ is the magnetic moment) and if the matter moves in spherical radial free fall

$$v = v_{\text{ff}} = \sqrt{2GM/r}.$$

$$\text{So : } \rho v_{\text{ff}} = \mathcal{M} / (4 \pi r^2)$$

$$\text{The free fall velocity } v_{\text{ff}} = (GM_x/2r)^{1/2}$$

And the Kinetic energy

$$E_{\text{kinetic}} = 1/2 \rho v_{\text{ff}}^2 = \mathcal{M} \sqrt{GM_x} r^{-5/2} / 8 \pi \sqrt{2}$$

The magnetic energy is

$$\bullet \quad E_{\text{mag}} = B^2 / 4 \pi = \mu^2 / 4 \pi r^6$$

• Balancing the two one finds that the Alfvén radius is

$$r_A \sim (\mu^4 / GM_x \mathcal{M}^2)^{1/7}$$

Or putting in typical numbers

$$r_A \sim 3.2 \times 10^8 \mathcal{M}_{17}^2 \mu_{30}^{4/7} M_{\odot}^{1/7} \text{ cm}$$

since a NS has a typical radius of 10^6 cm the magnetic field controls the flow at relatively large radii

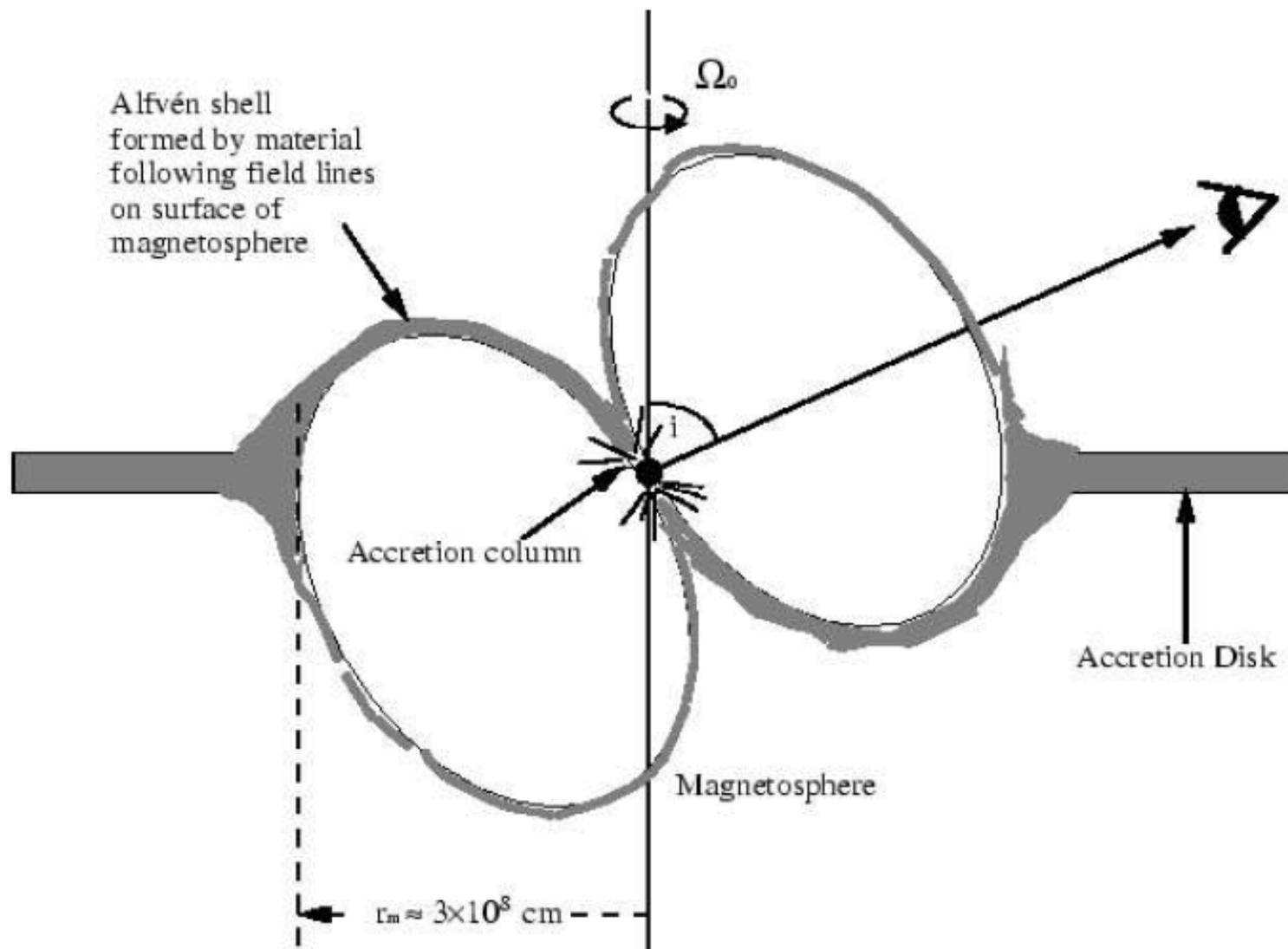


Fig. 1.— Rough sketch of the accretion flow in a disk being picked up by a strong neutron star magnetic field. From <http://heawww.gsfc.nasa.gov/users/audley/diss/img203.gif>

- Putting in typical numbers the radius where magnetic and material stresses are equal is the Alfvén radius

$$r_A = \left(\frac{\mu^4}{2GM\dot{M}^2} \right)^{1/7} = 3.2 \times 10^8 \dot{M}_{17}^{-2/7} \mu_{30}^{4/7} \left(\frac{M}{M_\odot} \right)^{-1/7} \text{ cm} .$$

\dot{M}_{17} is the accretion rate in units of 10^{17} gm/sec- why do we scale it this way?!

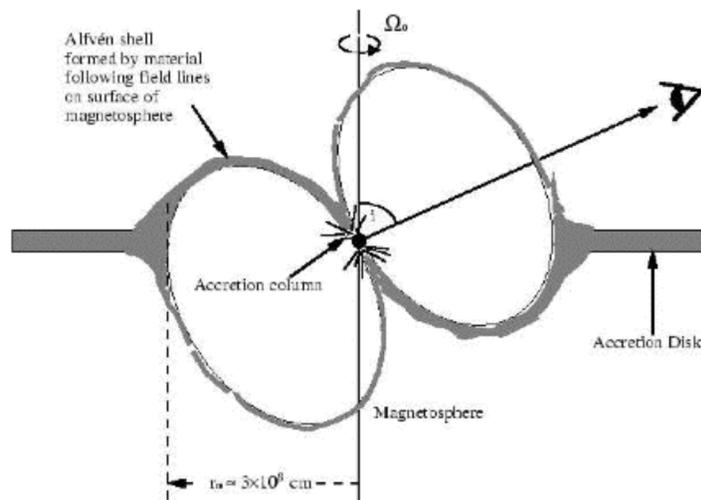
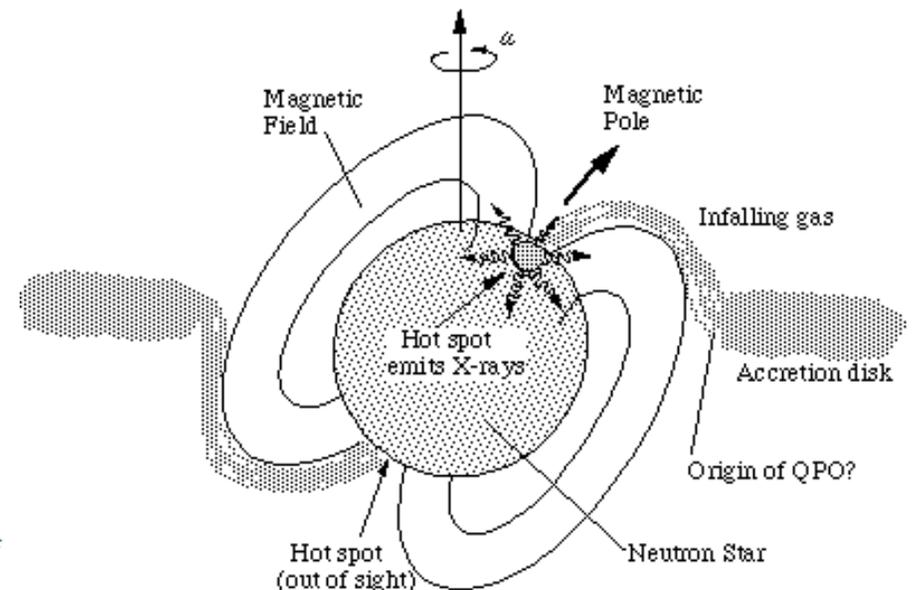


Fig. 1.— Rough sketch of the accretion flow in a disk being picked up by a strong neutron star magnetic field. From <http://hea-www.gsfc.nasa.gov/users/audley/diss/img203.gif>

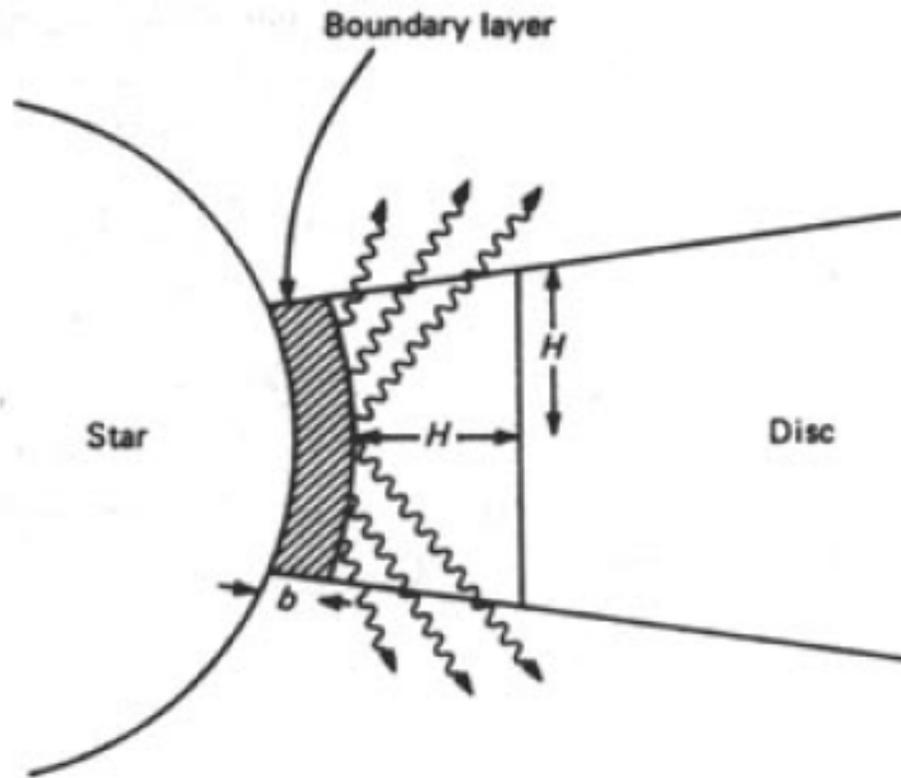


Violation of Eddington Limit ??

- The accretion rate of, ~ 0.1 the Eddington limited accretion rate falls onto a surface area only 10^{-3} of the star !
- So the local flux generated \gg Eddington limit
- For such accretion to persist, the radiation cannot escape back up the accretion funnel (remember the incoming material is interacting with the radiation for the Eddington limit to be defined).
- Instead the radiation has to come out where there is little or not accreting material (out the sides).
- The Eddington flux is a limit only for spherically symmetric systems, and in this case we have a system that is very aspherical
- the radiation pattern can be a “fan beam” (radiation escaping out the sides), so that we might get two peaks per cycle from the funnel (one from one side, one from the other) as opposed to the one peak we would expect if this were just a thermally glowing hot spot.

Where is the Accretion Occurring

- In a weak field neutron star (LMXB) has the accretion disk and the place where the material hits the star surface (boundary layer) produce the radiation



Origin of Field ?

- If the field is due to the 'original' star The fields in Main Sequence (MS) stars are $\sim 1\text{G}$.
- For a MS progenitor of radius $4 \times 10^{11}\text{cm}$ (the sun has a radius of $7 \times 10^{10}\text{cm}$), ($10M_{\odot}$) the star would contain a magnetic flux of $\sim 5 \times 10^{23} \text{ Gcm}^2$ ($\pi r^2 B$)
- If flux is conserved during the collapse then a neutron star with the same flux would have surface field strength of $5 \times 10^{11}\text{G}$, sufficient for a pulsar
- However no one really knows if flux is conserved in the formation of the NS during the Supernova explosion and collapse and there are good reasons to believe that this is not true

Eclipsing Pulsing Neutron Stars

- A breakthrough in the understanding of these objects was the discovery of eclipses and pulse timing.

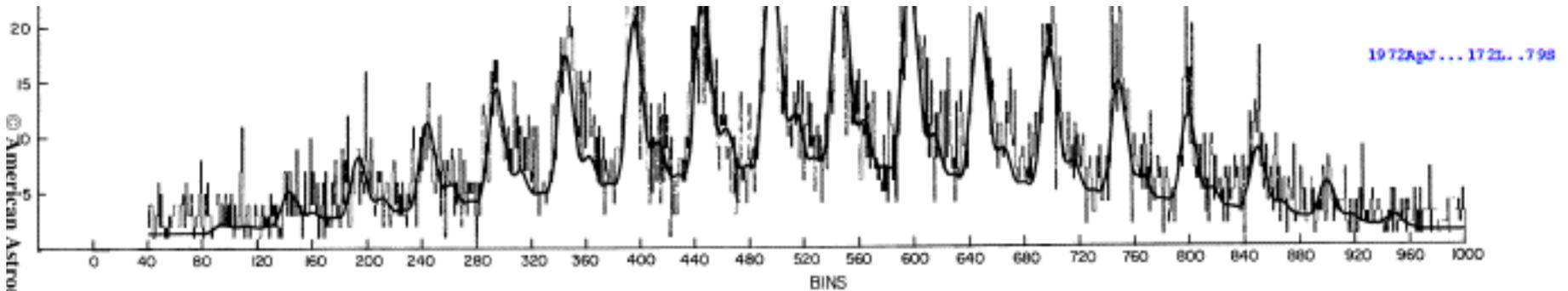
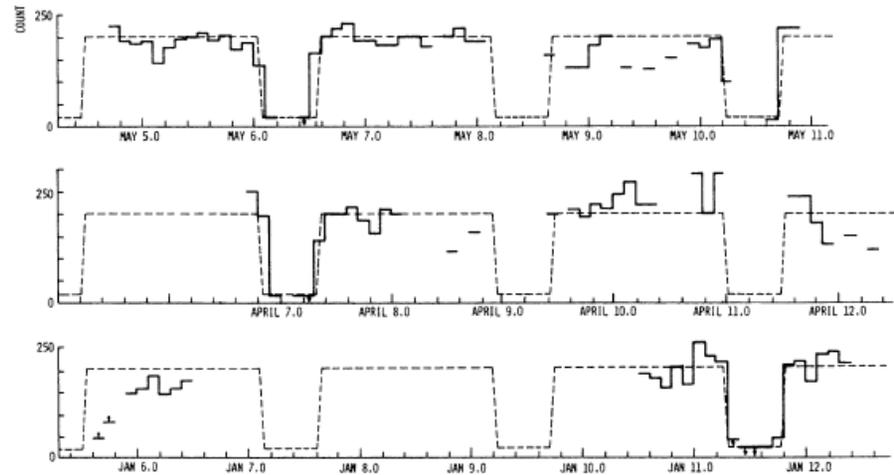


FIG. 3.—The counts accumulated in 0.096 bins from Cen X-3 during a 100-s pass on 1971 May 7 are plotted as a function of bin number. The functional fit obtained by minimizing is also shown.

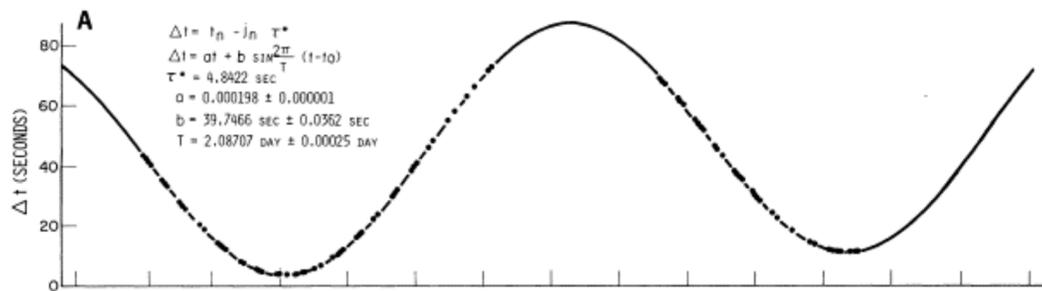


FIG. 4.—(a) The difference Δt between the time of occurrence of a pulse and the time predicted for a constant period is plotted as a function of time. A best-fit function and the values of the parameters are given. (b) The dependence of the pulsation period τ on time as derived from the best-fit phase function above is shown and the values of the parameters given. (c) The intensity observed and the light curve predictions are shown for the same set of data. Note the coincidence of the null points of the period function with the centers of the high- and low-intensity states.

Orbit

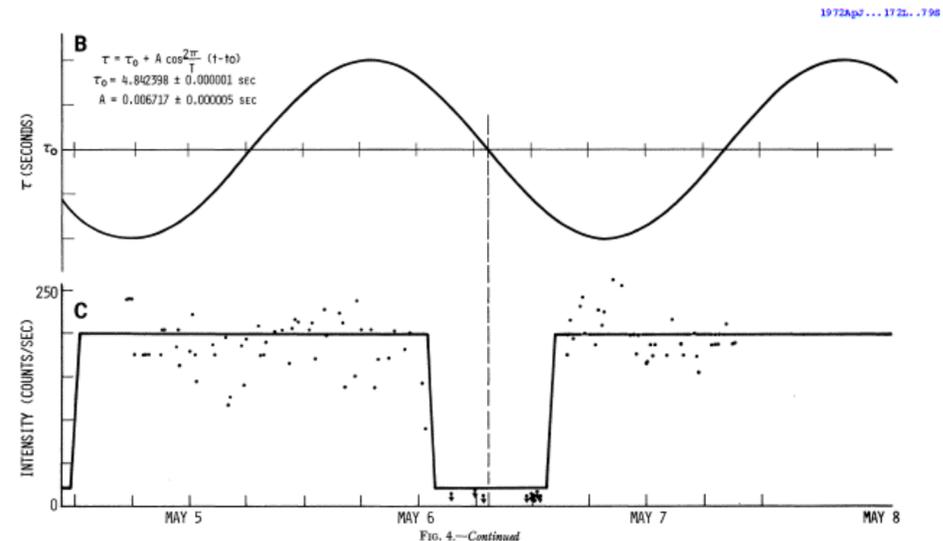
- Sign and phase of the pulses are due to the Doppler effect
- Amplitude of the sine pulse curve gives the size of the orbit (39.75 lt sec) with a 2.09 day orbit
- Eclipses are due to occultations of the NS by its companion
- Circular orbit from shape of time variation of pulses
- Get mass of system and orbital parameters
- Period of 4.8 sec shows that it must be a collapsed object (NS)

$$v \sin i \equiv \frac{Ac}{\tau_0} = 415.1 \pm 0.4 \text{ km s}^{-1},$$

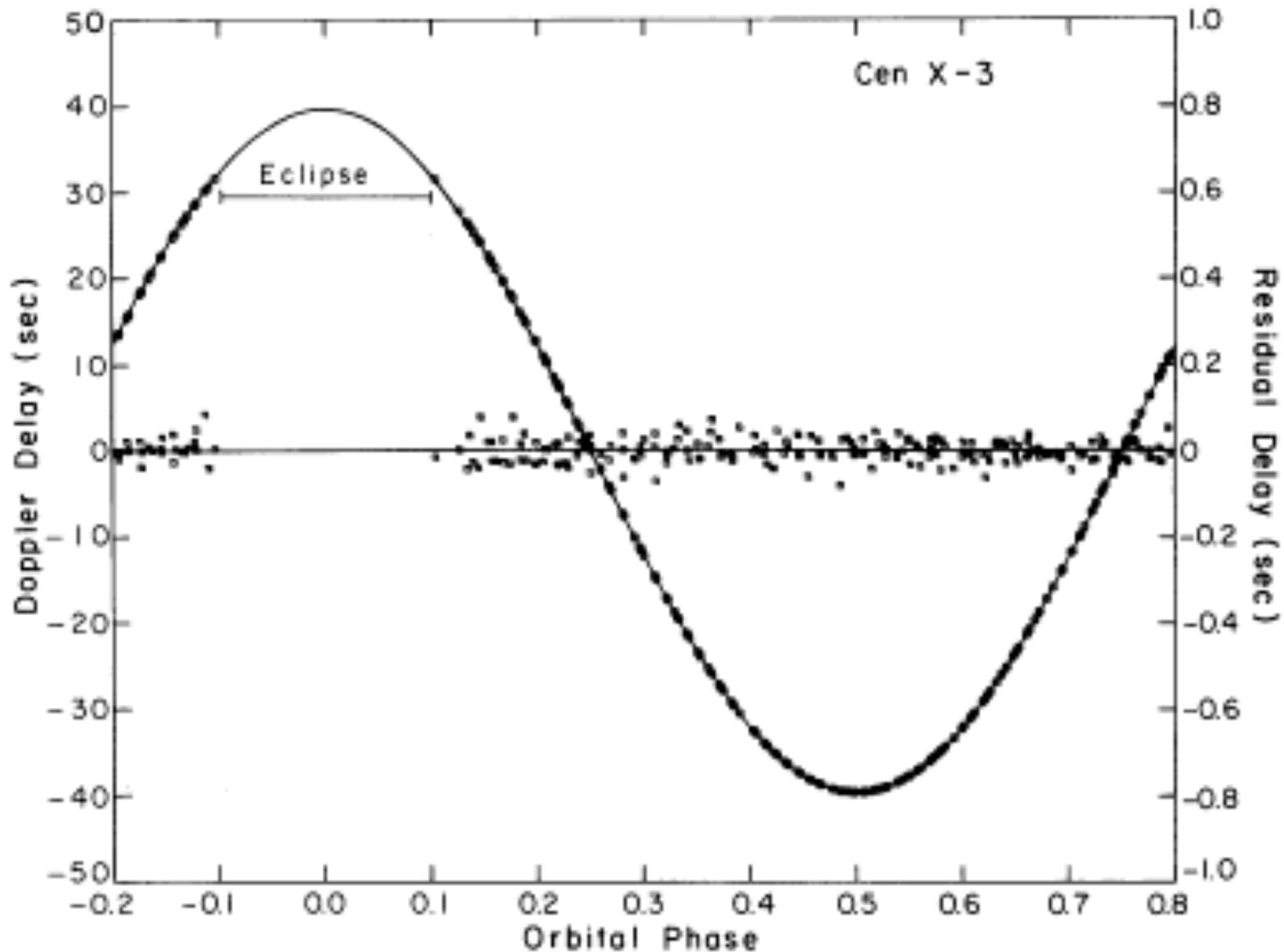
$$r \sin i \equiv \frac{T}{2\pi} v \sin i = (1.191 \pm 0.001) \times 10^{12} \text{ cm},$$

$$\frac{M^3 \sin^3 i}{(M + m)^2} \equiv \frac{(2\pi)^2}{GT^2} (r \sin i)^3 = (3.074 \pm 0.008) \times 10^{34} \text{ g}.$$

Solution for Cen X-3



Measurement of Orbit Via Pulse Timing



Mass of the NS Star

- In order to measure the mass of the neutron star and its optical companion we need to measure the mass function. For a circular orbit it can be shown that this is defined

$$M_X = K_O^3 P / 2\pi G \sin^3 i (1 + K_X / K_O)^2$$

- M_O and M_X are the mass of the optical component and the X-ray source, respectively, i is the inclination
 - K_X , K_O are the semi-amplitude of the radial velocity curve for the x-ray and optical companion, respectively
 - P is the period of the orbit and i is the inclination of the orbital plane to the line of sight.
- K_X and P can be obtained very accurately from X-ray pulse timing delay measurements with K_O is measured from optical spectra for the companion

Mass Function

- $F(m_1, m_2, i) = m_1^3 \sin^3 i / (m_1 + m_2)^2$
- Re-writing this as

$$M_x = F_x q (1+q)^2 / \sin^3$$
- q = ratio of the mass of the x-ray star to its companion

- Using Newton's laws

$$F(m_1, m_2, i) = (P / 2\pi G (1-e^2)^{3/2} (v_2 \sin i)^3$$

And

$$F(m_1, m_2, i) = (4\pi^2 / GP^2) (a_2 \sin i)^3$$

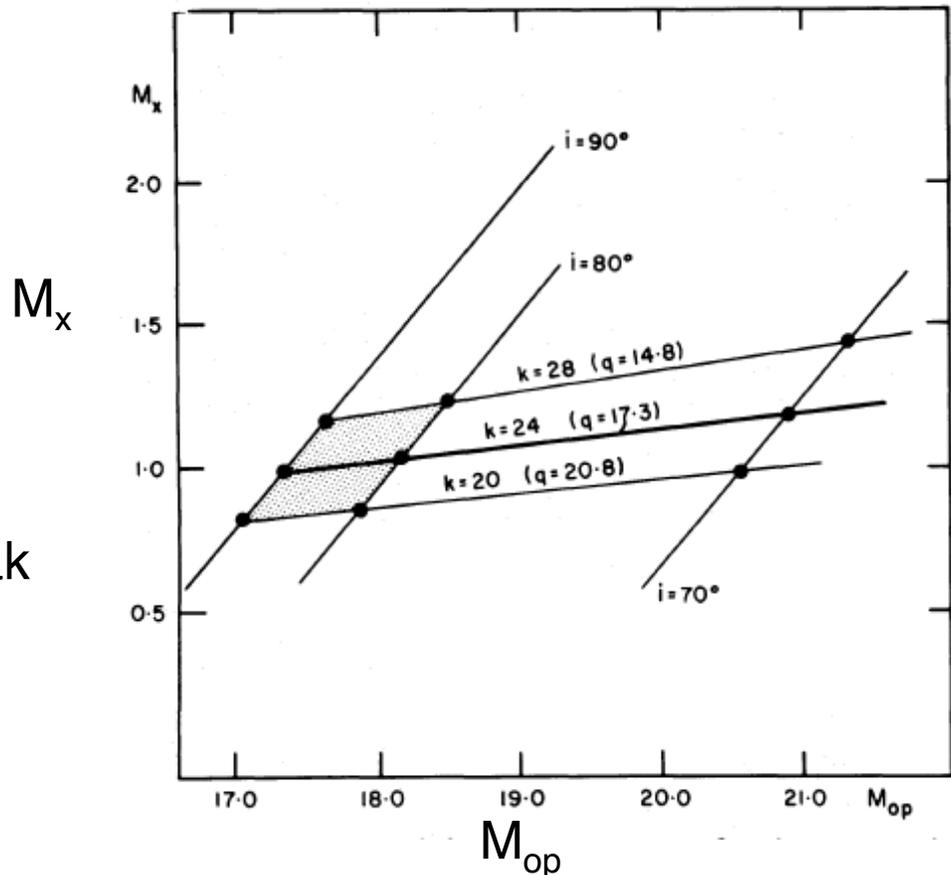
Where a_2 is the orbital semi-major axis of star 2, $v_2 \sin i$ is 1/2 the peak to peak orbital velocity of star 2

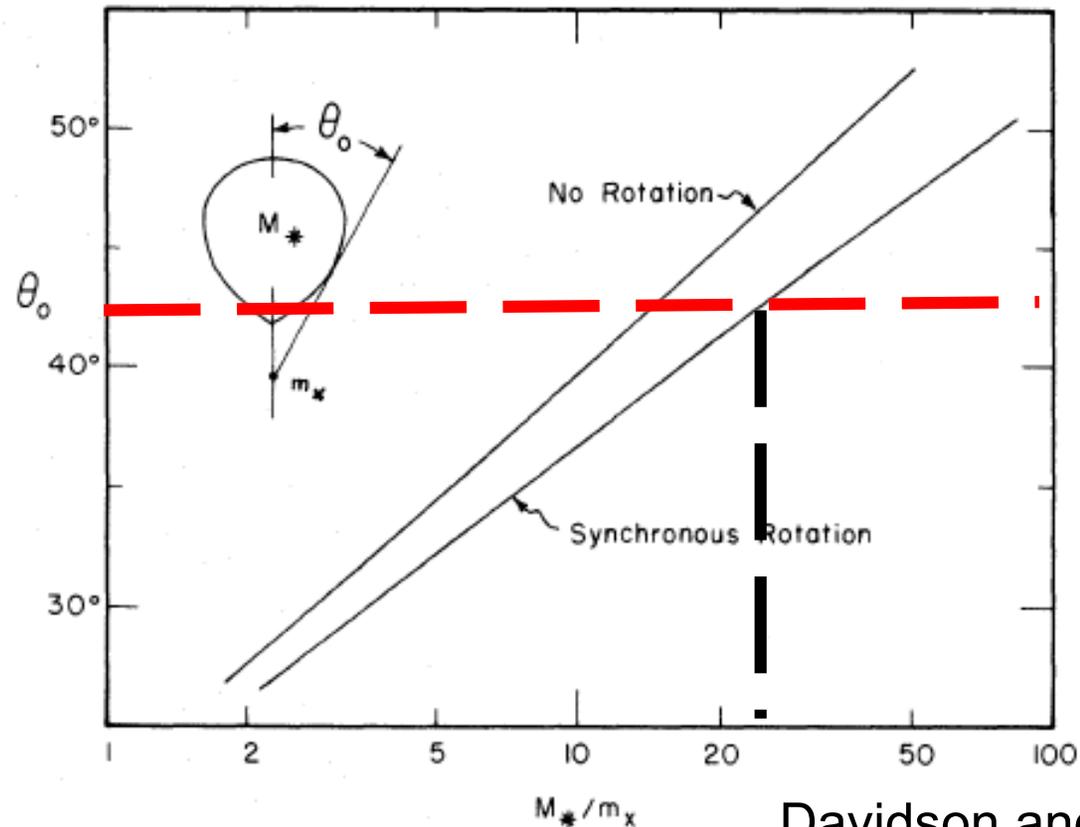
P is the period and e is the eccentricity

The delays in the observed arrival time of the pulses gives $a_2 \sin i / c$ and the period thus

$$F(m_1, m_2, i)$$

The duration of the eclipse tells us about the star size

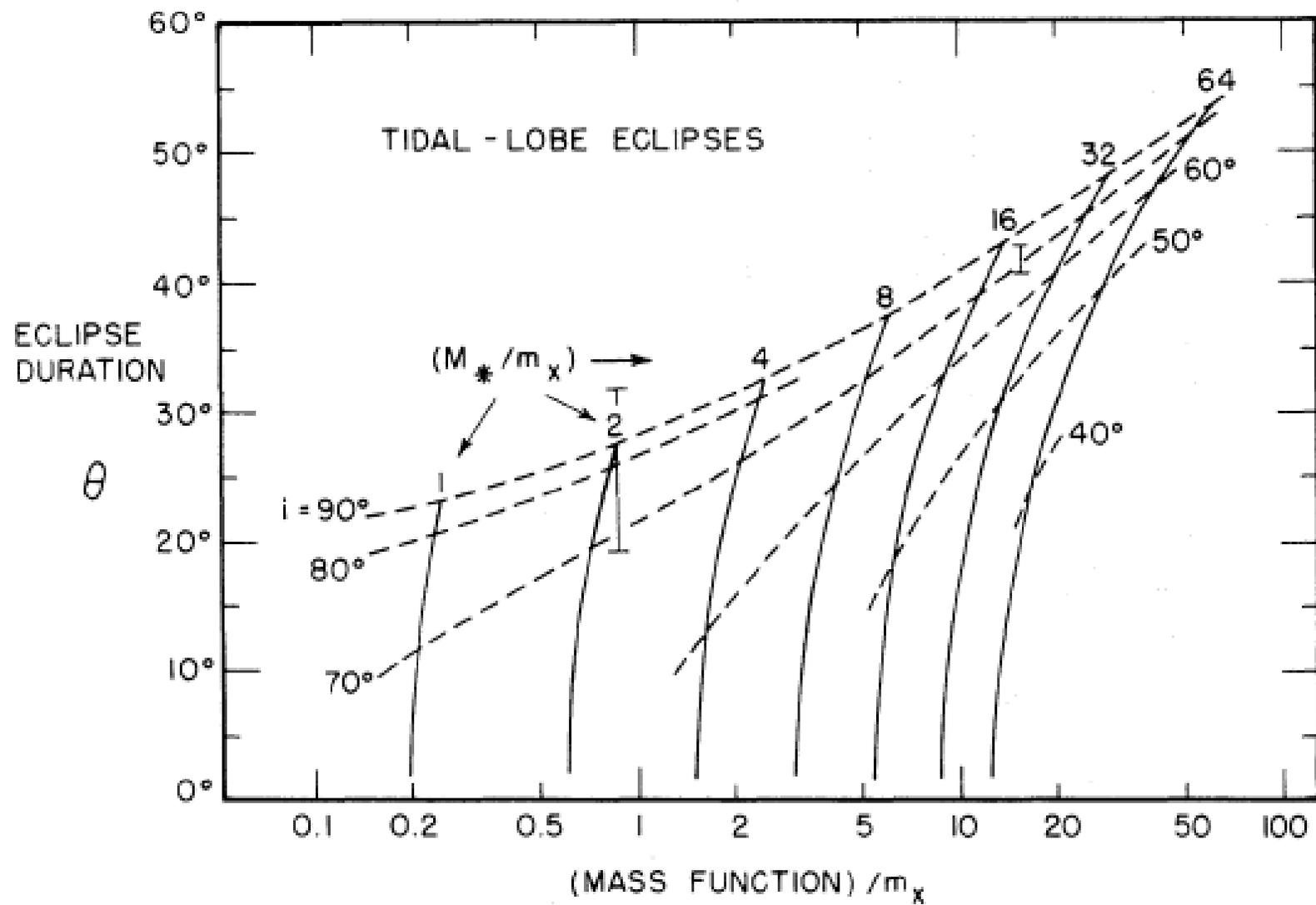




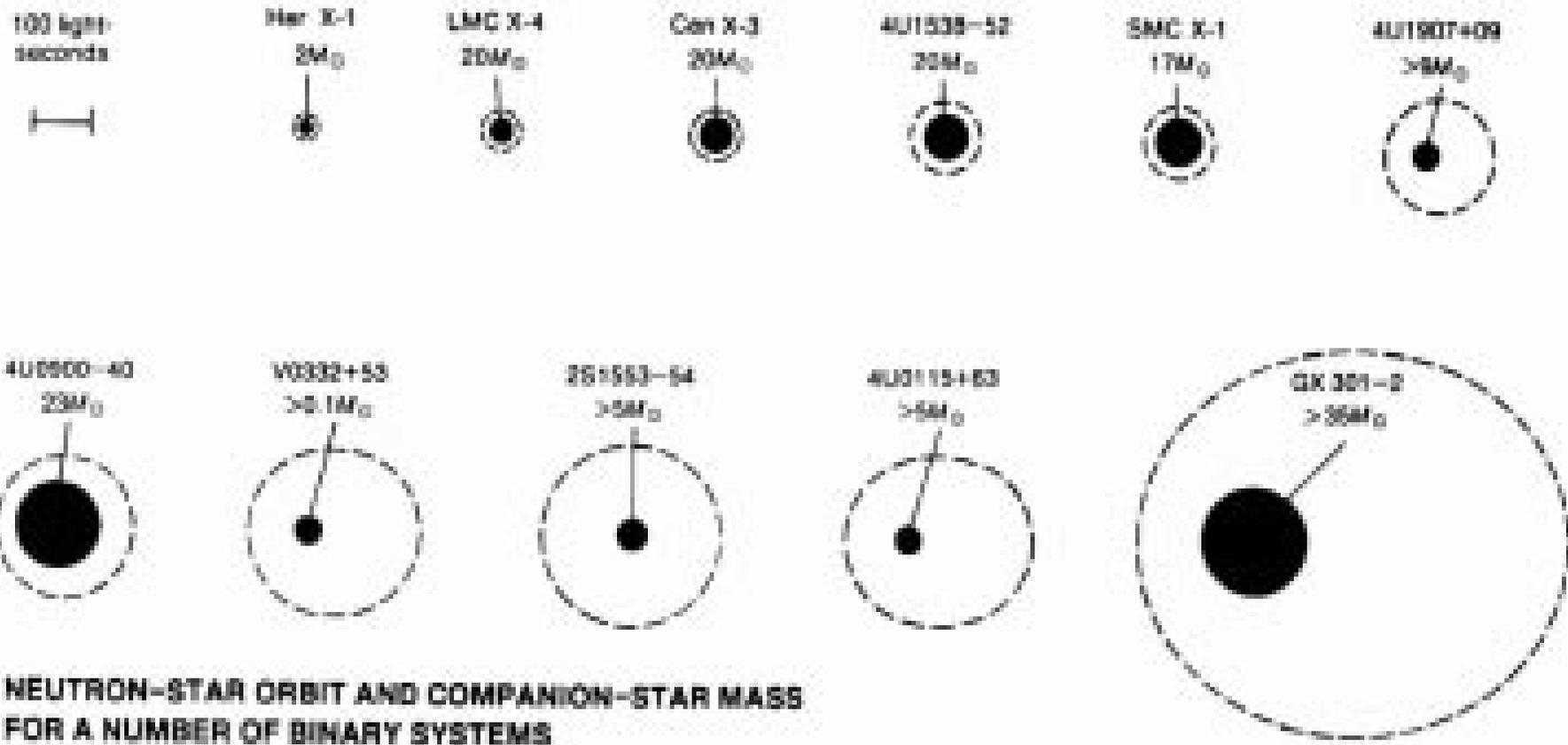
Davidson and Ostriker 1973

FIG. 1.—Eclipse half-angles in the equatorial plane, for cases in which the eclipsing star is nonrotating and fills its tidal lobe, and in which it rotates synchronously with the binary orbital period and fills its Roche lobe.

- For Cen X-3 the eclipse lasts 0.488 days out of the 2.1 day period or an opening angle of 43 degrees $((.488/2.1)/2)$. We know the mass function $M_x^3 \sin^3 i / (M_x + M_*)^2 = 15$ in this case) $M_x \sim 1$



Neutron Star Orbits



Charles and Seward

summary

- X-ray binaries exhibit a wide range of behaviors, but much of the interesting physics/astrophysics is common to all
- Understanding of accretion disks, accretion flows, X-ray induced winds, compact object evolution are all in an active state of research.

Maximum Mass of a Compact object (Kalogera and Baym 1996)

- The set of fundamental constraints, independent of the detailed physical properties of neutron matter, imposed on the equation of state of the inner core are
- (i) the mass density, ρ , is non-negative, i.e., gravity is attractive;
- (ii) the pressure, P , at zero temperature is a function of ρ only, i.e., neutron matter is a fluid
- (iii) $dP/d\rho \geq 0$, -sound speed of neutron matter $(dP/d\rho)^{1/2}$ is real and matter is stable against collapse;
- (iv) the sound speed does not exceed the speed of light, i.e., $dP/d\rho \leq c^2$, hence signals cannot be superluminal and causality is satisfied.

Under these conditions mass of NS is maximum for 'stiffest' equation of state -the sound speed is the speed of light $c_s^2 = dP/d\rho = c^2$.

A huge amount of messy nuclear physics define the equation of state and it is not well understood.

Using the equation of hydrostatic equilibrium in general relativity

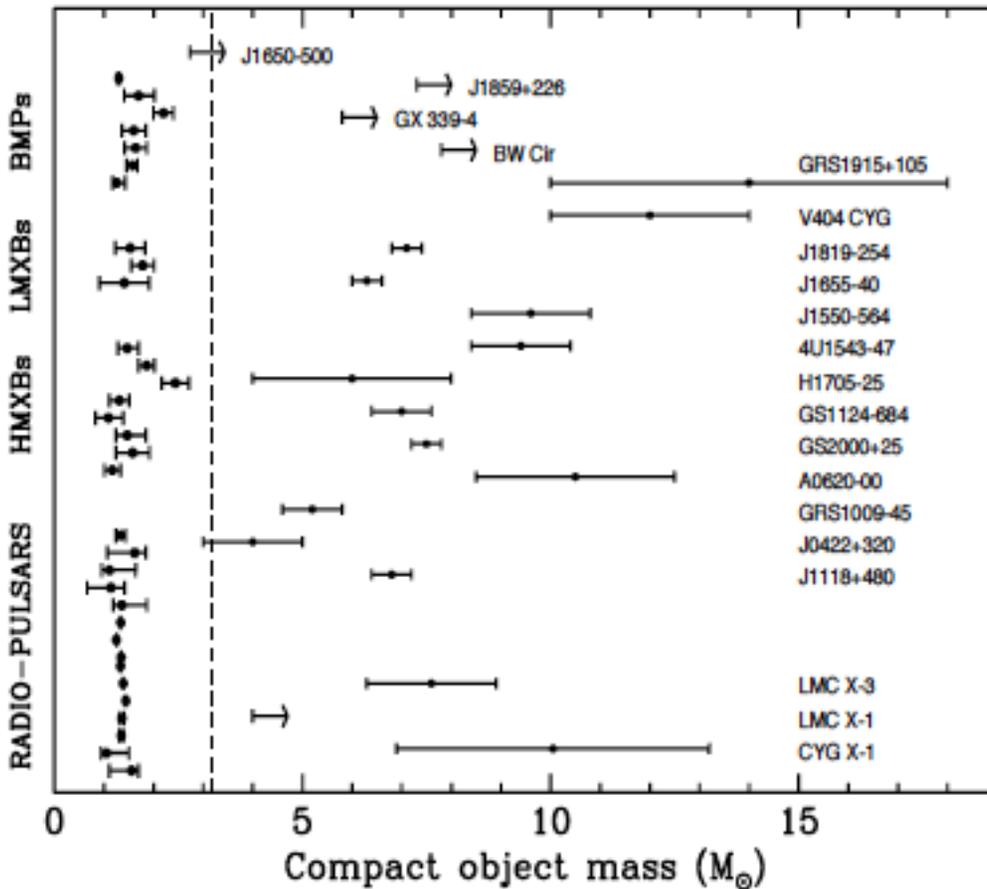
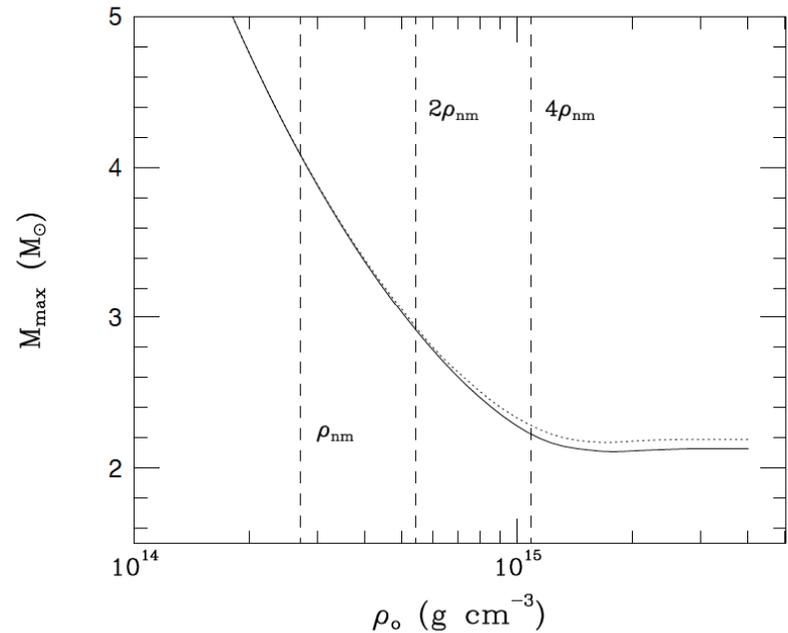
$$\frac{dP}{dr} = -\frac{G}{r^2} \left[\rho(r) + \frac{P(r)}{c^2} \right] \left[m(r) + 4\pi r^3 \frac{P(r)}{c^2} \right] \left[1 - 2G \frac{m(r)}{rc^2} \right]^{-1},$$

$$M_{max} = 6.7 M_{\odot} \left(\frac{\rho_0}{10^{14} \text{g cm}^{-3}} \right)^{-1/2}.$$

Maximum Mass (Cont)

nuclear matter density $\rho_{nm} = 2.7 \times 10^{14} \text{ g cm}^{-3}$.

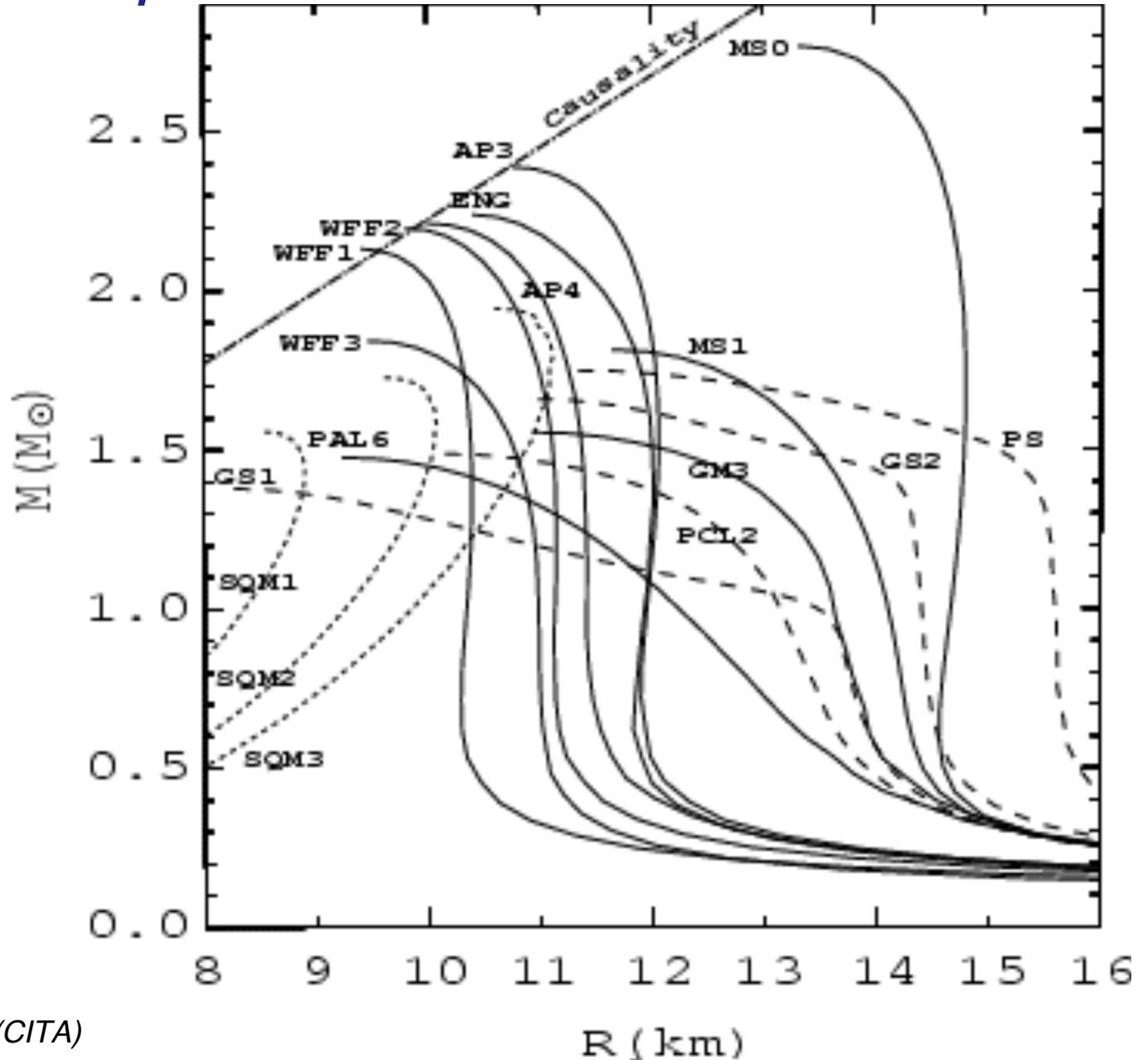
- None of the objects thought to be NS have a mass $>2.4M_{\text{sun}}$ but objects exist in the Milkyway way with a mass up to $19M_{\text{sun}}$



Possible Equations of State of a NS

Each line represents
a different possible
equation of state-
the relationship
between
Pressure and density

Maximum mass is
 $\sim 3 M_{\text{sun}}$



From website of Kaya Mori (CITA)