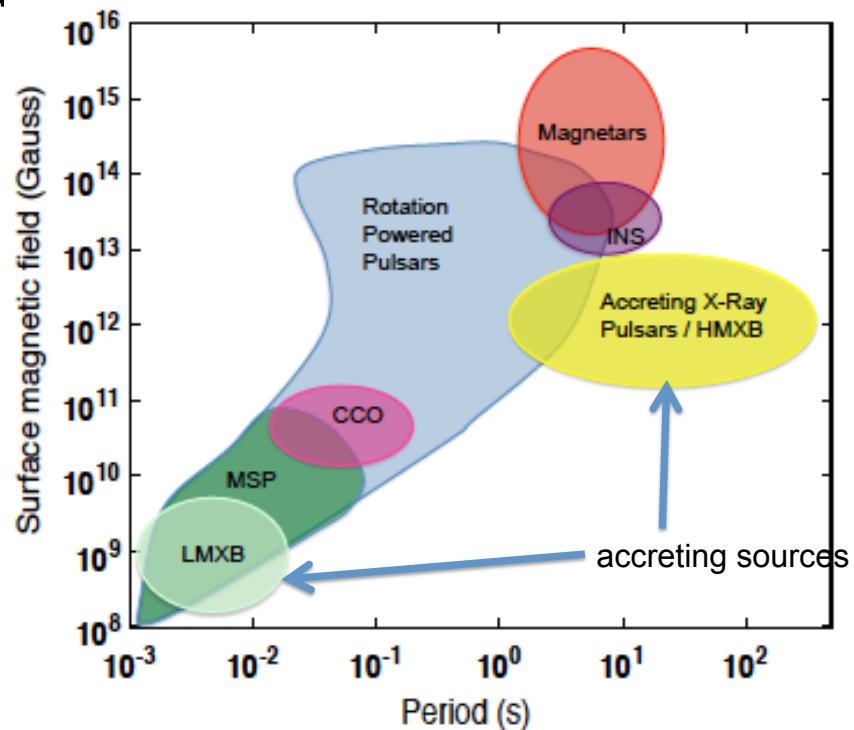


Next Paper(s)

- Modelling the behaviour of accretion flows in X-ray binaries Everything you always wanted to know about accretion but were afraid to ask Done, Gierlinski & Kubota [2007A&ARv..15....1D](#)
- Sec 1 and 2 ONLY-OR Sec 7 only this is a very long article!
- -----
- 2014MNRAS.437.1698 X-ray emission from star-forming galaxies - III. Calibration of the LX-SFR relation up to redshift $z \approx 1.3$ Mineo, S.; Gilfanov, M.; Lehmer, B. D

Period and Magnetic Field

A. Harding 2013



Population

- The most 'common' observational population are non-accreting pulsars
- Periods from .. 0.001-100 secs
- 22 orders of magnitude range in dP/dt
- dipole magnetic $B_s \sim 10^{19}(P/dP/dt)^{1/2}$ where P is in seconds, B in gauss

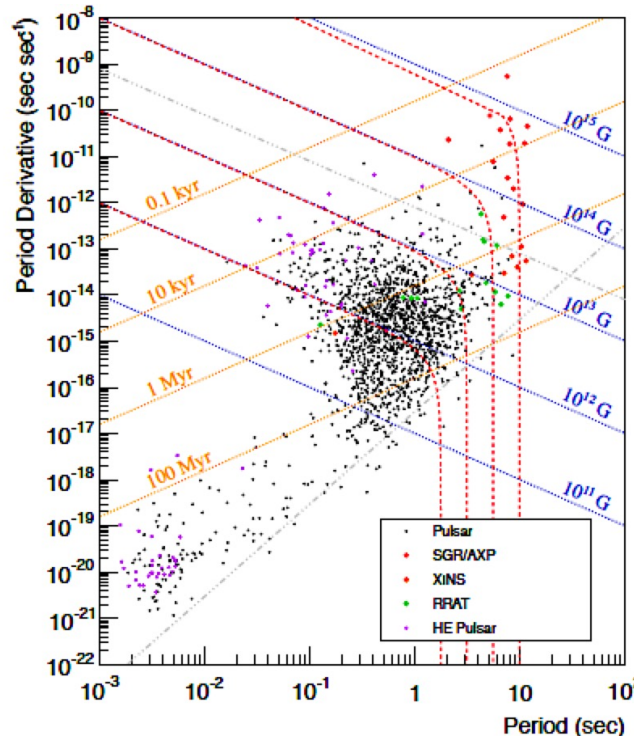
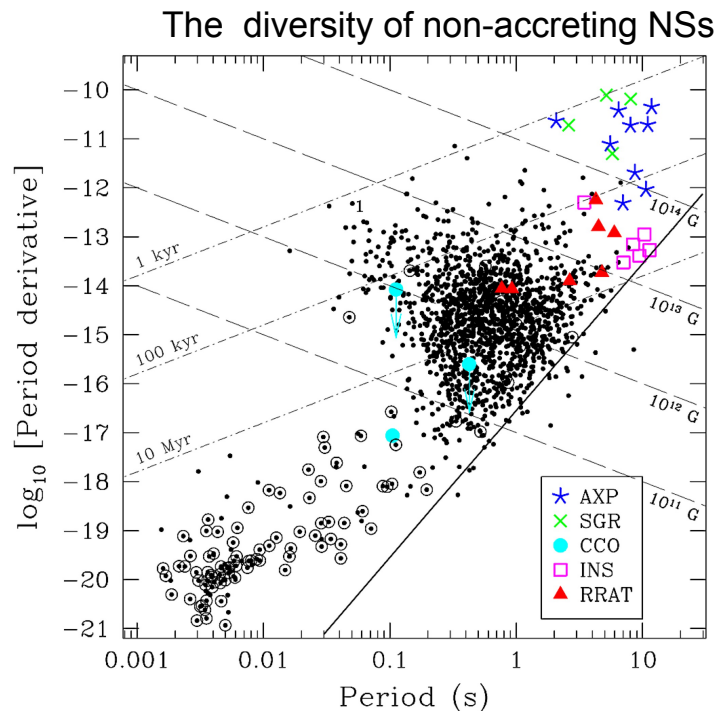


Figure 1: The $P-\dot{P}$ diagram of pulsars shown with lines of constant dipole magnetic field, B , and spin-down age ($\tau_c = \frac{P}{2\dot{P}}$). The black dots show the majority of radio-discovered pulsars believed to be rotation-powered, and the red circles show the X-ray or gamma-ray discovered magnetars addressed in this white paper (see §1.1). Source: ATNF pulsar catalog

- 'Millisecond pulsars' are rotation-powered, but have different evolutionary histories, involving long-lived binary systems and a 'recycling accretion episode which spun-up the neutron star and quenched its magnetic field
- We will not discuss
 - X ray-Dim Isolated NSs (XDINSs), Central Compact Object (CCOs) Rotating Radio Transients (RRATs), AXPS and Magnetars...



Open circles are in binaries

Degenerate Compact Objects- See Longair pg 394-398

- The determination of the internal structures of white dwarfs and neutron stars depends upon detailed knowledge of the equation of state of the degenerate electron and neutron gases
- In these objects degeneracy pressure is main force balancing gravity
- pressure is independent of the temperature for degenerate stars, only need the first two equations of stellar structure (2.6) to carry out the analysis,
- $dp/dr = -GM\rho/r^2$; $dM/dr = 4\pi\rho r^2$.
- In eqs 13.16-13.24 the eqs for a white dwarf are derived
- $M = 5.836/\mu_e^2 M_\odot$. $\mu_e = 2$ for white dwarf and thus the **Chandrasekar mass for a white dwarf $M_{ch} = 1.46M_\odot$** (eq. 13.24)
- For NS general relativity is important..

White Dwarfs... Courtesy of C. Reynolds

- Size, and pressure

$$R \sim \frac{K}{GM^{1/3}}$$

$$P_c = K\rho_c^{5/3} \quad P = \frac{3^{2/3}\pi^{4/3}\hbar^2}{5m_e\mu^{5/3}m_p^{5/3}}\rho^{5/3}$$

*Mass of particle
producing
degeneracy
pressure*

*Number of nucleons
per degenerate
particle*

- So, an approximate expression for radius of white dwarf is:

$$R \sim \frac{K}{GM^{1/3}}$$

$$R \sim 1.2 \times 10^4 \left(\frac{M}{M_{\odot}} \right)^{-1/3} \mu^{-5/3} \text{ km}$$

- Exact calculation gives

$$R \sim 1.13 \times 10^4 \left(\frac{M}{M_{\odot}} \right)^{-1/3} \left(\frac{\mu}{2} \right)^{-5/3} \text{ km}$$

Degneracy and All That- Longair pg 395 sec **13.2.1-2.**

- In *white dwarfs*, internal pressure support is provided by electron degeneracy pressure and their masses are roughly $<1M_{\odot}$
- the density at which degeneracy occurs in the non-relativistic limit is proportional to $T^{3/2}$
- This is a quantum effect: Heisenberg uncertainty says that $\delta p \delta x > h/2\pi$
 - Thus when things are squeezed together and δx gets smaller the **momentum, p**, increases, particles move faster and thus have more pressure
- Consider a box- with a number density, n , of particles are hitting the wall; the number of particles hitting the wall per unit time and area is $1/2nv$ (v is velocity)
 - the momentum per unit time and unit area (**Pressure**) transferred to the wall is $2nvp$; $\mathbf{P} \sim nv p = (n/m) p^2$ (m is mass of particle)

In other words

Heisenberg uncertainty principle, $\Delta x \Delta p \geq \hbar$

where Δx is the uncertainty of the position measurements, Δp is the uncertainty of the momentum measurements, and \hbar is $h/2\pi$.

- As pressure increases, system will be more compact and, for electrons within it, their delocalization, Δx , will *decrease*.
- Thus, the uncertainty in the momenta of the electrons, Δp , will *grow*.
no matter how low the temperature drops, the electrons must be travelling at larger velocities, contributing to the pressure
- When this term exceeds that of the pressure from the thermal motions of the electrons, the electrons are referred to as degenerate, and the material is termed degenerate matter.
 - This analysis is true even at almost zero temperature.

Degeneracy- continued

- The average distance between particles is the cube root of the number density and if the momentum is calculated from the Uncertainty principle $p \sim h/(2\pi\delta x) \sim \hbar n^{1/3}$
- and thus $P = h^2 n^{5/3} / m$ - if we define matter density as $\rho = [n/m]$ then
- $P \sim \rho^{5/3}$ **independent of temperature** ρ is mass density
- Dimensional analysis gives the central pressure as $P \sim GM^2/r^4$
- If we equate these we get $r \sim M^{-1/3}$ e.g **a degenerate star gets smaller as it gets more massive**

- At higher densities the material gets 'relativistic' e.g. the velocities from the uncertainty relation get close to the speed of light- this changes things and $P \sim \rho^{4/3}$;
- this is important because when we use $P \sim GM^2/r^4$ we find that the pressure does not depend on radius and just get an expression that depends on mass- this is the Chandrasekar mass. (see 13.2.2 in Longair)

When Does Relativity Become Important

- Particles are moving at relativistic speeds when density satisfies:

$$n_e > \frac{8\pi}{3} \left(\frac{m_e c}{h} \right)^3 \quad \rho > 2 \times 10^{12} \text{ kg m}^{-3}$$

Maximum Mass of a Compact Object- Longair 13.2.2

- The Chandrasekar limit (maximum mass of a white dwarf) is when it costs less energy for a electron to fuse with a proton to form a neutron then to climb higher in the Fermi sea.

– Above this limit the compact object becomes all neutrons (a neutron star)

An alternative way of looking at this is to calculate the equation of state (EOS) of degenerate matter and use hydrostatic equilibrium.

– $P_e = (1/20)(3/\pi)^{2/3}(h^2/m_e) (\rho/\mu_e m_p)^{5/3}$ ---- ρ is the total mass density and $\mu_e m_p$ is the mass per electron (composition of the material) - or more simply

– $P_e \sim \rho^{5/3}$ - non relativistic

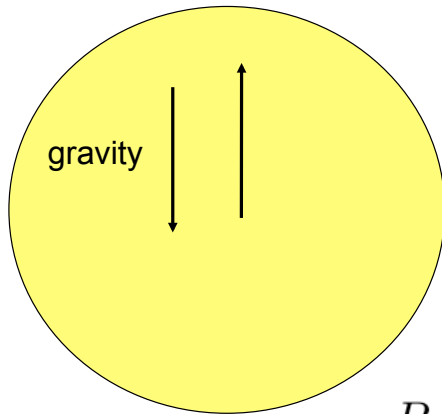
- for relativistic matter $P_e = (1/8)(3/\pi)^{1/3} c h (\rho/\mu_e m_p)^{4/3}$ - **notice the appearance of the speed of light**

– $P_e \sim \rho^{4/3}$

- in hydrostatic equilibrium (remember $dP(r)/dr = GM(r)\rho(r)/r^2$; $P \sim GM^2/R^4$)
- Setting the 2 pressures equal produces the Chandrasekhar limit at which a white dwarf collapses to a neutron star $M \sim 1.46 M_\odot$ (but depends on its composition μ_e , eg an iron core??)

Chandrasekar Limit eqs 13.15-13.25

- Electron degeneracy is responsible for balancing gravity in White Dwarfs



Pressure gradient = weight

$$\frac{dP}{dR} = -\frac{GM}{R^2}\rho$$

$$\frac{P_c}{R} \sim \frac{GM}{R^2}\rho_c$$

$$P_c = K\rho_c^{5/3} \quad \rho_c \sim M/R^3$$

Neutron Star Size Courtesy of C. Reynolds

- So, we can try to estimate radius of neutron star given what we know about white dwarfs

$$\frac{R_n}{R_{wd}} \sim \frac{m_e}{m_n} 2^{5/3}$$

– We know that

– So we expect

$$R_{wd} \sim 10^4 \text{ km}$$

$$R_n \sim 16 \text{ km}$$

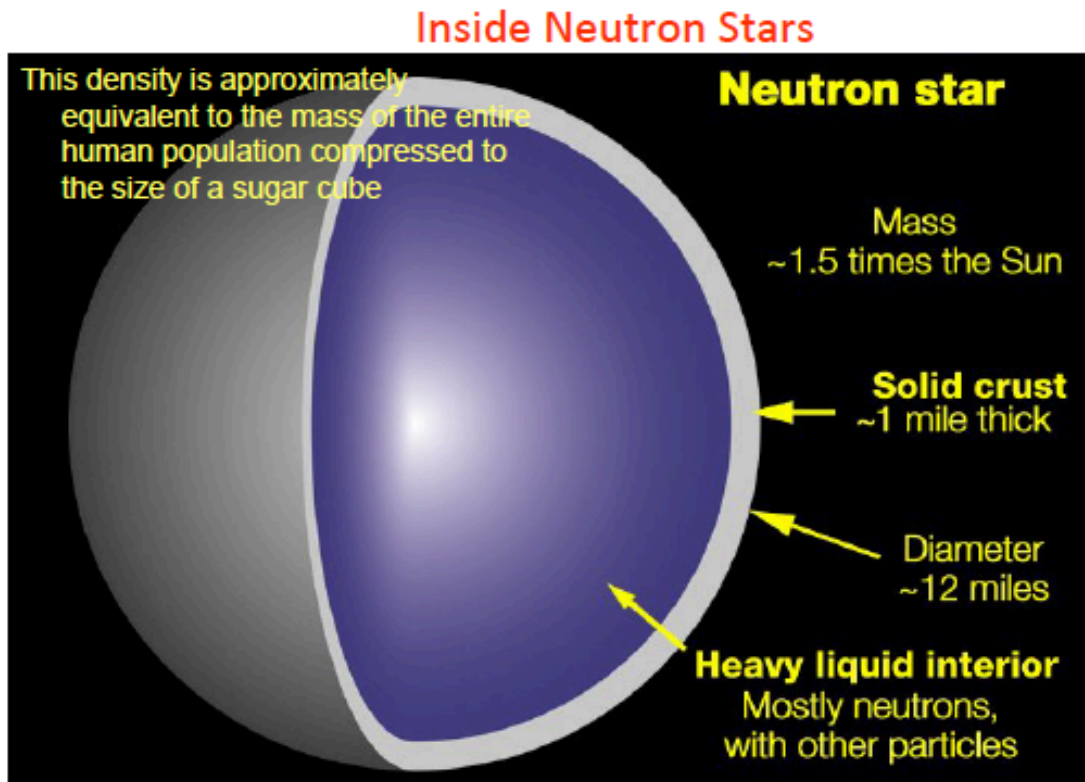
- By analogy to white dwarfs, neutron stars have (to a crude approximation)..

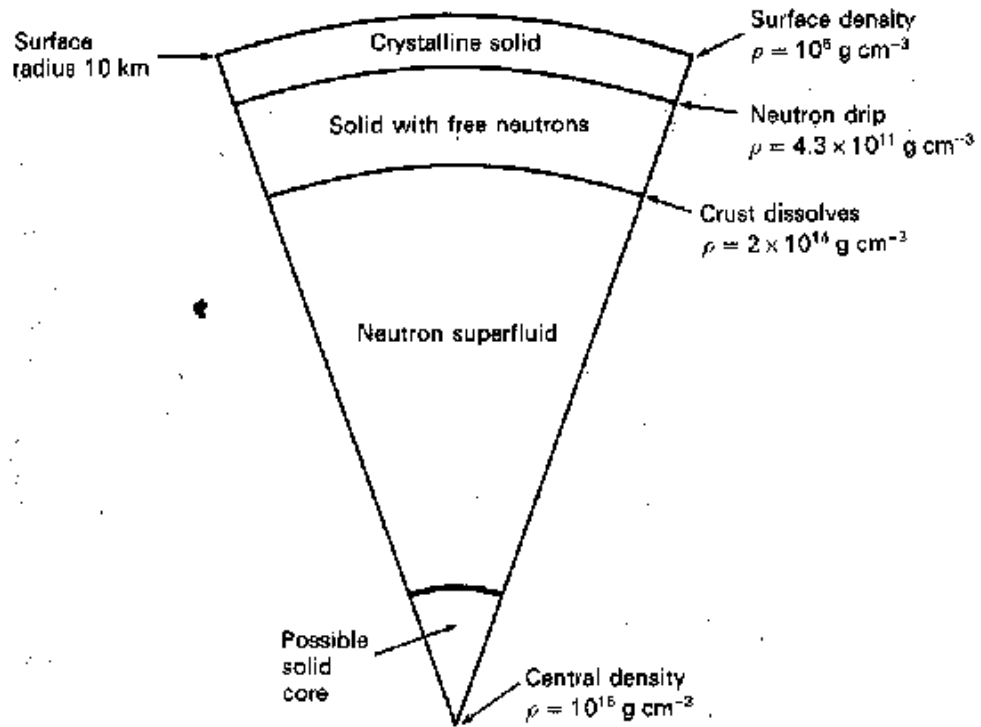
$$R_n \sim \frac{K_n}{GM^{1/3}}$$

- Where...

$$P_n = K_n \rho^{5/3} \qquad P_n = \frac{3^{2/3} \pi^{4/3} \hbar^2}{5m_n^{8/3}} \rho^{5/3}$$

- I.e., degenerate particles have mass m_n , and $\mu=1$





see fig 13.13 in Longair

Lots more going on... a lot is uncertain
Lattimer and Prakash 2004

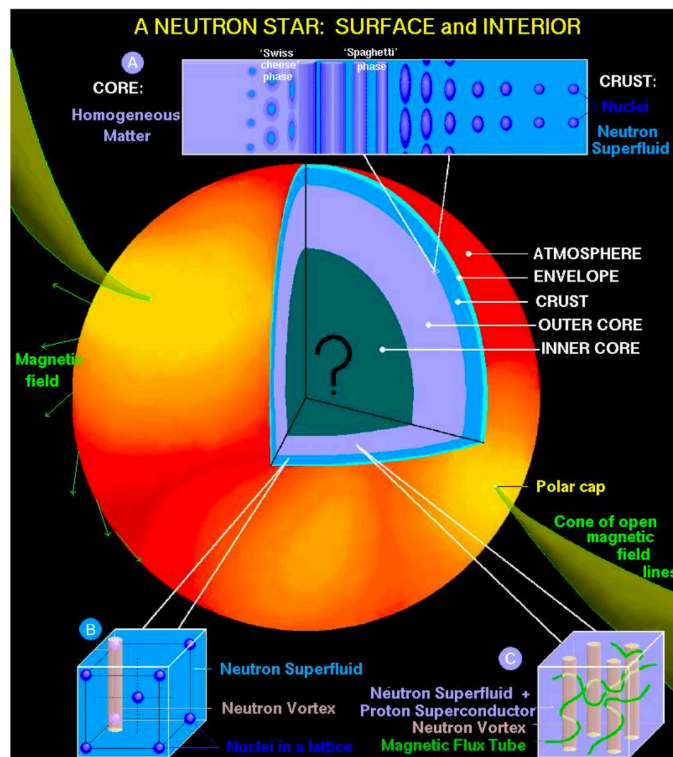


FIG. 3: The major regions and possible composition inside a normal matter neutron star. The top part illustrates expected geometric transitions from homogenous matter at high densities in the core

Radius of NS

- Use the 'known' density of nuclear matter ($\rho_{\text{Neutron}} \sim 1.2 \times 10^{14} \text{g/cm}^3$) and the Chandrasekar mass gives a radius
 - $R_{\text{NS}} \sim (3M_{\text{Chandra}}/4\pi\rho_{\text{Neutron}})^{1/3} \sim 10 \text{km}$
- consistency between the observed spin periods, and neutron stars

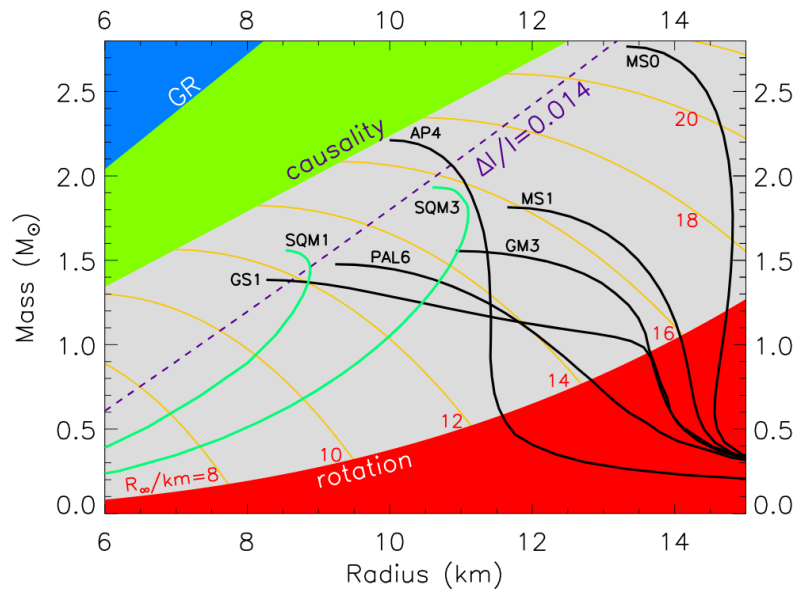
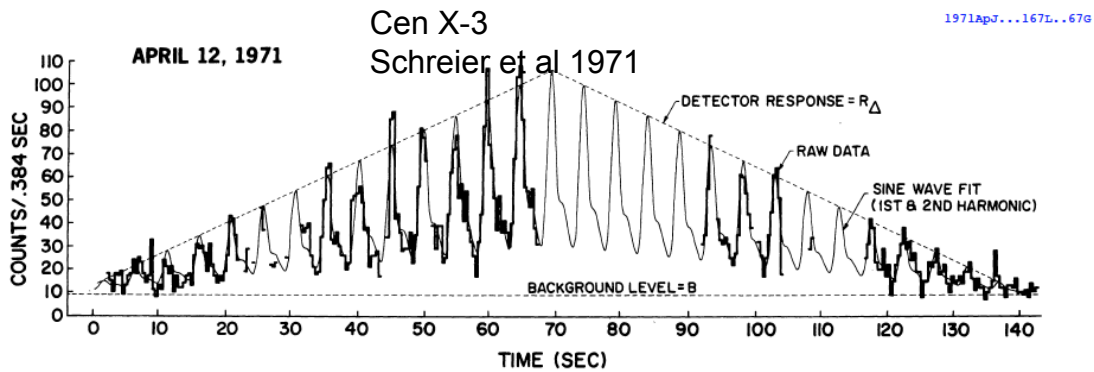
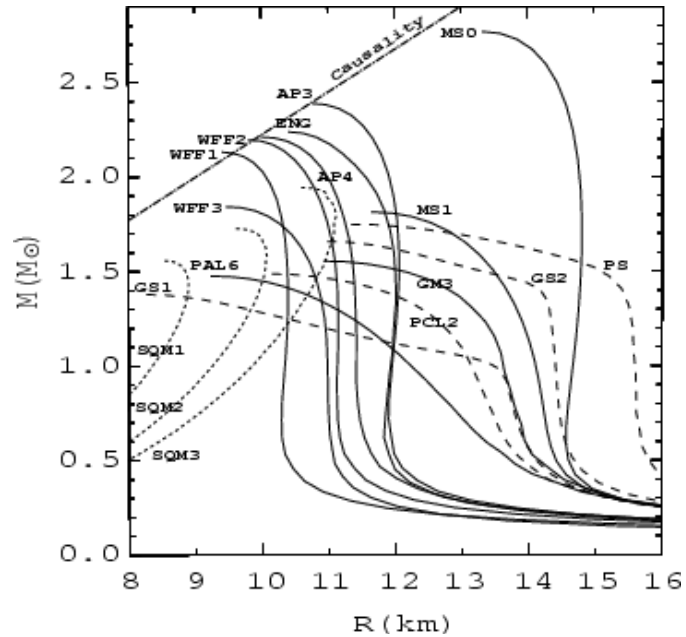


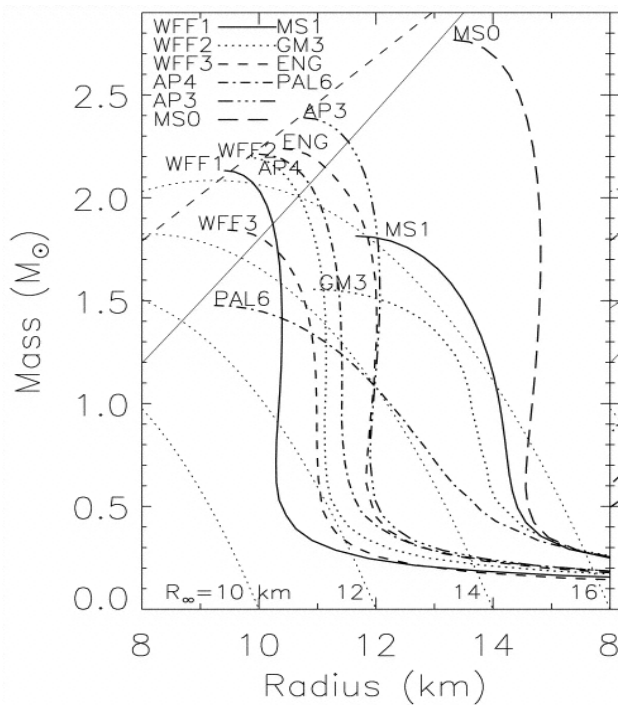
FIG. 2: Mass-radius diagram for neutron stars. Black (green) curves are for normal matter (SQM) equations of state [for definitions of the labels, see [27]]. Regions excluded by general relativity (GR), causality and rotation constraints are indicated. Contours of radiation radii R_∞ are given by the orange curves. The dashed line labeled $\Delta I/I = 0.014$ is a radius limit estimated from Vela pulsar glitches [27].

EOS of Neutron Star- Size/Mass Relation

- Rather Complex
 - Have to use General Relativistic form of hydrostatic equilibrium equation
 - Neutrons don't behave like an ideal degenerate gas...
- strong force interactions are crucial
 - There remain uncertainties about the “equation of state” of neutron stars



Fundamental Physics: The Neutron Star Equation of State (EOS)



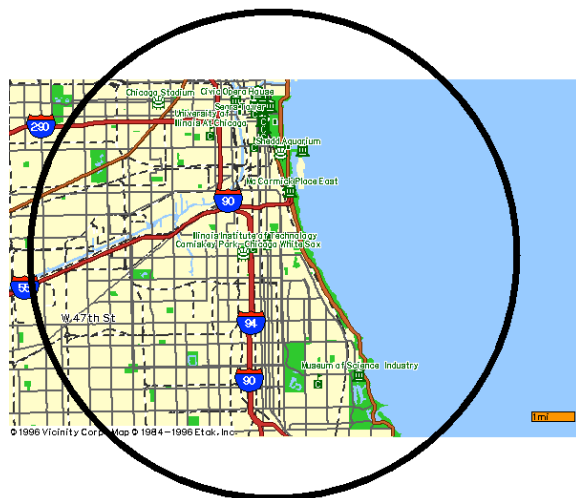
- High mass limit sets highest possible density achievable in neutron stars (thus, in nature, “the MOST dense”).
- Radius is prop. to $P^{1/4}$ at nuclear saturation density. Directly related to symmetry energy of nuclear interaction
- Other issues: have to use general relativistic eq for hydrostatic equilibrium
- Maximum mass measurements, limits softening of EOS from hyperons, quarks, other “exotica”.



Fun Fact for the Family

- one teaspoon of a neutron star has a mass of $\sim 5 \times 10^{12}$ kilograms.
- <http://videos.howstuffworks.com/nasa/13498-chandra-neutron-stars-video.htm>

C. Miller Neutron star vs. Chicago



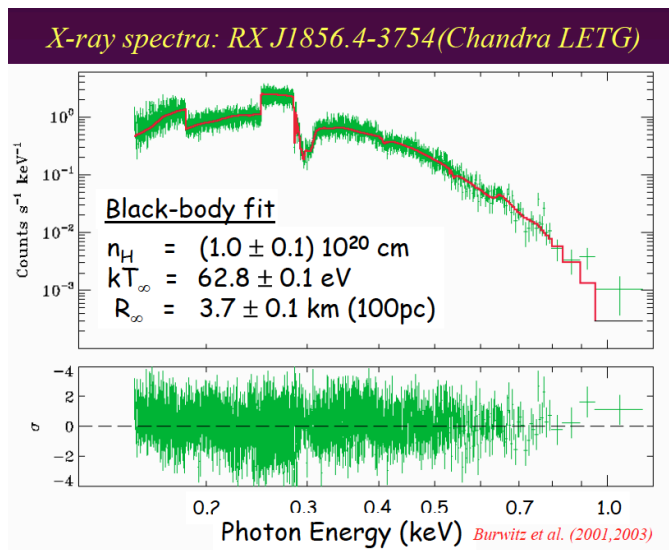
Mass= $1.4 M_{\text{sun}}$, Radius=10 km
Spin rate up to 38,000 rpm
Density $\sim 10^{14}$ g/cc, Magnetic field $\sim 10^{12}$ Gauss

A Quick Tour

- Some properties of the various 'types' of neutron stars....
- Isolated Neutron Stars
 - cooling
 - spinning (radio pulsars)
 - magnetars
- accreting neutron stars

Isolated Neutron Stars- Non Accreting

- These objects are cooling from the initial high temperature of the supernova explosion
- Recent results show that they have an almost pure black body spectrum-

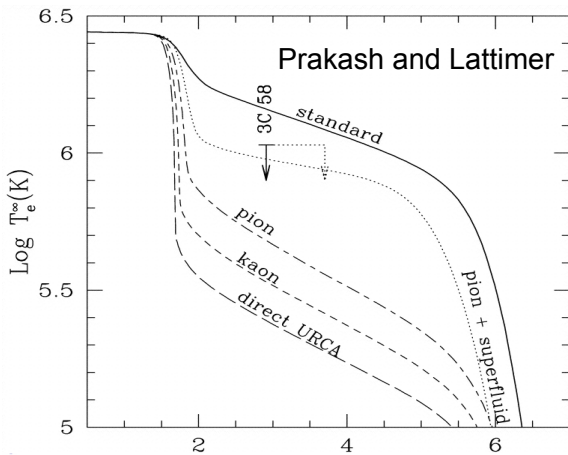


Burwitz et al 2001

Neutron Star Continuum Spectroscopy and Cooling

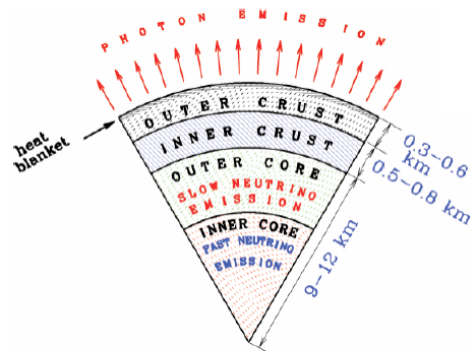
- After Neutron star is created in a supernova, if it is isolated it cools
- The rate at which it cools depends on the conductivity and heat capacity which depends on what it is made of and additional physics not well understood.

(L. Cominsky)



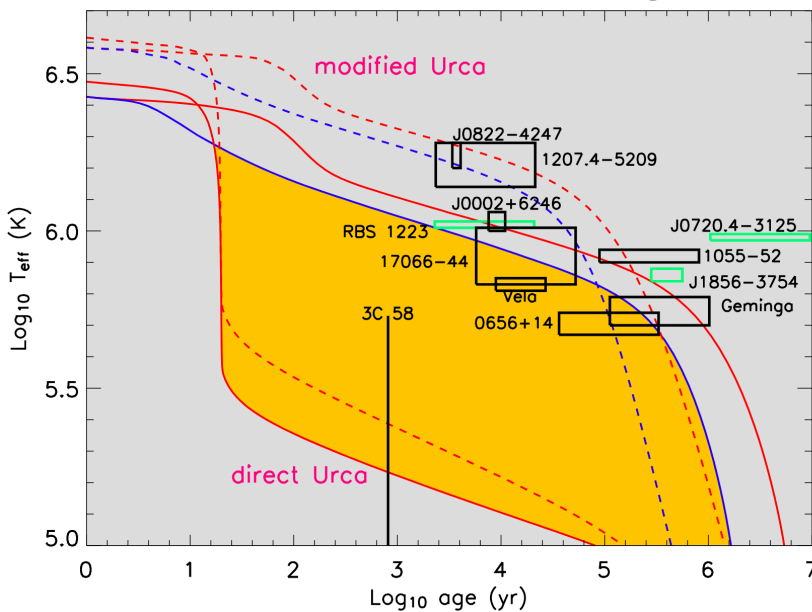
Main cooling regulators:

1. EOS
2. Neutrino emission
3. Superfluidity
4. Magnetic fields
5. Light elements on the surface



D.G. Yakovlev, O.Y. Gnedin*, M.E. Gusakov, A.D. Kaminker
K.P. Levenfish, A.Y. Potekhin

Cooling



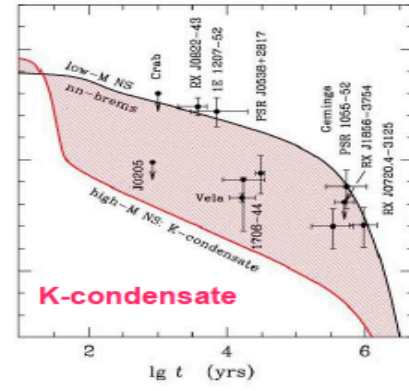
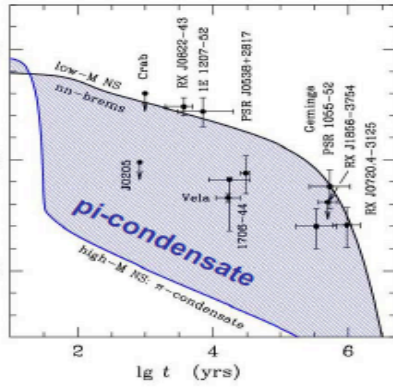
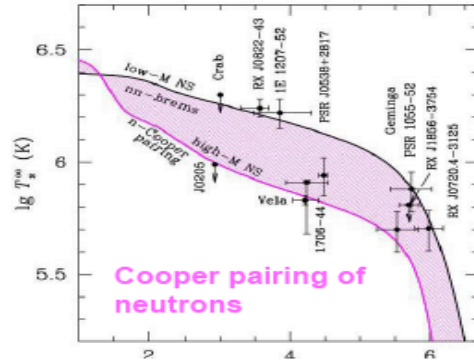
Observational estimates of neutron star temperatures and ages together with theoretical cooling simulations for $M = 1.4 M_{\odot}$.

- Lattimer and Prakash 2004

Interesting Physics- Will Not Discuss Further

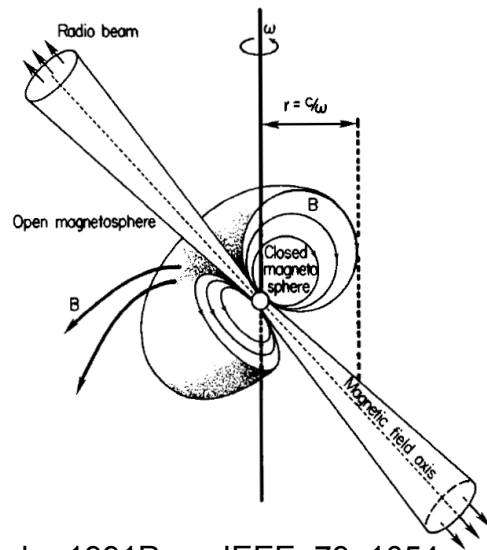
- The physics of how neutron stars cool depend critically on their exact composition

THEORY
versus
OBSERVATIONS



Isolated Neutron Stars Longair 13.5.1

- Most** isolated neutron stars that are **radio and γ -ray pulsars** –
 - rapidly spinning neutron stars that emit relativistic particles that radiate in a strong magnetic field
- The pulses originate from beams of radio emission emitted along the magnetic axis-the pulsar loses energy by electromagnetic radiation which is extracted from the rotational energy of the neutron star.
- to produce pulsed radiation from the magnetic poles, the magnetic dipole must be oriented at an angle with respect to the rotation axis and then the magnetic dipole displays a varying dipole moment
- Energy loss goes as $\Omega^4 B^2$**
- As they radiate the star spins down-visible for $\sim 10^7$ yrs



Taylor 1991Proc. IEEE, 79, 1054

- The shortest period (or angular velocity Ω) which a star of mass M and radius R can have without being torn apart by centrifugal forces is (approximately) $\Omega^2 R \sim GM/R^2$
- With an average density of the neutron star ρ , $\Omega \sim (G\rho)^{1/2}$
 - A rotation period of $P=2\pi/\Omega \sim 1$ sec requires density of 10^8 gm/cm^3
- To 'radiate' away the rotational energy $E_{\text{rot}} = 1/2 I\Omega^2 \sim 2 \times 10^{46} I_{45} P^{-2}$ ergs
 - $\tau_{\text{loss}} \sim E_{\text{rot}}/L \sim 60 I_{45} P^{-2} L_{37}^{-1} \text{ yr}$ ($I=2/5MR^2$)
 - Where the moment of inertia I is in units of 10^{45} gmcm^2
- If the star is spinning down at a rate $d\Omega/dt$ its rotational energy is changing at a rate $E_{\text{rot}} \sim I\Omega(d\Omega/dt) + 1/2(dI/dt)\Omega^2 \sim 4 \times 10^{32} I_{45} P^{-3} dP/dt$ ergs/sec
- However only a tiny fraction of the spindown energy goes into radio pulses- **a major recent discovery is that most of it goes into particles and γ -rays.**

Radiation Mechanism+ Magnetic Field

$$-dE/dt \sim \mu_0 \Omega^4 p_{m0}^2 / 6\pi c^3 \text{ .eq 13.33}$$

Where p is the magnetic moment

- This magnetic dipole radiation extracts rotational energy from the neutron star.
 - I is the moment of inertia of the neutron star,
- $-d[1/2I\Omega^2]/dt = -I\Omega d\Omega/dt = \Omega^4 p_{m0}^2 / 6\pi c^3$ and so $d\Omega/dt \propto \Omega^3$
- The age of the pulsar can be estimated if it is assumed that its deceleration can be described by a law $d\Omega/dt \propto \Omega^n$ if n is constant throughout its lifetime
- Using the relation $\tau = P / (2 dP/dt)$, the typical lifetime for normal pulsars is about $10^5 - 10^8$ years.
- If the loss of rotational energy is due to magnetic braking (see derivation in Longair 13.40-13.42) $B \approx 3 \times 10^{15} (P/dP/dt)^{1/2} \text{ T}$.

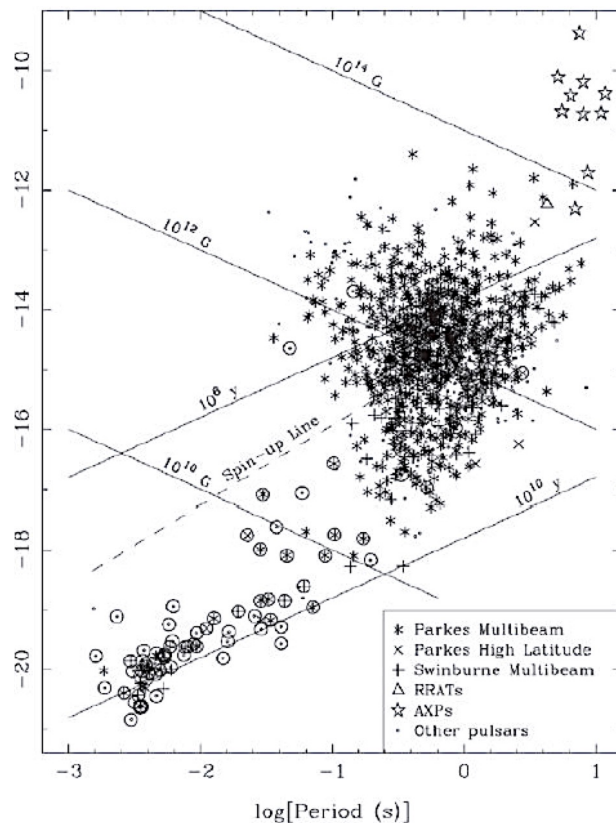
Radiation Mechanism

- It is conventional to set $n = 3$ to derive the age of pulsars and so $\tau = P/(2 dP/dt)$ (see derivation in Longair 13.35-13.37).
- Using this relation the typical lifetime for normal pulsars is about 10^5 – 10^8 years.
- extracting rotational energy $-dE_{rot}/dt = -I\Omega(d\Omega/dt) = -4\pi I(dP/dt)P^{-3}$ Longair 13.39
- In more useable units a rotating dipole has a Poynting flux of (Harding 2013)

$$\dot{E}_d = \frac{4\pi^2 I \dot{P}}{P^3} = \frac{2B_0^2 \Omega^4}{3R^6 c^5} = 10^{31} \text{ erg s}^{-1} B_{12}^2 P^{-4}$$

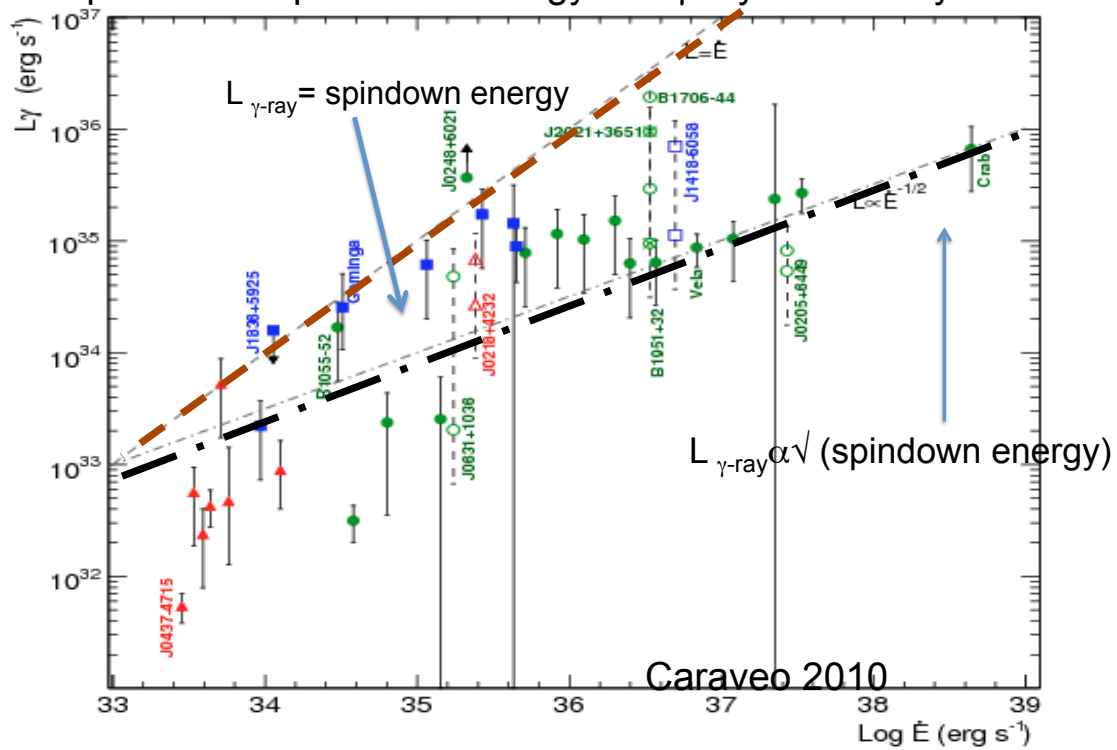
where $\Omega = 2\pi/P$ is the spin angular velocity, R the NS radius, B_0 is the surface magnetic field and $B_{12} \equiv B_0 / 10^{12}$ Gauss in units of 10^{12} Gauss

- Where radio pulsars lie in the $P, dP/dt$ plot.
 - the lines correspond to constant magnetic field and constant age.
- If magnetic braking mechanism slows-down the neutron star then (see eqs 13.40-13.42)
- $B_s \approx 3 \times 10^{15} (P/\{dP/dt\})^{1/2}$ T
B is in teslas



- The radio pulsations make up $\sim 10^{-4}$ or less of the spin-down power
- The pulsed radiation <10% of the total spin-down power.
- Most of the power in pulsed emission is in γ rays around a GeV
- γ -ray peaks are not in phase with the radio pulses, but typically arrive later in phase,

Graph
Comparison of Spin Down Energy and γ -ray Luminosity of Pulsars



Magnetars 13.5.5

Their defining properties occasional huge outbursts of X-rays and soft-gamma rays, as well as luminosities in quiescence that are generally orders of magnitude greater than their spin-down luminosities.

- Their are two classes: the ‘anomalous X-ray pulsars’ (AXPs) and the ‘soft gamma repeaters’ (SGRs)

Magnetars are thought to be young, isolated neutron stars powered by the decay of a very large magnetic field.

Their intense magnetic field, inferred via spin-down to be

- in the range 10^{14} - 10^{15} G is close to the ‘quantum critical field’
 $B_{\text{QED}} \equiv m_e^2 c^3 / \alpha h q = 4.4 \times 10^{13} \text{G}$, q = charge, α is the fine structure
- where the Landau level separation constant exceeds the rest mass energy of an electron, $m_e c^2 = 511 \text{keV}$.

In their most luminous outburst magnetars can briefly out-shine all other cosmic soft-gamma-ray sources combined [Kaspi 2010]

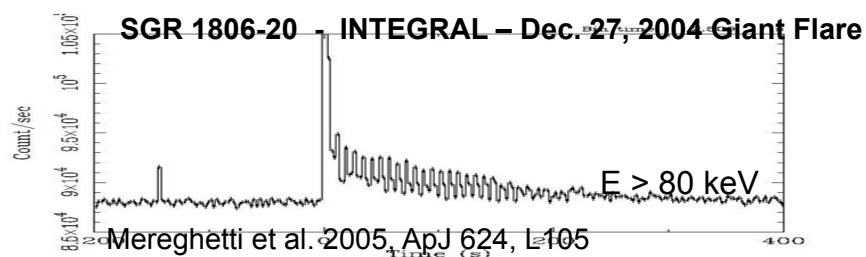
What is a Magnetar ?

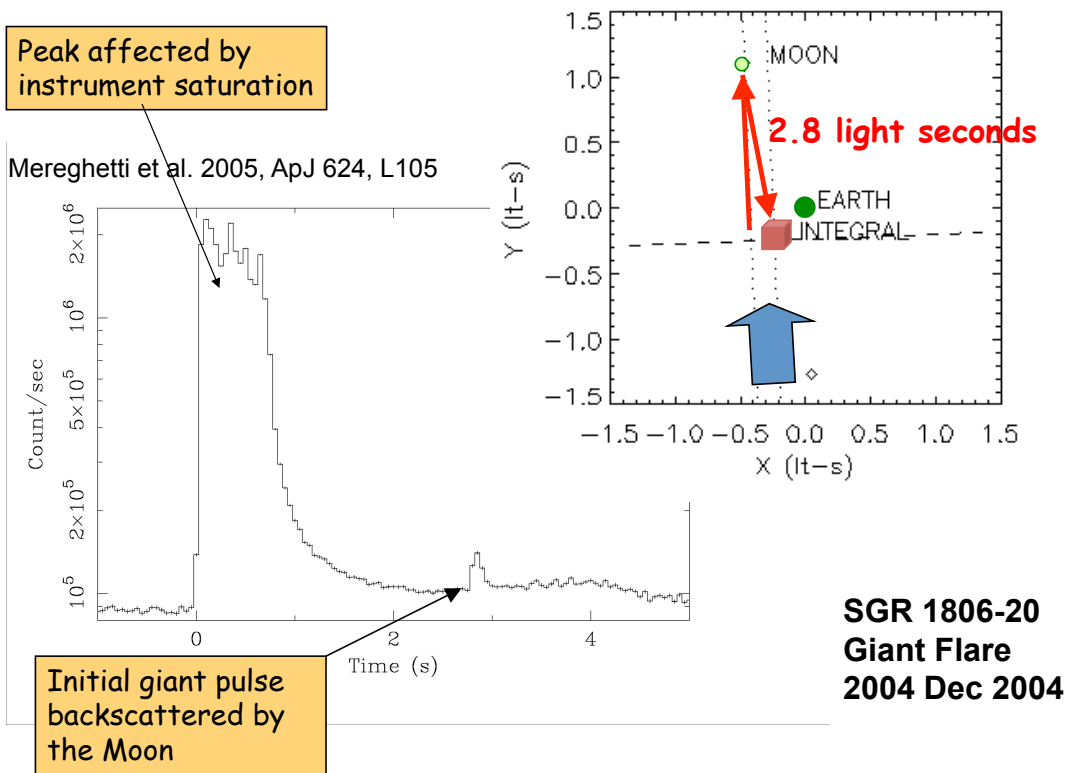
Isolated neutron star powered by magnetic energy $B \sim 10^{14}$ - 10^{15} Gauss-the origin of strong magnetism in neutron stars is not well understood

9 AXPs + 4 SGRs in our Galaxy and in Magellanic Clouds

Properties ?

- 1) “Persistent” X-ray emission
 $L_x \sim 10^{35} \text{erg/s}$; $kT \sim 0.5 \text{keV}$ +hard power law $P \sim 5$ - 12s
high spin-down 10^{-11} – 10^{-13}s/s ; $dE_{\text{ROT}} / dt \ll L_x$
- 2) Short (<1 s), super-Eddington bursts $kT \sim 30 \text{keV}$
- 3) Giant Flares - rare events! $L \sim 10^{44}$ – $>10^{46} \text{ergs}$





S.Mereghetti - Madrid 4-6 June 2007

Accreting Neutron Stars: Longair 13.5.2- Also Ch 14

- These are the brightest x-ray sources in the sky and were the first x-ray sources discovered
- They have a wide range of properties (spectral and temporal) and show an almost bewildering array of behaviors
- Their luminosities range over 6 orders of magnitude and are highly variable

