- The S-process is believed to occur mostly in asymptotic giant branch stars; occuring over time scales of thousands of years, passing decades between neutron captures. Producing heavy elements beyond Sr and Y, and up to Pb,
- R-process which is believed to occur over time scales of seconds in explosive environments (core-collapse supernovae). Radioactive isotopes must capture another neutron faster than they can undergo beta decay in order to create abundance peaks at germanium, xenon, and platinum

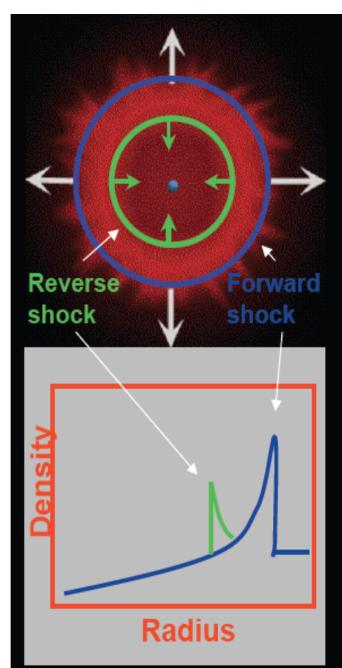
3 phases in SNR's life.

•Free expansion (less than 200-300 years)

•Adiabatic or "Taylor-Sedov" phase (about 20,000 years)

Radiative or Snow-plow phase (up to 500,000 years)

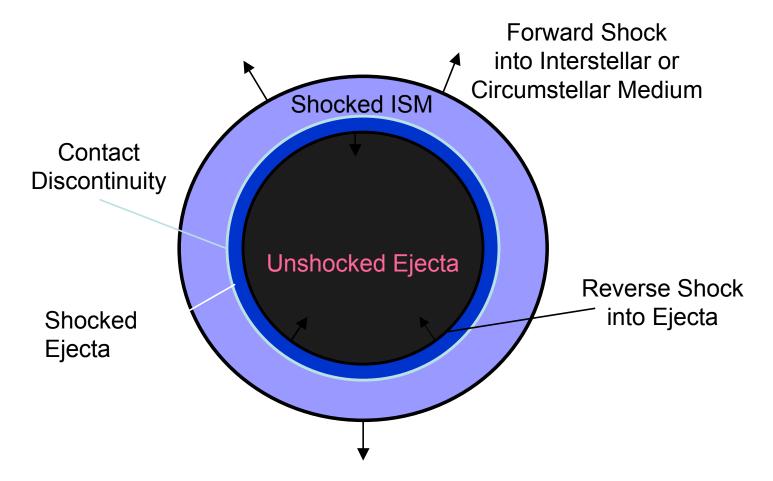
and then ... Merge with the ISM



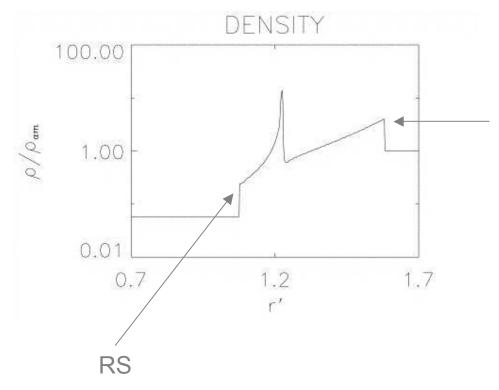
Supernova Remnants See R+B sec 4.4

- Explosion blast wave sweeps up CSM/ISM in forward shock
 - spectrum shows abundances consistent with solar or with progenitor wind
- As mass is swept up, forward shock decelerates and ejecta catches up; reverse shock heats ejecta
- spectrum is enriched w/ heavy elements from hydrostatic and explosive nuclear burning

Supernova Remnant Cartoon



Forward shock moves supersonically into interstellar/circumstellar medium Reverse shock propagates into ejecta, starting from outside



Shocks compress and heat gas

Mass, momentum, energy conservation give relations FS (for γ =5/3) $\rho = 4\rho_0$ $V = 3/4 v_{shock}$ T=1.1 m/m_H (v/1000 km/s) ² keV

X-rays are the characteristic emission

These relations change if significant energy is diverted to accelerating cosmic rays The shock is "collisionless" because its size scale is much smaller than the mean-free-path for collisions (heating at the shock occurs by plasma processes) coupled through the structure of turbulence in shocks and acceleration

Collisions do mediate ionizations and excitations in the shocked gas

The Shock

- A key ingredient in SNR dynamics is the strong (high Mach number) shock which is "collisionless"
- the effect of the shock is carried out through electric and magnetic fields generated collectively by the plasma rather than through discrete particle–particle collisions
- the shock system is given by the synonymous terms "adiabatic" and "non-radiative" to indicate that no significant energy leaves the system in this phase
- a "radiative" shock describes the case where significant, catastrophic cooling takes place through emission of photons
- For standard 'collionless' shocks $kT=(3/16)\mu m_p V_S^2 \sim 1.2(V_S/1000)^2 keV$

- Kinetic energy of expansion (KE) is transferred into internal energy total energy remains roughly the same (e.g. radiative losses are small)
- The temperature of the gas is related to the internal energy
- T~10⁶k $E_{51}^{1/2}$ n^{-2/5}(t/2x10⁴yr)^{-6/5}
- n is the particle density in cm⁻³ E_{51} is the energy of the explosion in units of 10⁵¹ ergs
- for typical explosion energies and life times the gas emits primarily in the x-ray band
- measuring the size (r), velocity (v) and temperature T allows an estimate of the age
- $t_{\text{Sedov}} \sim 3 \times 10^4 \text{T}_6^{-5/6} \text{E}_{51}^{1/3} \text{n}^{-1/3} \text{yr}$
- at T ~ 10^{6} - 10^{7} k the x-ray spectrum is line dominated

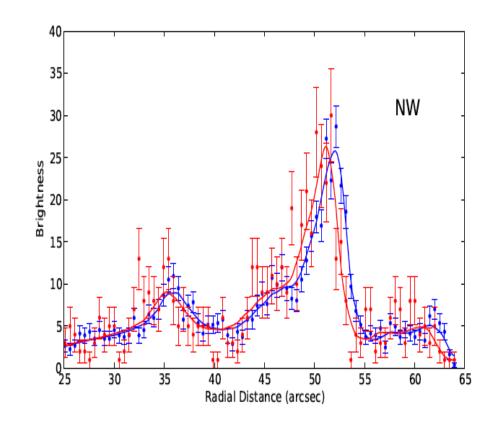
- If the density (gm/cm³) in the ISM/circumstellar gas is ρ_{ism}
- then the radius of the shock when it has swept up an equal mass to the eject M_{ejecta} is simply
- $r_1 = 2pc(M_{ejecta}/M_{\odot})^{1/3} (\rho_{ism}/10^{-24} \text{ gm/cm}^3)^{-1/3}$
- to get an estimate of the time this occurs assume that the shock has not slowed down and the total input energy remains the same (radiation losses are small) and travels at a velocity $v_{ejecta} = (v_{ejecta}/10^4 \text{ km/sec})$ to get
- $t_1 = r_1 / (v_{ejecta} / 10^4 \text{ km/sec}) = 200 \text{ yr}(E_{51})^{-1/2} (M_{ejecta} / M_{\odot})^{5/6} (\rho_{ism} / 10^{-24} \text{ gm/cm}^3)^{-1/3}$
- To transform variables total energy $E=1/2M_{ejecta}v^2 \sim r^3 \rho_{ism}v^2$ to get $r\sim (E/\rho_{ism})^{1/5} t^{2/5}$

Limit of Strong Shocks

- Ratio of temperatures behind and in front of the shock is related to the Mach speed, *M*, of the shock (shock speed/sound speed in gas)
- $T_2/T_1 = (2\gamma(\gamma-1)\mathcal{M}^2)/(\gamma-1)^2 = 5/16\mathcal{M}^2$ if the adiabatic index $\gamma = 5/3$ (ideal gas)

Expansion of G1.9+0.3

- The simplest Type Ia SNR model : ejecta with an exponential density profile, ρ_e ~exp(v/v_e)t⁻³, expanding into a uniform ambient ISM (Dwarkadas, Chevalier 1998)
- Velocity scale of ejecta $v_e = 2440E_{51}^{1/2}(M_e/1.4M_{\odot})^{-1/2}$ km/sec



Sedov-Taylor phase

This solution is the limit when the swept-up mass exceeds the SN ejecta mass -the SNR evolution retains only vestiges of the initial ejecta mass and its distribution.

The key word here is SELF SIMILAR (solutions can be scaled from solutions elsewhere)

===> f(r, t) becomes $f(r/r_{ref}) * f(r_{ref})$

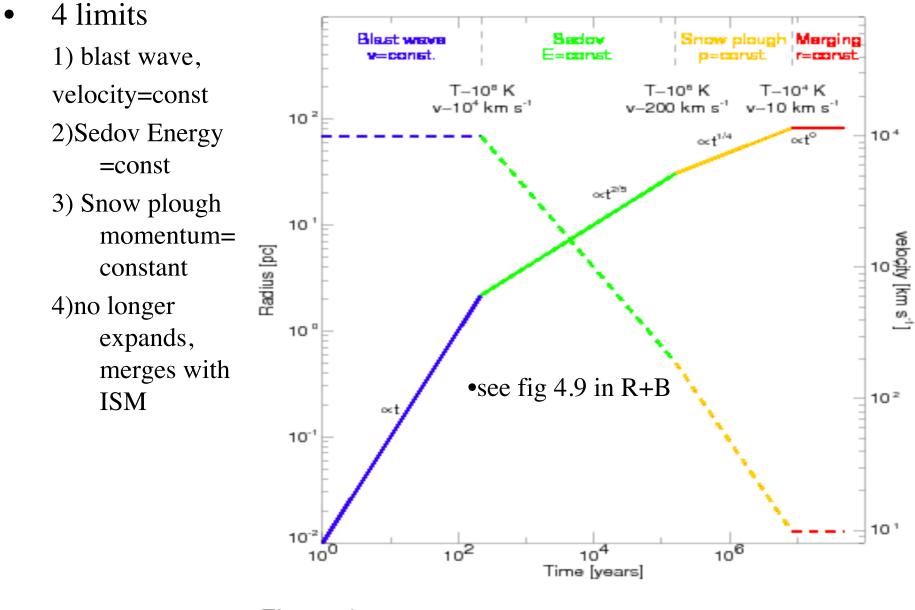
(skipping the equations) $R_s = 12.4 \text{ pc} (KE_{51}/n_1)^{1/5} t_4^{2/5}$

t = 390 yr $R_s T_{meas}^{-1/2}$ R_s is the shock radius, T is the temperature

In the Sedov-Taylor model one expects thermal emission coming from a thin shell behind the blast wave. As the shock expends the pressure drops between the shock wave and the material ejected.

- Forward shock into the ISM- is a 'contact discontinuity'outside of this the ISM does not yet 'know' about the SN blast wave
- Reverse shock- information about the interaction with the ISM travels backwards into the SN ejecta
- Shell like remnants
- Shell velocity much higher than sound speed in ISM, so shock front of radius R forms.

The 4 Phases in the Life of a SNR



t, Fig. 4.6)

3-D Structure

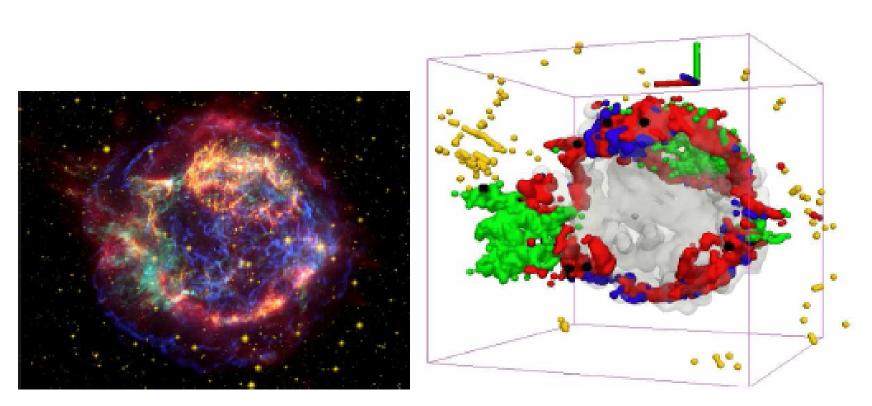


Fig. 2 Cas A present. Left: Composite view of Cas A in X-ray (*Chandra*, green & blue), visible (HST, yellow), and the IR (Spitzer, red); from *Chandra* Photo album, released June 2005. Right: A detailed 3D reconstruction of Cas A also in X-ray (black & green), optical (yellow), and IR (red, blue & gray.) Doppler shifts measured in X-ray and infrared lines provided the third dimension, from DeLaney et al. (2010).

Velocity data allows an inversion of the 2-D to 3-D structure

• Free Expansion Phase

the ejecta expands freely into the interstellar medium. The expanding envelope compresses the ISM, creates a shock wave because of its high velocity, and sweeps up the ISM. During this initial phase, the mass of gas swept up is <<mass of the ejecta and the expansion of the envelope is not affected by the outer interstellar gas andit keeps its initial speed and energy.

• Adiabatic Expansion Phase

When mass of gas swept up > mass of ejecta the kinetic energy of the original exploded envelope is transferred to the swept up gas, and the swept up gas is heated up by the shock wave roughly independent of the physics of the explosion. The radiative losses from the swept up gas are low (energy is conserved) - adiabatic expansion phase.

The evolution during this phase is determined only by the energy of explosion E_0 , the density of interstellar gas, and the elapsed time from the explosion t. A self similar solution relating the density, pressure, and temperature of the gas, and the distribution of the expansion velocity exists (Sedov-Taylor) 15

Sedov-Taylor Solution

- nice discussion in Draine sec 39.1.2
- assume a spherical shock of radius R and it has a a power law dependence on energy of the explosion, E, time since explosion,t, and density of the medium into which it is running ρ.
- $R=AE^{\alpha}\rho^{\beta}t^{\eta}$
- with dimensional analysis (e.g. the powers to which mass lenght and time appear to get
- mass $\alpha+\beta=0$, length $1-2\alpha-3\beta=0$, $-2\alpha+\eta=0$ time

one solves this to get $\alpha = 1/5$, $\beta = -1/5$, $\eta = 2/5$

```
or R=AE<sup>1/5</sup>\rho^{-1/5} t^{2/5}
```

putting in the physics and numbers R=1.54x10¹⁹cm $E_{51}^{1/5}n^{-1/5} t_3^{2/5}$ (we have switched units, n is particle density, t_3 is in units of 10^3 years, E_{51} is in units of 10^{51} ergs. $v_s = 1950 \text{ km/s } E_{51}^{1/5} n^{-1/5} t_3^{-3/5}$ T=5.25x10⁷k E₅^{2/5}n^{-2/5} t₃^{-6/5}

Sedov-Taylor Solution

- $R \sim (E/\rho)^{2/5} t^{2/5}$
- $v \sim (2/5)(E/\rho)^{2/5} t^{-3/5}$
- Just behind the shock wave
 ρ₁=ρ₀ (γ+1/γ-1); γ is the adiabatic index

 $v_1 = (4/5)(1/\gamma + 1) (E/\rho_0)^{2/5} t^{-3/5}$

Pressure

P1=(8/25)(ρ_0/γ +1)(E/ ρ_0)^{2/5}t -6/5

Next 2 Phases

• Constant Temperature Expanding Phase

The expansion velocity decreases with time and, radiative cooling behind the shock front becomes important. When the radiative cooling time of the gas becomes shorter than the expansion time, the evolution deviates from the self similar one. In this phase, the SNR evolves, conserving momentum at a more or less constant temperature and the radius of the shell expands in proportion to the 1/4 power of the elapsed time since the explosion.

Radiative/Snow plough phase

T drops as a steep function of radius

===> at some point, T is below T_{recomb} ~ 1 keV- the cooling function increases steeply and the gas recombines rapidly; v_{shock} ~150km/sec

Age of SNR when this happens depends on models for cooling functions, explosion energy and density.

roughly t_{cool} ~nkT/n² Λ (T) -~4x10⁴yrT₆^{3/2}/n

 $(\Lambda(T)$ is the cooling function)

phase starts when t_{cool} < t_{Sedov} ; T_6 < $E^{1/7}n^{2/7}$

Between 17,000 and 25,000 years (assuming standard E_o and n_1) -

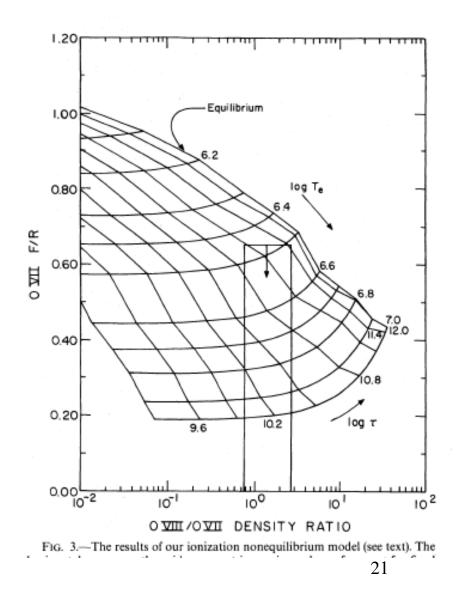
Then: THE END... SNR merges with surrounding medium

End of Snowplough Phase- Draine sec 39.1.4

- The strong shock gradually slows (radiative losses and accumulation of 'snowplowed' material)
- Shock compression declines until v_{shock}~c_s (sound speed); no more shock
- Using this criteria the 'fade away' time
- $t_{fade} \sim ((R_{rad}/t_{rad})/c_s)^{7/5} t_{rad}$
- $t_{fade} \sim 1.9 \times 10^6 \text{ yrs } E_{51}^{0.32} \text{ n}^{-0.37} (c_s/10 \text{ km/sec})^{-7/5} c_s = 0.3 \text{ km/sec} (T/10 \text{ k})^{1/2}$
- $R_{fade} \sim 0.06 kpc E_{51}^{0.32} n^{-0.37} (c_s/10 km/sec)^{-2/5}$

Plasma takes time to come into equilibrium

- particle ("Coulomb") collisions in the post-shock plasma will bring the temperature of all species, including the free electrons, to an equilibrium value:
- $kT = 3/16 \mu m_p v_s^2$
- However it takes time for the system to come into equilibrium and for a long time it is in nonequilibrium ionization (NEI) $\tau \sim n_e t \sim 3 \times 10^{12} \text{ cm}^{-3} \text{ s}$
- if the plasma has been shocked recently or is of low density it will not be in equilibrium



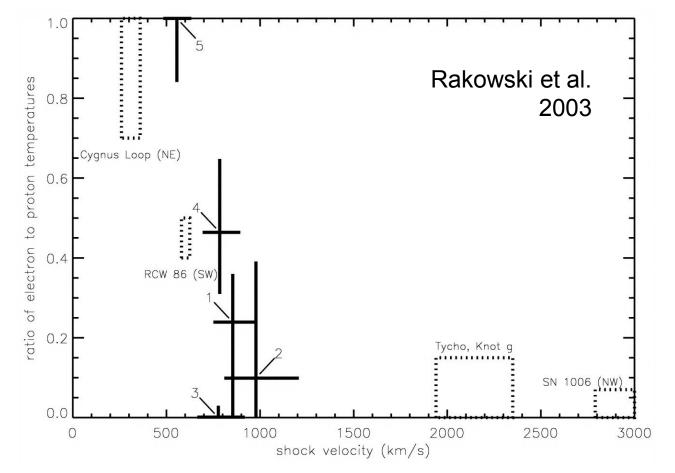
Temperatures behind shock are proportional to mass

 $kT_{i,e} \sim m_{i,e} v_{sh}^2$

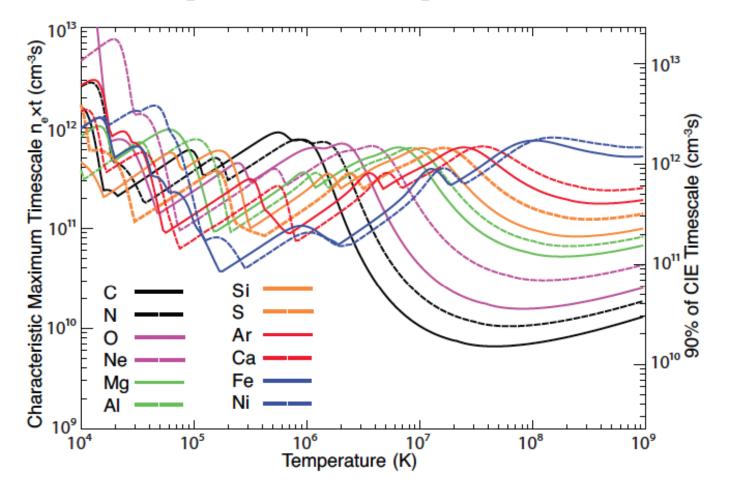
Electrons and ions will equilibrate their temperatures by Couloumb collisions, but possibly more quickly by complicated collisionless plasma processes

The efficiency of heating depends on the Mach number (shock velocity): faster electron heating in slower shocks

Electron Heating at SNR Shocks Compare T_e to T_p



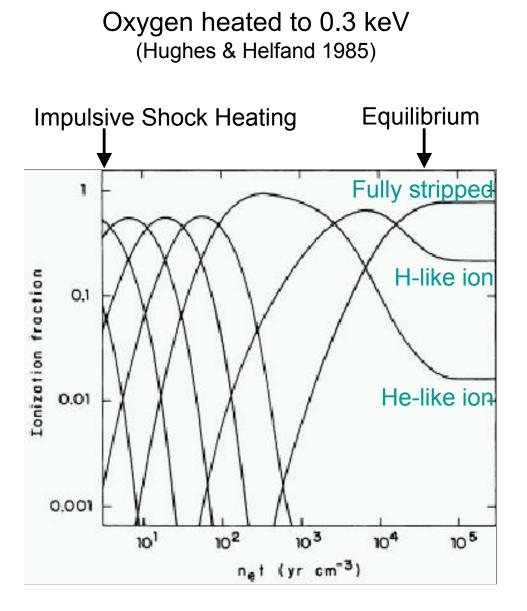
• Timescale to reach equilibrium depends on ion and temperaturesolution of coupled differential equations.



axis] Density-weighted timescales (in units of cm⁻³ s) for C, N, O, Ne, Mg, Al, S, Si, Ar, Ca, Fe, and Ni owards ionization equilibrium in a constant temperature plasma. [Right axis] Density-weighted timescale for eir equilibrium value.

Smith and Hughes 2010

Time-Dependent Ionization



lonization is effected by electron-ion collisions, which are relatively rare in the $\sim 1 \text{ cm}^{-3}$ densities of SNRs

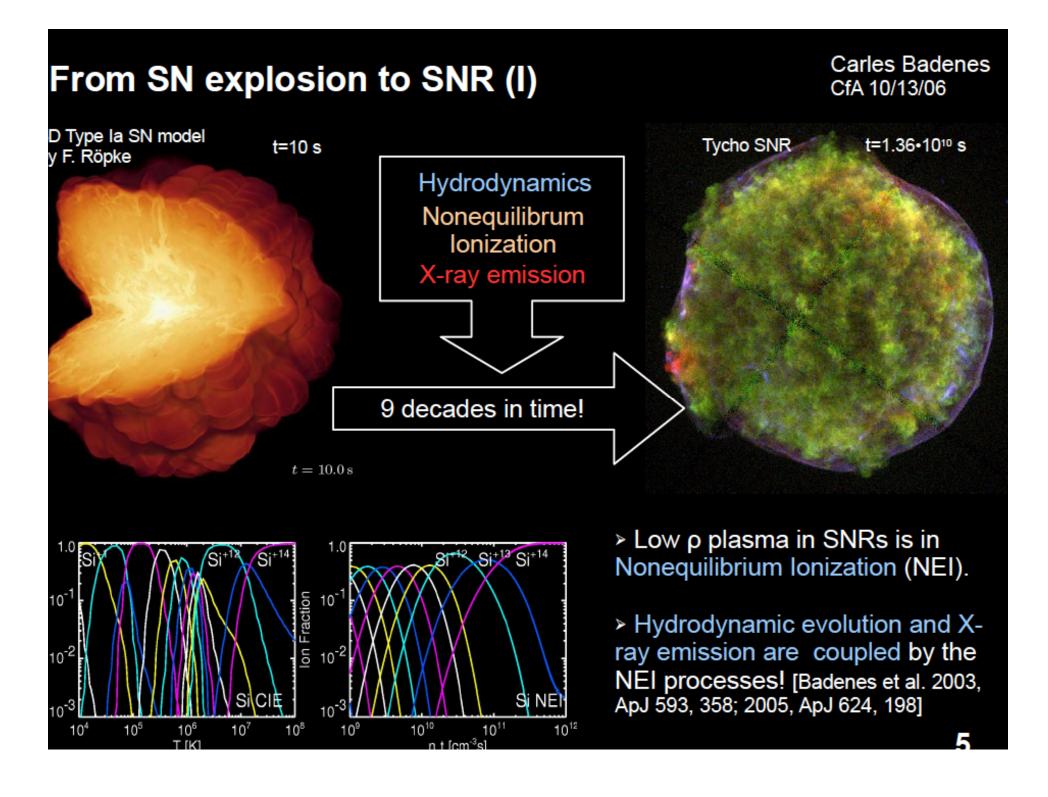
Ionization is time-dependent

lonization timescale = n_et electron density x time since impulsively heated by shock

lonization equilibrium attained at $n_e t \sim 10^4 \, cm^{\text{-3}} \, yr$

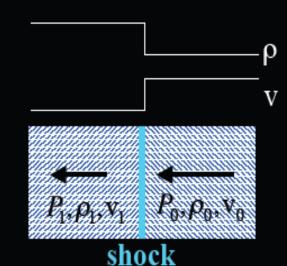
Ionizing gas can have many more H- and He- like ions, which then enhances the Xray line emission

Inferred element abundances will be too high if ionization equilibrium is inappropriately assumed for an ionizing gas 24



Shocks in SNRs

- Expanding blast wave moves supersonically through CSM/ISM; creates shock
 - mass, momentum, and energy conservation across shock give (with γ=5/3)



$$\rho_1 = \frac{\gamma + 1}{\gamma - 1}\rho_0 = 4\rho_0$$

$$v_1 = \frac{\gamma - 1}{\gamma + 1} v_0 = \frac{v_0}{4}$$
$$v_{ps} = \frac{3v_s}{4}$$

$$T_{1} = \frac{2(\gamma - 1)}{(\gamma + 1)^{2}} \frac{\mu}{k} m_{\rm H} v_{0}^{2} = 1.3 \times 10^{7} v_{1000}^{2} \text{ K}$$

X-ray emitting temperatures

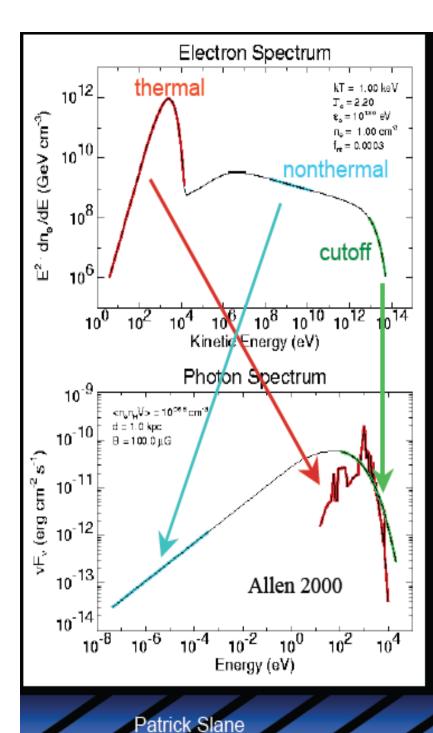
Shock velocity gives temperature of gas

V

- note effects of electron-ion equilibration timescales
- If another form of pressure support is present (e.g. cosmic rays), <u>the</u> <u>temperature will be lower than this</u>

Patrick Slane

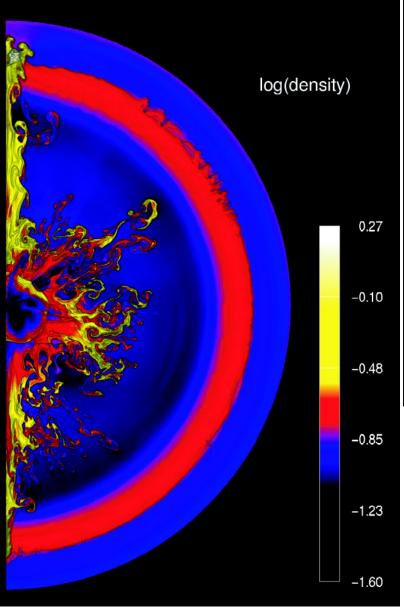
Harvard-Smithsonian Center for Astrophysics



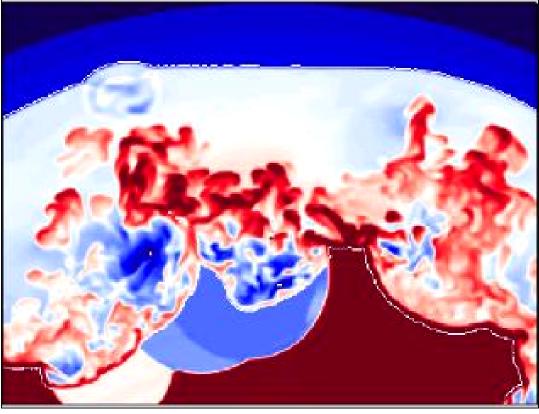
Shocked Electrons and their Spectra

- Forward shock sweeps up ISM; reverse shock heats ejecta
- Thermal electrons produce line-dominated x-ray spectrum with bremsstrahlung continuum
 - yields kT, ionization state, abundances
- nonthermal electrons produce synchrotron radiation over broad energy range
 responsible for radio emission
- high energy tail of nonthermal electrons yields x-ray synchrotron radiation
 - rollover between radio and x-ray spectra gives exponential cutoff of electron spectrum, and a limit to the energy of the associated cosmic rays
 - large contribution from this component modifies dynamics of thermal electrons

Harvard-Smithsonian Center for Astrophysics



Kifonidis et al. 2000



Fe bubbles Blondin et al. 2001

Instabilities

irregular shock boundaries mixing between ejecta layers mixing between ejecta and ISM

- Remember Project
- Due the week before the last day of classes.. April 30

• Evalutions:

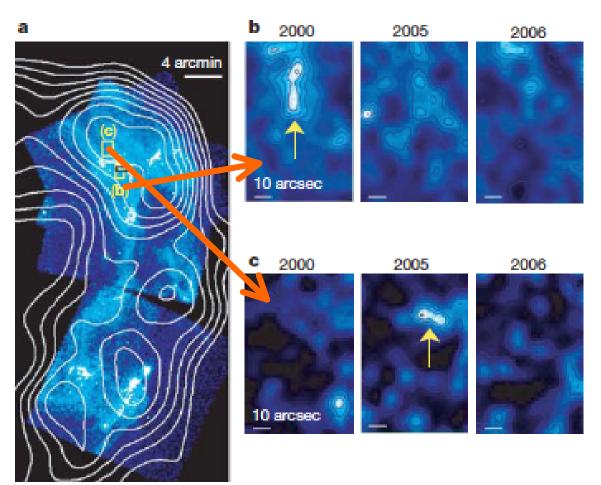
CourseEvalUM will be open for student evaluations for this semester

- Apr. 23 through May 10 https://courseevalum.umd.edu/
- The evaluations are important to me, the department and the university: please take a little bit of time and fill them out!



SNR are Thought to Be the Source of Galactic cosmic

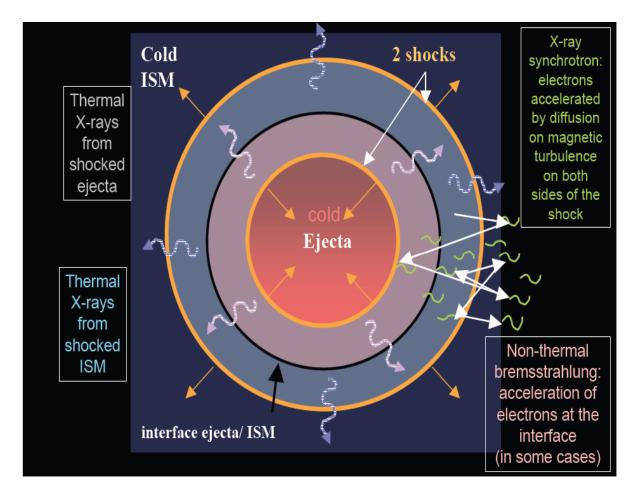
- rays
- They need to put ~ 5-20% of their energy into cosmic rays in order to explain the cosmic-ray energy density in the Galaxy (~2 eV/cm³ or $3x10^{38}$ erg/s/kpc²), the supernova rate (1-2/100yrs), the energy density in SN (1.5x10⁴¹ ergs/sec~2x10³⁹ erg/s/kpc²)
- particles are scattered across the shock fronts of a SNR, gaining energy at each crossing (Fermi acceleration)
- Particles can travel the Larmor radius
- $R_L \sim E_{17} / B_{10\mu G} Z kpc$



many young SNRs are actively accelerating electrons up to 10-100TeV, based on modeling their synchrotron radiation

- Fermi acceleration-1949:
- charged particles being reflected by the moving interstellar magnetic field and either gaining or losing energy, depending on whether the "magnetic mirror" is approaching or receding. energy gain per shock crossing is proportional to velocity of shock/speed of light spectrum is a power law

See Melia sec 4.3

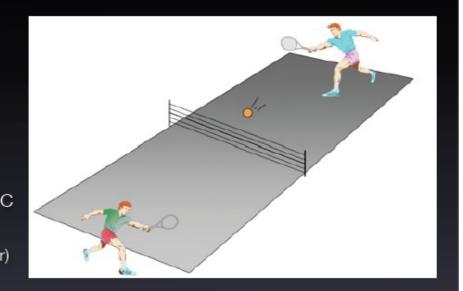


DeCourchelle 2007

Nice analogy- ping pong ball bouncing between descending paddle and table

Particle Acceleration sec 4.4.2 in R+B Spitovsky 2008

Particle acceleration: $u \quad u \quad r$ $\Delta E/E \sim V_{shock}/C$ $B \quad (E) \sim N_0 E^{-K(r)}$



Free energy: converging flows

Acceleration mechanisms:

First order Fermi
 Diffusive shock acceleration
 Shock drift acceleration
 Shock surfing acceleration

Second order Fermi

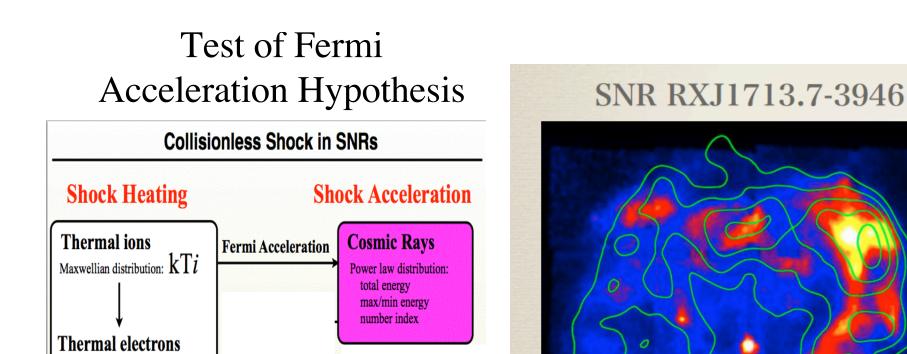
Efficient scattering of particles is required. Monte Carlo simulations of rel. shocks show that this implies very high level of turbulence $\delta B/B$ (Ostrowski et al). Is this realistic? Are there specific conditions?

Requires turbulence for injection into acceleration process and to stay near the shock

Needs spectrum of turbulent motions (waves) downstream.

Fermi Acceleration

- 2nd Order energy gained during the motion of a charged particle in the presence of randomly moving "magnetic mirrors". So, if the magnetic mirror is moving towards the particle, the particle will end up with increased energy upon reflection.
- energy gained by particle depends on the mirror velocity squared. - also produces a power law spectrum
- First order: energy gained is proportional to shock velocity

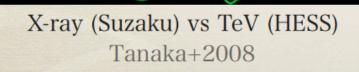


•Uchiyama

Maxwellian distribution: kTe

kTi > kTe

in young SNRs

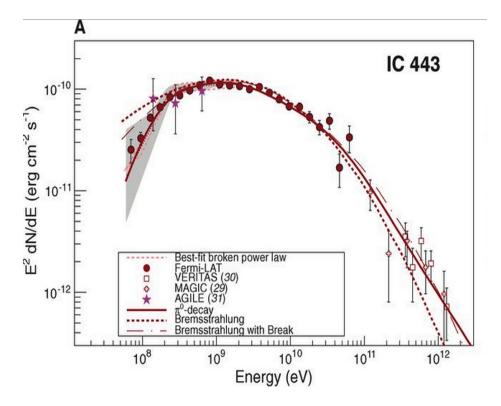


• Shock waves have moving magnetic inhomogeneities - Consider a charged particle traveling through the shock wave (from upstream to downstream). If it encounters a moving change in the magnetic field, it can reflect it back through the shock (downstream to upstream) at increased velocity. If a similar process occurs upstream, the particle will again gain energy. These multiple reflections greatly increase its energy. The resulting energy spectrum of many particles undergoing this process turns out to be a power law:

How Does the Fermi γ-ray Signal 'Prove' CRs are Accelerated ?

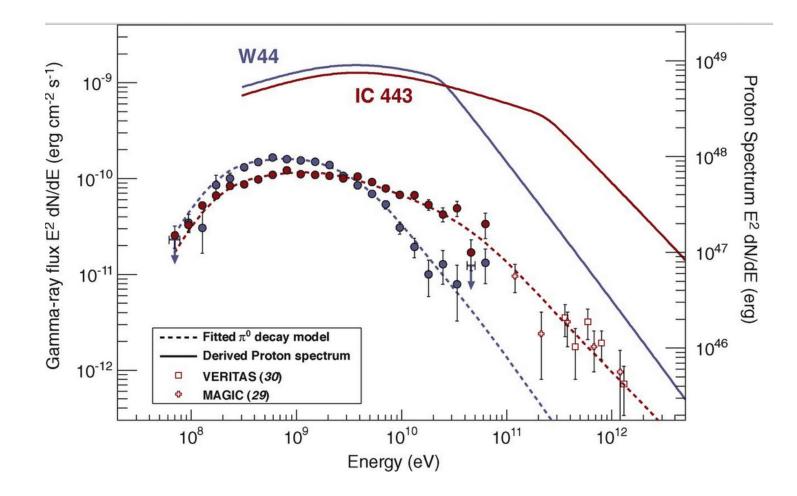
- γ-rays can originate in SNR in 3 separate ways
 - Inverse Compton scattering of relativistic particles
 - Non-thermal bremmstrahlung
 - Decay of neutral pions into 2 γ-rays
- the first 2 have broad band
 ~power law shapes
- pion decay has a characteristic energy E_{γ} =67.5 MeV- need to convolve with energy distribution of CR protons

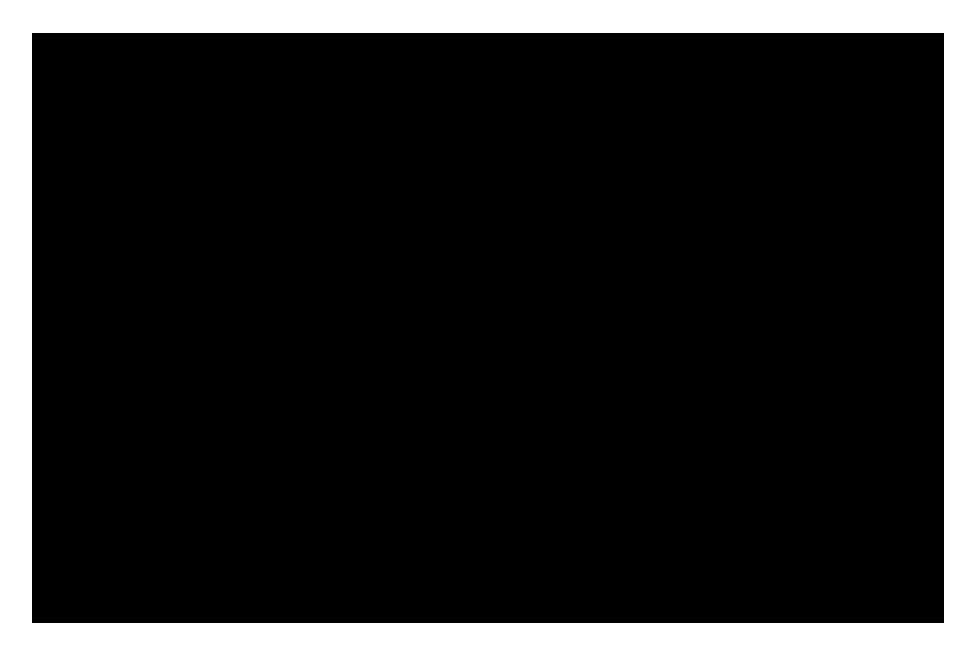
When cosmic-ray protons accelerated by SNRs penetrate into high-density clouds, π_0 decay γ -ray emission is expected to be enhanced because of more frequent pp interactions (p+p ρ p+p+ π_0 2p+2 γ



Fit of Fermi y-ray data to Pion model

- One of the fitted parameters is the proton spectrum need to product the g-ray spectrum via pion decay.
- This indicates that the proton spectrum is not a pure power law but has a break (change in slope) at high energies

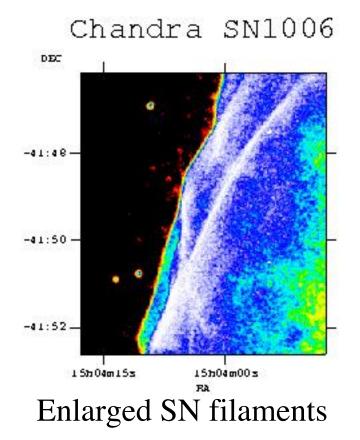


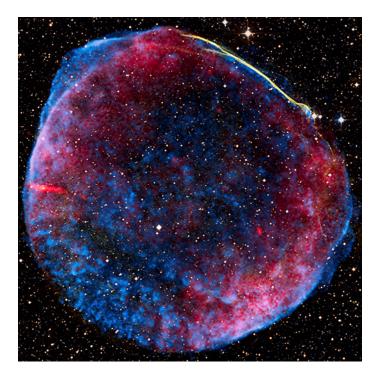


- an incoming proton with 135 MeV of kinetic energy cannot create a neutral pion in a collision with a stationary proton because the incoming proton also has momentum, and the collision conserves momentum, so some of the particles after the collision must have momentum and hence kinetic energy.
- Assume initially two protons are moving towards each other with equal and opposite velocities, thus there is no total momentum. in this frame the least possible K.E. must be just enough to create the π_0 with all the final state particles (p,p, π_0) at rest. Thus if the relativistic mass of the incoming protons in the center of mass frame is m, the total energy $E=2m_pc^2+m_{\pi 0}c^2$ and using total energy $=m_p/sqrt(1-v^2/c^2)$
- energy of proton is 931 meV gives v/c=0.36c ; use relativistic velocity addition to get total velocity or a needed 280Mev of additional energy-- threshold for π_0 production

Sn1006

• The first SN where synchrotron radiation from a 'thermal' remnant was detected- direct evidence for very high energy particles

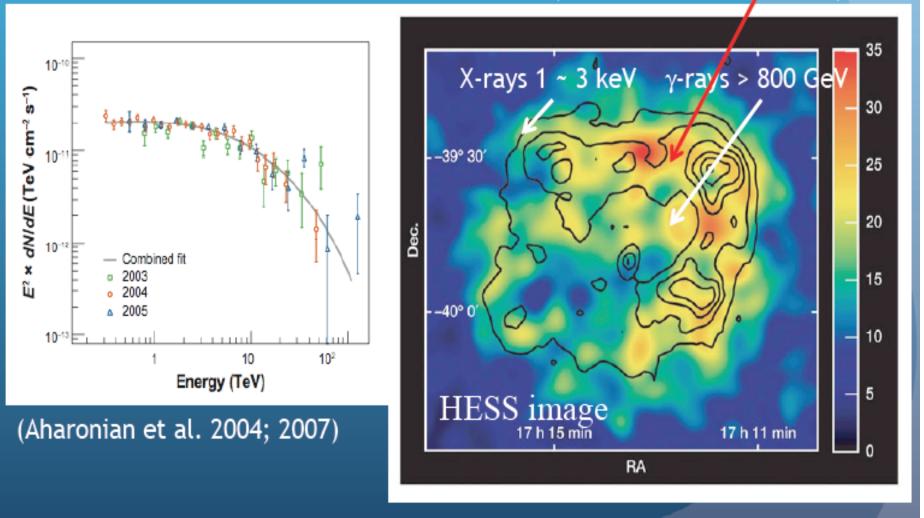




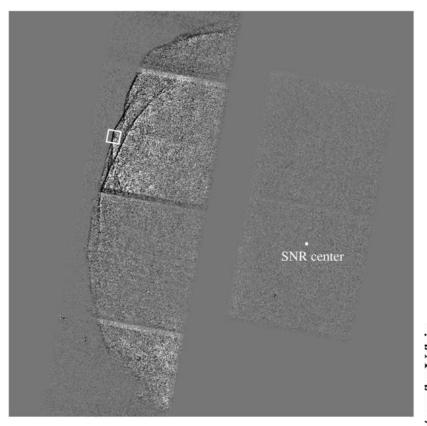
direct evidence is the energy of the photons emitted (~TeV)+ the needed particle energies to produce synchrotron x-rays $v_{synch} \sim 16 \text{keV}(\text{BE}_{\text{Tev}}^2) \text{Hz}$ loss time of the particles $t_{synch} \sim 400 \text{s} \text{ B}^{-2} \text{ E}_{\text{tev}}^{-1}$ for field of 100µG one gets $\text{E} \sim 100 \text{Tev}, t_{synch} \sim 15 \text{ years-- so need}$ continual reacceleration

Evidence for Particle Acceleration- Tev Emission + X-ray Synch

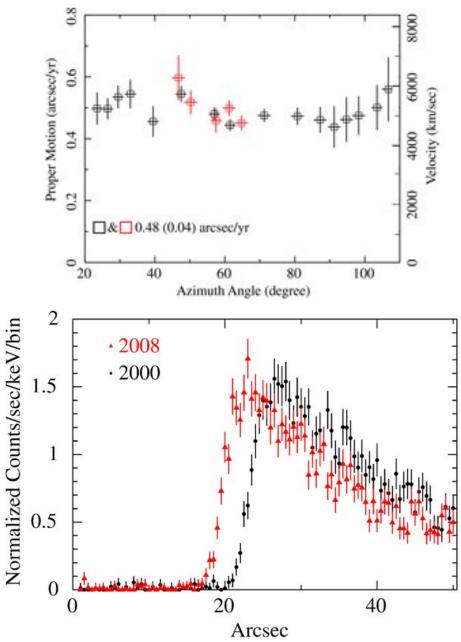




SN1006



Difference Image



Type I SN and Cosmology

- One of the main problems in astrophysics is understanding the origin and evolution of the universe: how old is the universe, how fast is it expanding, how much material and of what type is in it, what is its fate?
- A major step in this is to determine the relationship between distance and redshift
- In General relativity there are 3 distances of relevance
 - The proper distance D_p that we measure to an object is the distance we would get if we were to take a snapshot of the universe and directly measure the distance between where we are and where the object is, at some fixed time
 - The luminosity distance D_L is how far an object of known luminosity L (measured in energy per time) would have to be in Euclidean space so that we measure a total flux F (measured in energy per area per time), $D_L = \text{sqrt}(L/(4\pi F))$.
 - The angular diameter distance D_A is the distance an object of known size \angle would have to be in Euclidean space so that it appeared to be its measured angular size θ ; $D_A = \angle/\theta$.

More Cosmology

- Each of these distances depends on cosmological parameters * in a different way
 - * in classical cosmology -the Hubble constant (H_0)
 - The density of the universe in baryons Ω_B and total matter Ω_M
 - And 'cosmological constant' Λ or alternatively the equation of state parameter w=P/ ρ ; where P and ρ are the pressure and energy density of the universe (in GR the scale factor a and d²a/d²t {(da/dt)/a}²={8 π G ρ /3-k/a²}
- Back to type Ia SN-
 - It turns out (when I say that it means a huge amount of work by many people over many years) that type Ia SN are 'standardizable candles'- one can use their brightness, color and speed of decay to determine an 'absolute' luminosity to ~10% accuracy.
 - With a measured redshift and absolute luminosity one can get the luminosity distance

- Use of SN Ia as 'standardizable candles'
- What we want to know is the absolute distance to the source (luminosity distance)

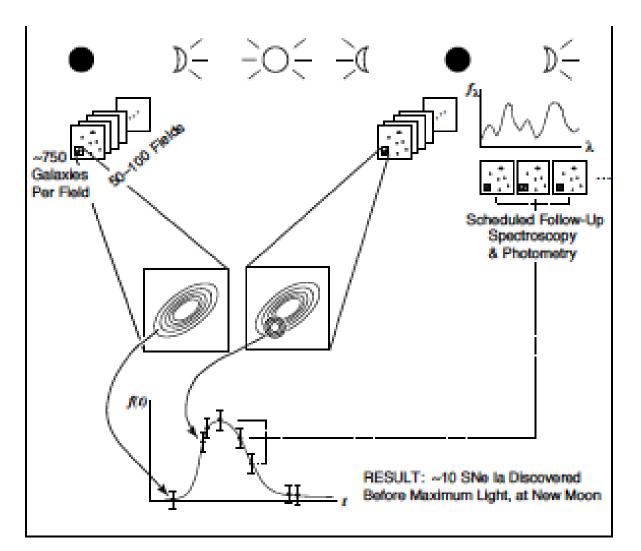
m(z)=M+5 log d_L(z,H₀, Ω_m , Ω_Λ)+25

 $d_{L}(z,H_{0,}\Omega_{m},\Omega_{\Lambda}) = \{c(1+z)/H_{0} \operatorname{sqrt}(\boldsymbol{k})\} \operatorname{sqrt}(\boldsymbol{k}) \int [(1+z')^{2}(1+\Omega_{m}z') - z'(2+z')\Omega_{\Lambda}]^{-1/2} dz'\}$

$$\boldsymbol{k=1-}\boldsymbol{\Omega}_{\mathrm{m}}\boldsymbol{\boldsymbol{-}}\boldsymbol{\Omega}_{\mathrm{\Lambda};}$$

the luminosity distance depends on z, Ω_m , Ω_Λ and H_0 and in principle seeing how it changes with redshift allows one to constrain these parameters

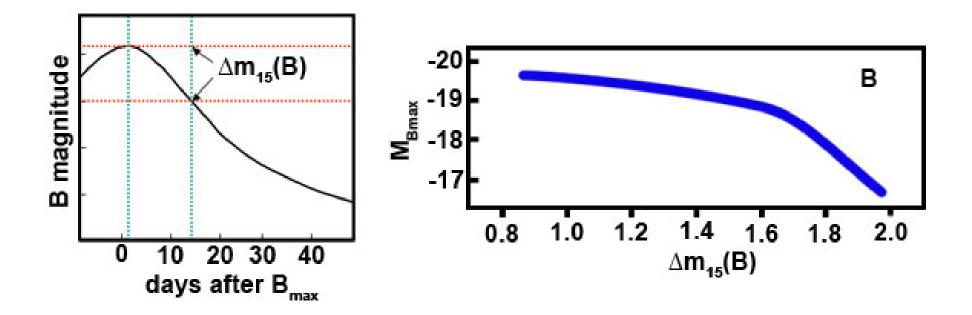
- Supernova on demand
 - we know the average SNIa rate per galaxy/yr (1/100 yrs for a L(*) galaxy)
 - To obtain ~10 SN
 Ia per 1 week of
 observing have to
 observe ~50,000
 galaxies about ~2
 weeks apart and
 see what has
 changed



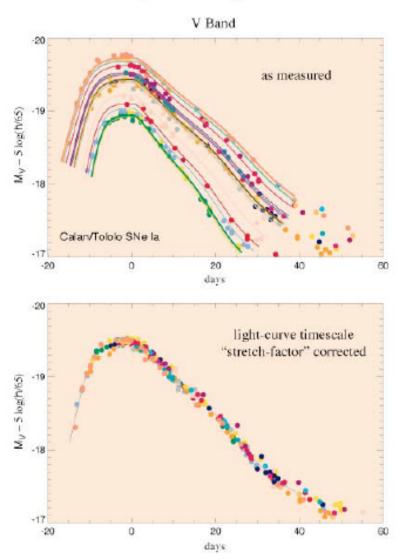
Pperlmuter et al 1997

Several Correlations Allow A Standard Candle to be Created

• Phillips et al 1993 notce that the change in brightness of the SN Ia light curve at a fixed timescale was related to the absolute brightness of the SN



Low Redshift Type Ia Template Lightcurves



The Phillips Relation (post 1993)

Broader = Brighter

Can be used to compensate for the variation in observed SN Ia light curves to give a "calibrated standard candle".

Note that this makes the supernova luminosity at peak a function of a single parameter – e.g., the width.

Woosely 2010

Standardizing Them

- There are a variety of SNIa light curves and brightness-
- However luminosity correlates strongly with light curve shape

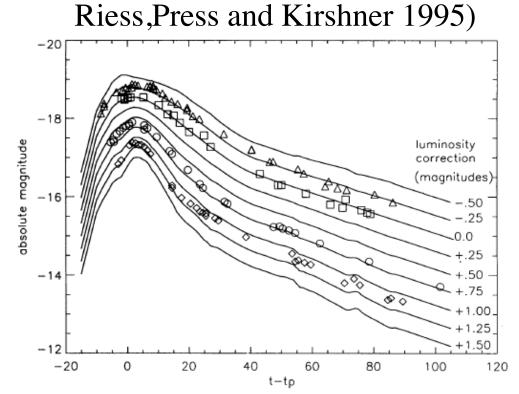
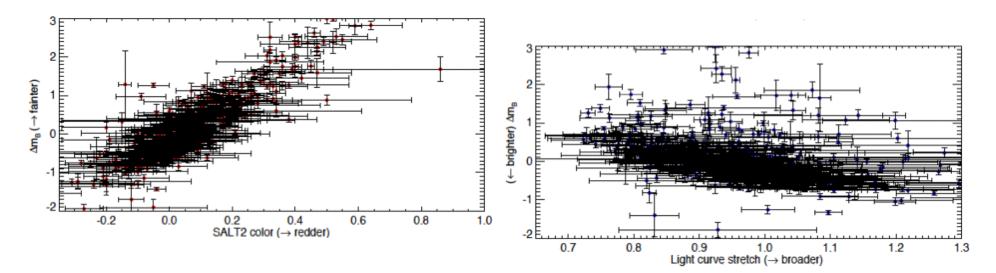
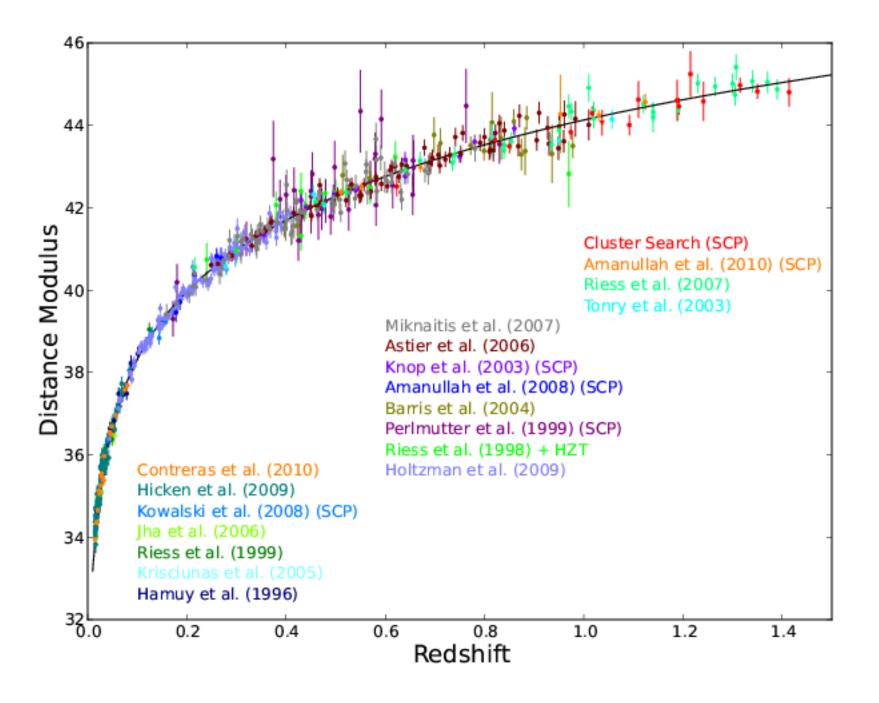
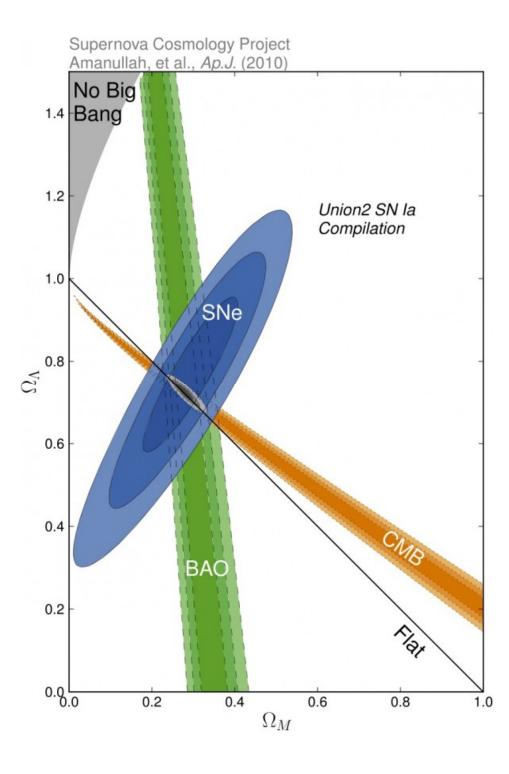


FIG. 1.—Empirical family of visual band SN Ia light curves. This sample of

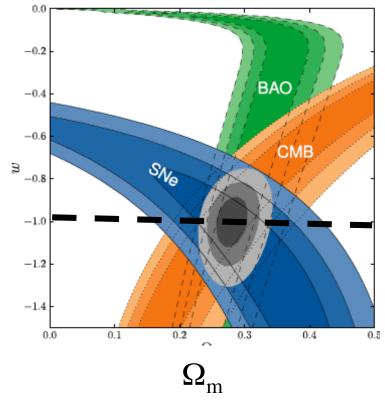




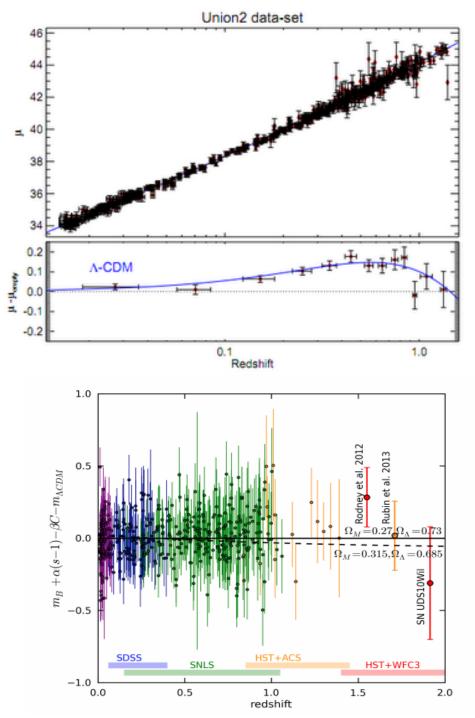


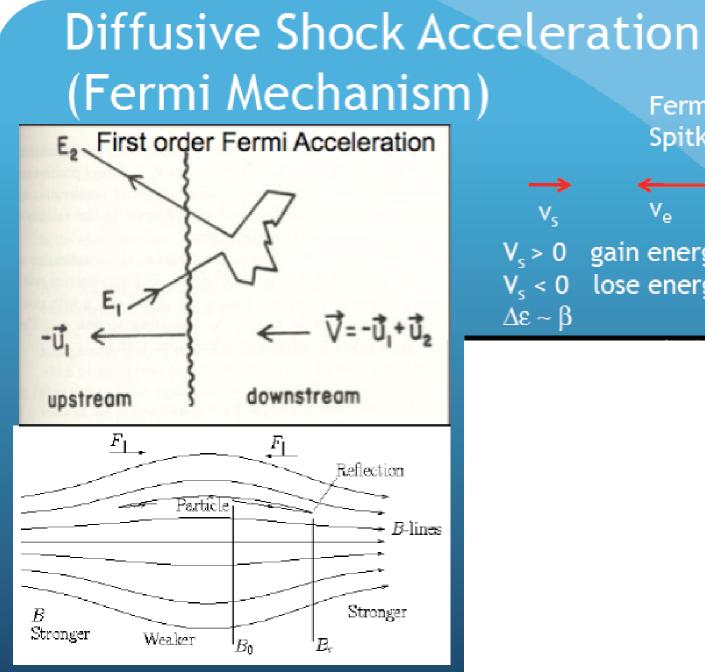
It Works Pretty Well

The formal errors in the cosmological parameters for this method only are a good fit to Λ-CDM (cold dark matter)



w=-1 ; a cosmological constant





Fermi 1949; Spitkovsky 2008; Ve ٧c $V_{s} > 0$ gain energy $V_{s} < 0$ lose energy $\Delta \epsilon \sim \beta$

Janfei Jang