Lecture 5:

More Newton…

- Newton's Universal Law of Gravity-refresh
- Acceleration in circular orbits
- Weak equivalence principle
- Kepler's laws from Newtonian gravity
- The power of Newton's laws
- Age of the earth

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And if we have time reference frames and some puzzles…

- Real and fictitious forces

RECAP

- **Newton's 1st law** – $\underline{V}$ = *constant* if ***F*** = *0*
- **Newton's 2nd law** – $\underline{F} = M\underline{a}$
- **Newton's 3rd law** – for every action there is an equal and opposite reaction.
- **Galilean Transformation** – the "usual" velocity addition/subtraction rule for changing frames of reference ($v_{tot}=v_1+v_2$).
- **Galilean Relativity** – the idea that the laws of nature are the same for a moving observer as for a stationary observer.

V= velocity, **a**= acceleration, **F**=force, M=mass

I: NEWTON'S LAW OF UNIVERSAL GRAVITATION

Newton's law of Gravitation: A particle with mass m_1 will attract another particle with mass m_2 and distance r with a force F given by

$$F = \frac{G m_1 m_2}{r^2}$$

- "G" is called the Gravitational constant ($G = 6.67\times10^{-11}$ N m^2 kg^{-2} in mks units)
- This is a **universal** attraction. Every particle in the universe attracts every other particle! Gravity often dominates in astronomical settings.

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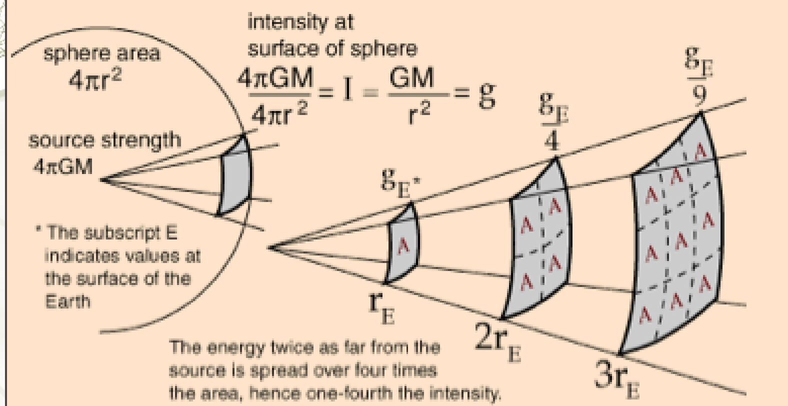
Inverse Square Law, Gravity

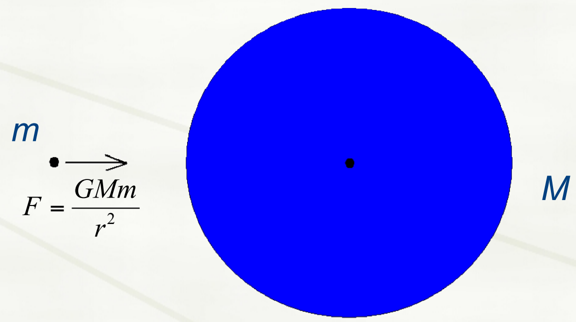
As one of the fields which obey the general inverse square law, the gravity field can be put in the form shown below, showing that the acceleration of gravity, g, is an expression of the intensity of the gravity field.



Other applications of the inverse square law	Electric field	Light	Sound	Radiation

http://hyperphysics.phy-astr.gsu.edu/hbase/forces/isq.html

- Newton's Law of Gravitation <u>defines</u> the "gravitational mass" of a body
- Using calculus, it can be shown that a spherical object with mass *M* (e.g. Sun, Earth) creates the same gravitational field as a particle of the same mass *M* at the sphere's center.



While it is rather difficult to prove, the only force laws that permits closed and stable orbits are $f(r) \sim r^{-2}$ (inverse square) and $f(r) \sim r$ (Hooke's law- works for springs)!

Inertial and gravitational mass: the weak equivalence principle

Newton's 2nd law says:

$$F = m_I a \qquad m_I\text{=inertial mass}$$

Newton's law of gravitation says:

$$F = \frac{GMm_G}{r^2} \qquad m_G\text{=gravitational mass}$$

So, acceleration due to gravity is:

$$a = \boxed{\left(\frac{m_G}{m_I}\right)} \frac{GM}{r^2}$$

So, if the ratio (m_G/m_I) varies, the rate at which objects fall in a gravitational field will vary…

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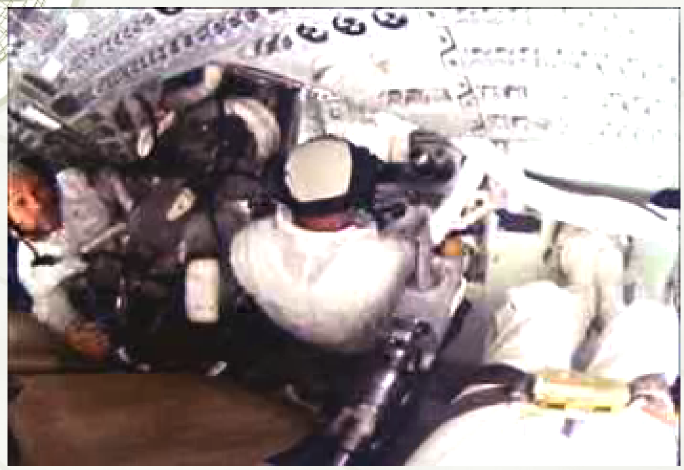
Equivalence of inertial and gravitational mass

- *Experimentally*, if all forces apart from gravity can be ignored, all objects fall at the same rate (first demonstrated by **Galileo**)
- So, m_I/m_G must be the *same* for all bodies- this has now been experimentally verified to 1 part in 10^{13}
- And we can choose the constant "G" such that $m_I = m_G$, and acceleration in a gravitational field is

$$a = GM/r^2$$

- This is the **weak equivalence principle**

"Weightlessness"



Apollo 10, in orbit (May 18-26, 1969)

No weight, or free-fall?

- Space Station orbits about 500km above Earth's surface. Radius of Earth is 6300km.
- Newton's inverse square law:
  - Gravitational acceleration at location of space station is 86% $(6300/6800)^2$ of what it is on the Earth's surface!
- So, why do the astronauts feel weightless?
  - The astronauts "fall" toward Earth at the same rate as the space station – another example of the equivalence principle (just like an elevator falling)

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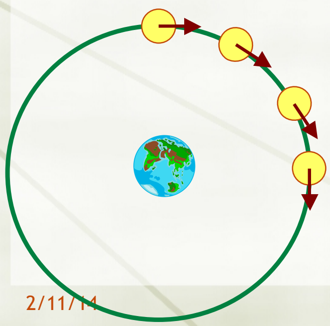
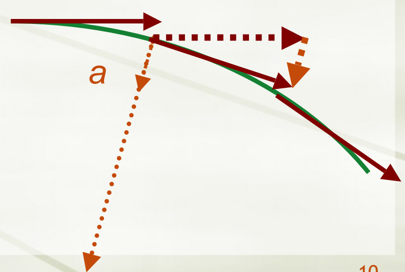
“Falling” in a circular orbit

- Newton arrived at the theory of gravity by imagining that the same force causes an apple to fall towards the Earth as the Moon to orbit Earth (continually “falling”), with a decrease in the magnitude of the force with distance squared
- In what sense is a body in orbit *“falling”*?
- *Direction* of acceleration (= rate of change of vector velocity) is always *directly towards center of orbit*
- *Acceleration* must be toward center because *gravitational force* is toward center, and **F=ma** is a **vector** equation

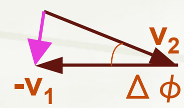


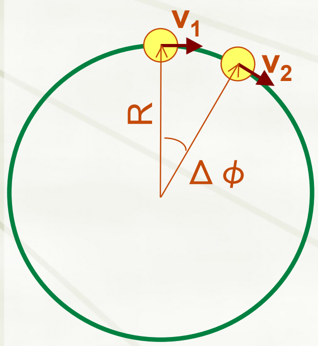


Acceleration in a circular trajectory

http://galileo.phys.virginia.edu/classes/109N/lectures/newtongl.html

$\Delta \mathbf{v} = \mathbf{v}_2 + (-\mathbf{v}_1)$





For small ϕ, $\Delta v \sim v \sin\phi \sim v \Delta\phi$

For constant speed v, the time it takes to go around is

$T = \text{perimeter}/v = 2\pi R/v$

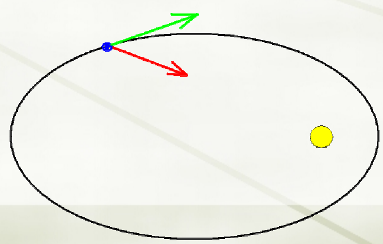
$\Delta\phi = \Delta t \, 2\pi/T = \Delta t \, v/R$

So $\Delta v/\Delta t = v \, \Delta\phi/\Delta t = v \, v/R$

$\underline{\mathbf{a}} = \Delta v/\Delta t = v^2/R$

KEPLER’S LAWS EXPLAINED!

- Kepler’s laws of planetary motion
  - Can be derived from Newton’s laws
  - IF the planets are attracted to the Sun by gravity with inverse square law (Newton’s breakthrough).
  - Full proof requires calculus (or very involved geometry)



- We're not going to prove this, but…
  - Newton's gravity law (*1/R^2*) is exactly what's needed to make this path be a perfect ellipse – hence Kepler's 1st law (see http://www.math.utk.edu/~freire/teaching/fall2006/m142f06NewtonKepler.pdf for the detailed math- for a very extended discussion see the file orbits.pdf on the web page)
  - The fact that the force is always towards Sun gives Kepler's 2nd law (equal areas in equal times ⇔ conservation of angular momentum).
  - Newton's 2nd law (**F** = m**a**) combined with his gravity law gives Kepler's 3rd law -- the relation between orbit period and semimajor axis

$$P^2 = \frac{4\pi^2}{GM_{sun}} R^3$$

Applications and impact of Newtonian physics

- With Newton's laws, it was possible to make **predictions** about orbits of solar system bodies
  - Halley argued that several comet appearances separated by 76 years were actually the same comet, and predicted its recurrence in 1758 (last return in 1910 1986)
  - Planets have near-elliptical orbits, but they are not *exact* ellipses due to gravity of *other* planets and thus show small deviations from Kepler's laws.

    Herschel, in 1781, discovered Uranus; its orbit showed enough variations to predict there must be another as-yet-unknown planet. In 1845, John Adams and Urbain Leverrier, predicted the existence of an unseen planet, to account for the fact that Uranus was being pulled slightly out of position in its orbit by the gravitational effect of an unknown body, and calculated its position and motion in the sky.

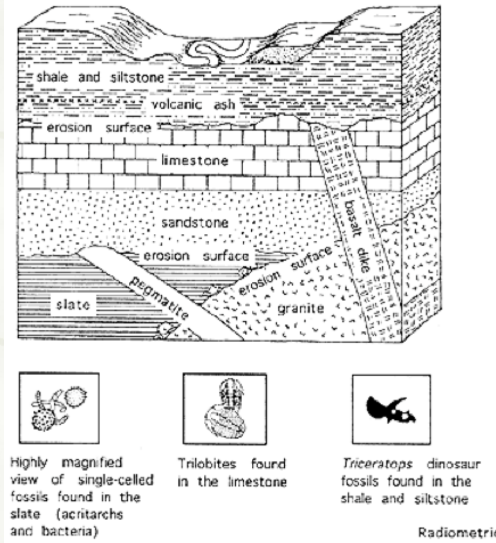
    observations confirmed this leading to the discovery of Neptune in 1846

Applications and impact of Newtonian physics

- Newton's laws can be applied to stars in galaxies, galaxies in clusters, etc., to understand orbits and "weigh" the system, since the mass is proportional to the inverse-square of the typical orbital period and cube of the orbital distance.
  - Its the *deviation* of the observations from our understanding of how much mass objects have that has led to the idea of dark matter.
- As Newton's physics came to be widely known, there was a huge cultural impact. With the Universe describable by precise mathematical laws, it supported the idea of "rationality" in other arenas -- including architecture, government, history, etc. Key to shift in thought known as the Enlightenment. The universe as a giant machine! (?)

The time domain

- With Copernicus, Kepler, Galileo, and Newton, ideas of the vast magnitude of space beyond Earth opened up
- What about *time*?
- Evidence from biology:
  - Important was finding progressions of fossils of (often extinct) creatures in successive layers of rock (late 1700's)
  - Physical progression in fossil features (invertebrates, fish, mammals) implied a biological transformation, in response to environmental changes.
  - Charles Darwin (1859) published *The Origin of Species* - evolution proceeds via natural selection. Much time would be required, since species are **observed** to change slowly. Darwin estimated at least "300 million" years based on ***geological*** evidence.



http://www.ucmp.berkeley.edu/fosrec/McKinney.html

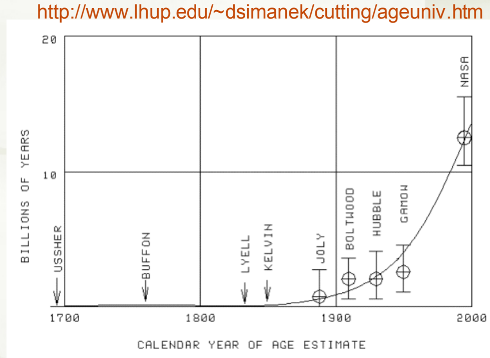
The time domain

- What about *evidence from geology and physics*
  - From ~1700-1850, the idea took hold that Earth was very old
  - Important was the realization that strata of rock take their observed physical form due to weathering, volcanism, etc, acting over very long periods (Charles Lyell: Uniformitarianism)

(earlier a French scholar, Bernard Palissy (1510-1589) believed the Earth was very old based on his observations that rain, wind, and tides were the cause for much of the present-day appearance of the Earth. **He was burned at the stake** in 1589)

http://abyss.uoregon.edu/~js/lectures/age_of_the_earth/age_of_the_earth.html

http://www.lhup.edu/~dsimanek/cutting/ageuniv.htm

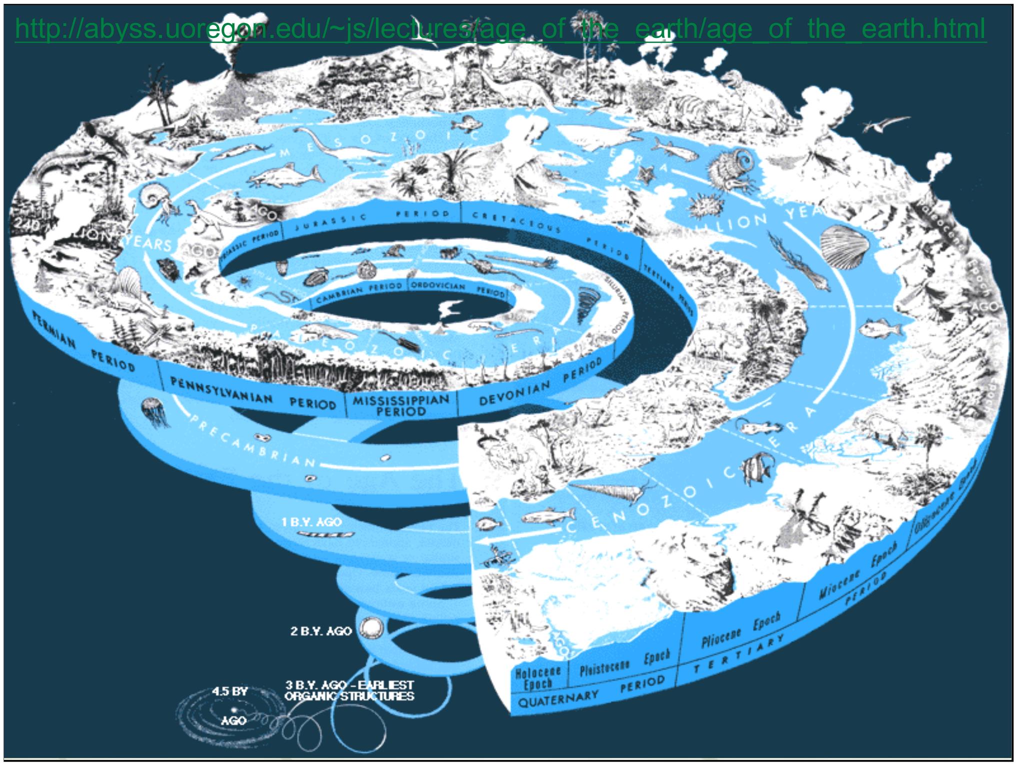


1760: Buffon ~1750 estimated the earth's age ~ 75,000 years by calculating its cooling from the molten state.

1831: Charles Lyell (~1830) estiamated an age of 240 Myrs based on fossils

1897: William Thomson (1824-1907) re- calculated the earth's cooling rate, age was between 20 and 400 Myrs.

1901: John Joly (1857-1933) calculated the rate of delivery of salt from rivers to oceans, determining the earth's age to be 90 to 100 Myrs



http://abyss.uoregon.edu/~js/lectures/age_of_the_earth/age_of_the_earth.html
MESOZOIC ERA
JURASSIC PERIOD
CRETACEOUS PERIOD
TERTIARY PERIOD
CAMBRIAN PERIOD
ORDOVICIAN PERIOD
PALEOZOIC ERA
PERMIAN PERIOD
PENNSYLVANIAN PERIOD
MISSISSIPPIAN PERIOD
DEVONIAN PERIOD
PRECAMBRIAN
CENOZOIC ERA
1 B.Y. AGO
2 B.Y. AGO
4.5 BY AGO
3 B.Y. AGO - EARLIEST ORGANIC STRUCTURES
Holocene Epoch
Pleistocene Epoch
Pliocene Epoch
Miocene Epoch
QUATERNARY PERIOD
TERTIARY PERIOD

"Red herrings" from physics, and a resolution

- In 1840-50's physicists Kelvin and Helmholtz argued that the only possible way for the Sun to power itself was by gravitational contraction (energy is released as the sun contracts)
- Comparing the rate energy is produced by the Sun (total observed luminosity) to the available gravitational energy, an age of 30 million years for the Sun was estimated

On the Age of the Sun's Heat

By Sir William Thomson (Lord Kelvin)

Macmillan's Magazine, vol. 5 (March 5, 1862), pp. 388-393.

http://zapatopi.net/kelvin/papers/on_the_age_of_the_suns_heat.html#fn2b

The Age of the Earth Debate, by Lawrence Badash, Scientific American, August 1989, p. 90.

Lord Kelvin was one of the great theoretical physicists of the 1800s. His work in thermodynamics led to the notion of absolute zero and the temperature scale is named in his honor.

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What Kelvin Actually Said

That some form of the meteoric theory is certainly the true and complete explanation of solar heat can scarcely be doubted, when the following reasons are considered:

(1) No other natural explanation, except by chemical action, can be conceived.

(2) The chemical theory is quite insufficient, because the most energetic chemical action we know, taking place between substances amounting to the whole sun's mass, would only generate about 3,000 years' heat.

(3) There is no difficulty in accounting for 20,000,000 years' heat by the meteoric theory

"Red herrings" from physics, and a resolution

- With development of nuclear physics in early 1900's, it was understood what was wrong with the Kelvin-Helmholtz argument: Sun's energy source is *fusion, not gravity*

  $4\,^{1}H \Rightarrow \,^{4}He + 2e^{+} + 2\nu$ +energy (Nobel Prize to H.Bethe)

  energy is 27 Mev (4×10^{-12} Joules/fusion)~2.5×10^{15}J/kg

- Based on our modern understanding of fusion the sun is now 4.5B years old and will live ~5B more years.
- http://www.nobelprize.org/nobel_prizes/physics/articles/fusion/

Chemistry vs Nuclear Fusion

- Kelvin dismissed the possibility that the Sun might be burning, because the fuels he knew could only maintain the Suns power output for a few thousand years.
- We can estimate the age of the Sun as follows. The chemical reaction that produces the most energy per unit mass is $2H + O \rightarrow H_2O$. Burning hydrogen can provide up to 143 MJ/kg (1 MJ = 10^6 Joules) per kilogram of fuel
- Sun is made out of hydrogen (ignoring where the oxygen comes from), then it could release a maximum energy

  $E = (143 \times 10^6$ J/kg$) \times (2 \times 10^{30}$ kg$)$* $= 3 \times 10^{38}$ J. The power radiated by the Sun, P, roughly equal to the total energy divided by the total lifetime. The Sun's lifetime is therefore $t = E/P = (3 \times 10^{38}$ J$)/(4 \times 10^{26}$ J/s$) = 7.5 \times 10^{11}$ s. **The Sun would last for only about 20,000 years at most- which caused Kelvin to doubt that this would work.**
- **Nuclear fusion is ~1×10^7 times more efficient leading to a life of 2×10^{11} yrs (!!!) if all the Hydrogen gets 'burned' into Helium**

* mass of sun

RadioActive Dating
totally unknown to Kelvin

- Starting in 1920's radioactive dating became possible
- Radioactive elements "decay" (that is, change into other elements) by "half lives." -the amount of time one half of the radioactive element will have decayed in ; one half of the remainder will decay in the next year (leaving one-fourth remaining), and so forth.

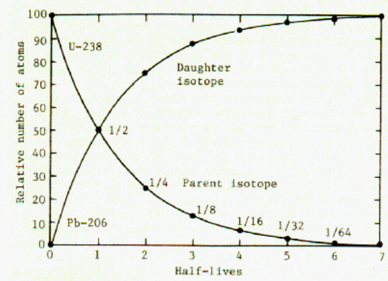
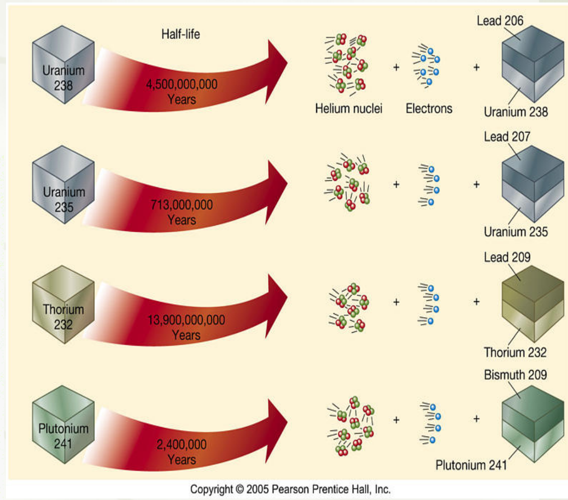


Figure 17.2 Relative decrease in parent isotope and increase in daughter isotope through seven half-lives.



http://www.fsteiger.com/radioact.html

hendrix2.uregon.edu

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Radioactive dating

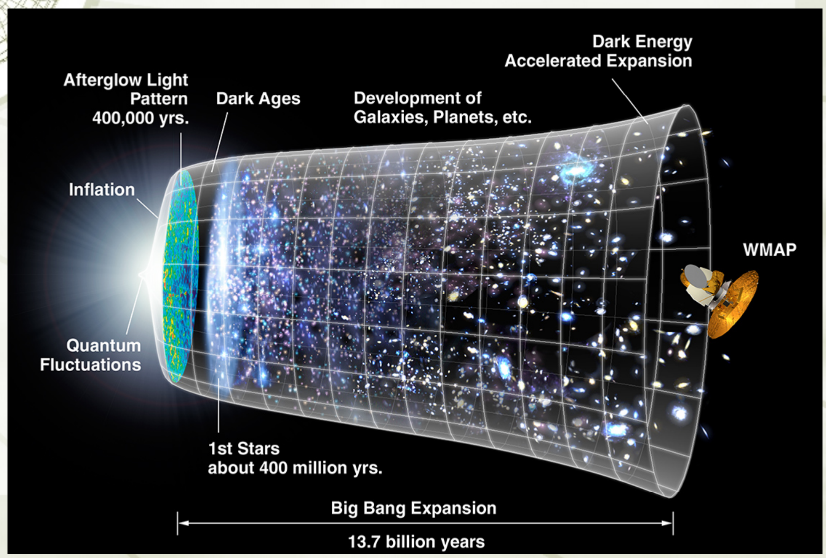
- Radioactive decay transforms one element into another, with a characteristic half-life
  - It is not affected by the physical conditions
- Age equation: $t = T_{1/2} \log(1+(D-D_0)/P)/\log 2$
  - D is the number of stable "daughter" atoms
  - P is the number of radioactive "parent" atoms
  - $T_{1/2}$ is the half-life (time that takes to convert 1/2 of the P atoms into D atoms)

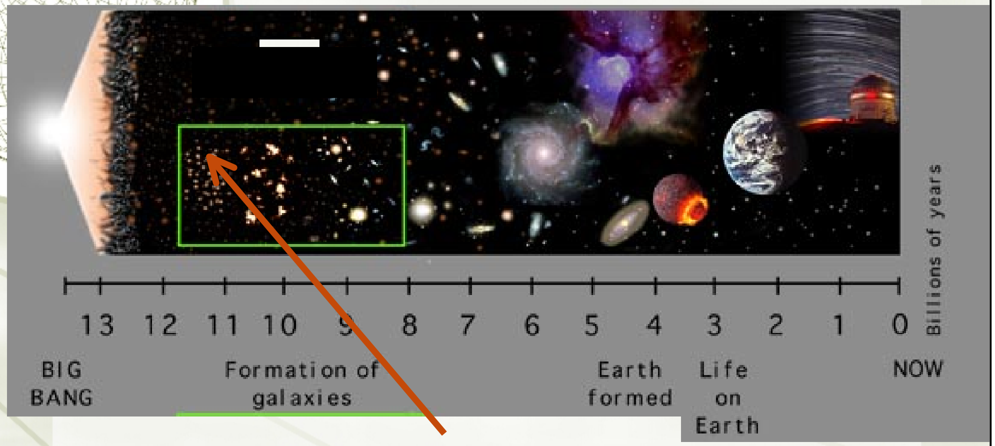
- Some reactions useful for planetary radioactive dating:
  - $^{238}U \rightarrow {}^{206}Pb$ ($T_{1/2}$ =4.47 billion years)
  - $^{235}U \rightarrow {}^{207}Pb$ ($T_{1/2}$ =704 million years)

RadioActive Dating

- To determine the fraction still remaining, we must know both the amount now present and also the amount present when the mineral was formed (and assume that nothing has happened to pollute the sample since)
- This is possible with several radioactive elements - the longest 'clock' is rubidium-87 which decays, with a half life of 48.8 billion years, to strontium-87. Strontium-87 is a stable element; it does not undergo further radioactive decay.

- Ages of meteorites, etc., establishes that the formation of the Solar system occurred about 4.6 billion years ago. Humans, 250,000 years (0.25 Myr); dinosaurs 100 Myr.
- Universe 13.6 Gyrs





Where we are going

INERTIAL AND NON-INERTIAL FRAMES OF REFERENCE

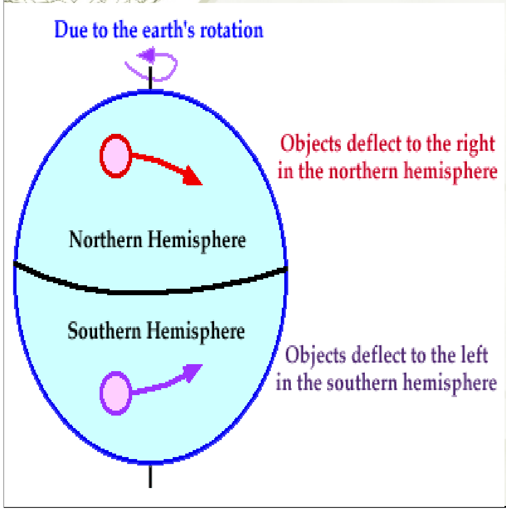
- Newton's laws were clearly powerful. But they also led to some puzzles, particularly relating to **reference frames**.
- We have already come across idea of **frames of reference that move with constant velocity**. In such frames, Newton's laws (esp. N1) hold. These are called **inertial frames of reference**.
- Suppose you are in an accelerating car looking at a freely moving object (i.e., one with no forces acting on it). You will see its velocity changing because you are accelerating! *In accelerating frames of reference, N1 doesn't hold* – this is a **non-inertial frame of reference**.

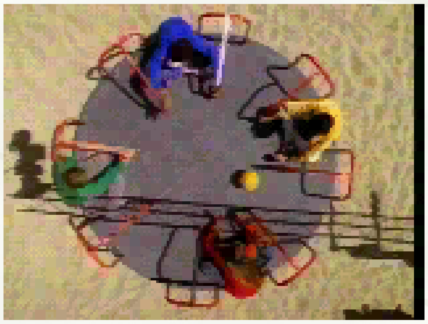
Real and fictitious forces

- In non-inertial frames you might be fooled into thinking that there were forces acting on free bodies.
- Such forces are call "fictitious forces". Examples –
  - G-forces in an accelerating vehicle.
  - Centrifugal forces in amusement park rides.
  - The Coriolis force on the Earth-a deflection of moving objects when they are viewed in a rotating reference frame. .
- Fictitious forces point opposite to the direction of acceleration
- Fictitious forces are always proportional to the inertial mass of the body.

- see pg 168 of text, figure 6.6

Coriolis 'Force'





Next lecture

Start reading Ch 6 PRINCIPLES OF SPACE AND TIME