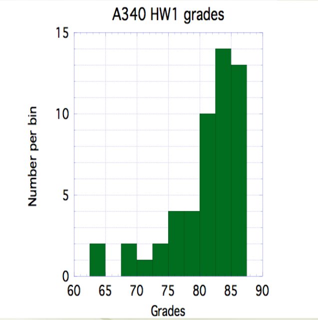
Homework

- 7 students did not hand it in ... if for some reason they were misplaced please tell me
- people did pretty well
- Homework 2 is due next Tuesday



Examples of time dilation

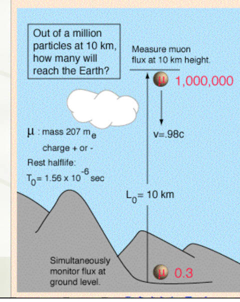
- The Muon Experiment
  - Muons are created in upper atmosphere from cosmic ray hits
  - Typical muon travel speeds are 0.99995×c, giving γ=100
  - Half-life of muons in their own rest frame (**measured in lab**) is t_h= 1.56 microseconds =0.00000156s
  - Traveling at 0.99995×c for t_h=0.00000156s, the muons would go only 468 m
  - But traveling for $\gamma \times t_h$= 0.000156s, the muons can go 46 km
  - *They easily reach the Earth's surface, and are detected!*
  - *Half-life can be measured by comparing muon flux on a mountain and at sea level; result agrees with $\gamma \times t_h$*
- Why muons- have comparatively long decay life time (the second longest known) and are relatively weakly interacting so they can penetrate the atmospher

- What would we observe if special relativity was not true

Muon Experiment

The measurement of the flux of muons at the Earth's surface produced an early dilemma because many more are detected than would be expected, based on their short half-life of 1.56microseconds. This is a good example of the application of relativistic time dilation to explain the increased particle range for high-speed particles.

Non-Relativistic



Distance: $L_0 = 10^4$ meters

Time: $T = \frac{10^4 \text{ m}}{(0.98)(3 \times 10^8 \text{ m/s})}$

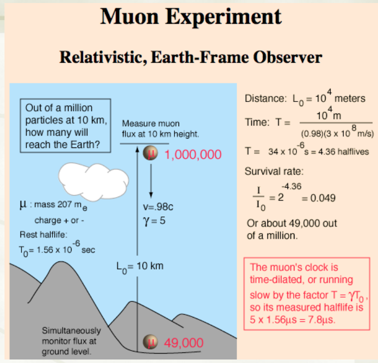
$T = 34 \times 10^{-6}$ s = 21.8 halflives

Survival rate:

$\frac{I}{I_0} = 2^{-21.8} = 0.27 \times 10^{-6}$

Or only about 0.3 out of a million.

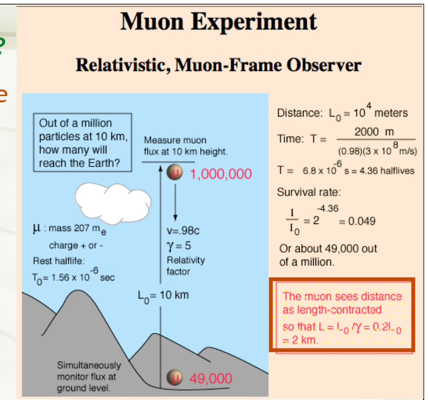
- Using special relativity what do 'we' see and predict



http://hyperphysics.phy-astr.gsu.edu/hbase/relativ/muon.html#c3

Muon frame

- what does the muon 'see'
- Different frame



Rossi & Hall Muon Experiment

- Classic experiment verifying time dilation was performed by Rossi & Hall in 1941…
  - Muons are "electron-like" particles… when at rest, they decay with a half-life of about 1.56μs
  - Muons are produced when cosmic rays slam into upper atmosphere, then rain down to Earth
  - Rossi & Hall measured the number of muons detected at the top of a 2000m mountain, and compared it to the number at sea-level…
    - Find 560 muons/hour at top of mountain
    - Even at v=c, will take 6.5μs for muon to travel 2000m
    - More than 4 half lives… less than 1/16th of particles should be left by the time they reach the bottom
    - BUT, they measured 422 muons/hour at bottom
    - It seems like only 0.64μs have passed in the muon's frame of reference… so time dilation formula says they are moving with $\gamma \approx 10$

Other Experimental Tests of Time Dilation

- Hafele and Keating, in 1971, flew caesium atomic clocks east and west around the Earth in commercial airliners, to compare the elapsed time against that of a clock that remained at the US Naval Observatory. Results were within 4% of the predictions of relativity.
- In 2010 time dilation was observed (Chou et al) at speeds of less than 10 meters per second using optical atomic clocks connected by 75 meters of optical fiber.
- More than 20 more experiments with decaying particles (pion,kaon,muons) in accelerators

I : More about time dilation…

the Twin's paradox

one of two twins travels at near the speed of light to a distant star and returns to the earth. Relativity dictates that when he comes back, he is younger than his identical twin brother.

BUT..."Why is the traveling brother younger?"-

relativity says that there is no absolute motion, wouldn't the brother traveling to the star also see his brother's clock on the earth move more slowly? If this were the case, wouldn't they both be the same age?

http://feegics.blogspot.com/2009/12/how-does-relativity-theory-resolve-twin.html

Twin Paradox

- The Earth and the ship are not in a symmetrical relationship: the ship has a turnaround - it undergoes non-inertial motion, while the Earth has no such turnaround.
- Special relativity does not claim that all observers are equivalent, only that all observers at rest in inertial reference frames are equivalent
  - Since there is no symmetry, it is not paradoxical if one twin is younger than the other.

Experimentally confirmed by Bailey et al. (1977), who measured the lifetime of positive and negative muons in the CERN Muon storage ring, muons were sent around a loop, so this experiment also confirms the twin paradox- agrees with Special relativity to accuracy of $2x10^{-3}$

Twin Paradox

- Its rather involved to do the math and it requires a particular type of diagram (Minkowski space time diagrams- which we will do a bit later) **please see the extra slides at the end of this lecture** (text pg 203-205)

  or

- http://www.einsteins-theory-of-relativity-4engineers.com/twin-paradox-2.html or

  http://www.oberlin.edu/physics/dstyer/Einstein/SRBook.pdf

  for a detailed solution

II : Length (Fitzgerald) contraction

- Think again about the muon experiment… but now from a muon's perspective!

- Fitzgerald contraction…
  - A moving object **contracts** by a factor γ (the same Lorentz factor) in the direction of motion
  - This is really ***a contraction of space itself***… the object does not experience forces or stresses that make it contract
  - Again, everything is relative… if someone watches you travel past them at high speed, you will appear to be contracted in the direction of motion

II: LENGTH CONTRACTION

- Consider two “markers” in space.
- Suppose spacecraft flies between two markers at velocity V.
- A flash goes off when front of spacecraft passes each marker, so that anyone can record it
- Compare what would be seen by observer at rest with respect to (w.r.t.) the markers, and an astronaut in the spacecraft…
- Observer at rest w.r.t. markers says:
  - Time interval is t_R; distance is $L_R=V\times t_R$
- Observer in spacecraft says:
  - Time interval is t_S; distance is $L_S=V\times t_S$
- We know from before that $t_R = t_S\,\gamma$
- Therefore, $L_S=V\times t_S= V\times t_R\times(t_S/t_R)=L_R/\gamma$
- *The length of any object is contracted in any frame moving with respect to the rest frame of that object, by a factor* γ
- In addition to time, length depends on your frame of reference !

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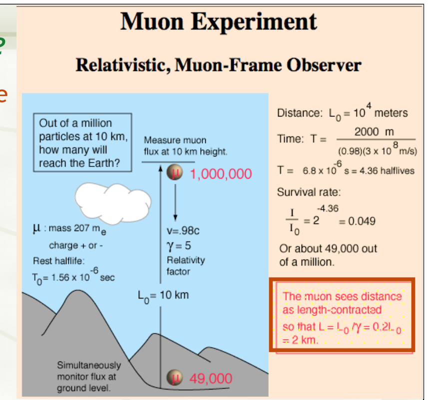
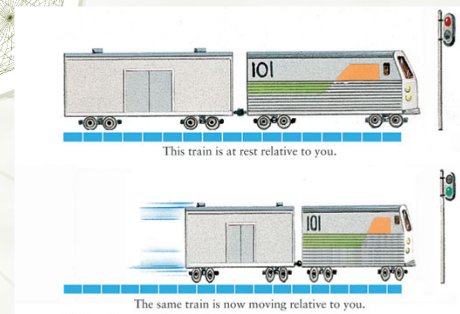
Muon frame

- what does the muon 'see'
- Different frame



Muon experiment, again

- Consider atmospheric muons again, this time from point of view of the muons
  - i.e. think in frame of reference in which muon is at rest
  - Decay time in this frame is 2 μs (2/1,000,000 s)
  - How do they get from top of the atmosphere to sea level before decaying?
- From point of view of muon, the atmosphere's height *contracts by factor of* γ
  - Muons can then travel reduced distance (at almost speed of light) before decaying.



(a) Length contraction

- A good youtube to watch is http://www.youtube.com/watch?v=xWST2gpbnvw- "Physics-X" lecture series, taught at Michigan Technological University by Dr. Robert Nemiroff

- Relativity In 5 Minutes

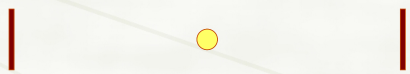
http://www.youtube.com/watch?v=KYWM2oZgi4E

- So, moving observers see that objects contract *along the direction of motion*.
- **Length contraction…** also called
  - Lorentz contraction
  - FitzGerald contraction

- Note that there is no contraction of lengths that are **perpendicular** to the direction of motion
  - *Recall M-M experiment: results consistent with one arm contracting*

III: RELATIVITY OF SIMULTANEITY

- Consider an observer in a room. Suppose there is a flash bulb exactly in the middle of the room.
- Suppose sensors on the walls record when the light rays hit the walls.



- **Since speed of light is constant**, light rays will hit opposite walls at precisely the same time. Call these events A and B.
- *This is what the movie last time showed*

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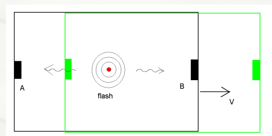
Change frames…

- Imagine performing same experiment aboard a moving spacecraft, and observing it from the ground.
- For the observer on the ground, the light rays will not strike the walls at the same time (since the walls are moving!). Event A will happen before event B.



- But astronaut in spacecraft thinks events are simultaneous.
- Concept of “events being simultaneous” (i.e. simultaneity) is different for different observers (**Relativity of simultaneity**).

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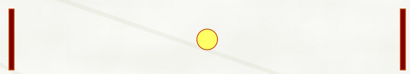

- But astronaut in spacecraft thinks events are simultaneous.
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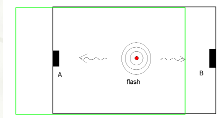
Change frames again!

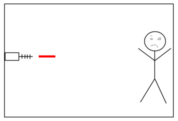
- What about perception of a 3rd observer who is moving faster than spacecraft?



- 3rd observer sees event B before event A
- So, order in which events happen can depend on the frame of reference.

The laser gun experiment

- Suppose there is a laser gun at one end of spacecraft, targeted at a victim at the other end.



- Laser gun fires (event A) and then victim gets hit (event B).
- Can we change the order of these events by changing the frame of reference? i.e., can the victim get hit **before** the gun fires?

- This is a question of **causality**.
- The events described are **causally-connected** (i.e. one event can, and does, affect the other event).
- In fact, it is **not possible** to change the order of these events by changing frames, according to Special Relativity theory.

- This is true provided that
  - The laser blast does not travel faster than the speed of light
  - We do not change to a frame of reference that is going faster than the speed of light
- To preserve the Principle of Causality (cause precedes effect, never vice versa), the speed of light must set the upper limit to the speed of anything in the Universe. Anything? Well, **anything that transmits any information.**

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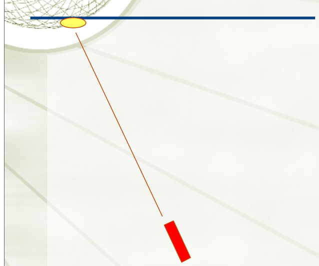
Some things "move" faster than light



- But they transmit no information
- E.g., light spot on a distant screen

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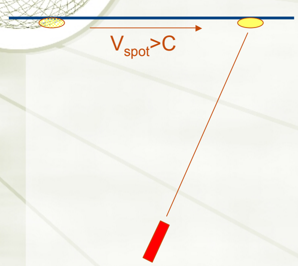
Some things "move" faster than light



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Causality

- Can causality be proved?
  - No, it is an axiom of physics
- What if causality doesn't hold?
  - Then the Universe returns to being random, unconnected events that can't be understood or predicted.
  - This would be a true "end of science."
- So we will *insist* on causality as we continue to explore relativity.

World Line and Light Cone

- In the short story 'Life-Line', Robert A. Heinlein describes the world line of a person
- He stepped up to one of the reporters. "Suppose we take you as an example. Your name is Rogers, is it not? Very well, Rogers, you are a space-time event having duration four ways. You are not quite six feet tall, you are about twenty inches wide and perhaps ten inches thick. In time, there stretches behind you more of this space-time event, reaching to perhaps nineteen-sixteen, of which we see a cross-section here at right angles to the time axis, and as thick as the present. At the far end is a baby, smelling of sour milk and drooling its breakfast on its bib. At the other end lies, perhaps, an old man someplace in the nineteen-eighties.
- "Imagine this space-time event that we call Rogers as a long pink worm, continuous through the years, one end in his mother's womb, and the other at the grave..."

Distances in space

- Two events A and B separated by distance Δs in space (x, y, z):

  $$\Delta s = [(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2]^{1/2}$$

  (Thanks, Pythagoras!)

  where $\Delta x = x_A - x_B$, $\Delta y = y_A - y_B$, $\Delta z = z_A - z_B$

*Distances in time **and** space*

- Two events A and B separated by distance Δs in time (t):

  $\Delta s = [(c\Delta t)^2]^{1/2}$

  where $\Delta t = t_A - t_B$, and we've multiplied by c to make the units of Δs come out as a distance

- Two events A and B separated in x and t:

  $\Delta s = [(c\Delta t)^2 - (\Delta x)^2]^{1/2}$

  one space dimension+time

Space-time intervals

- Two events A and B in space-time are separated by an **invariant interval**, given by (3-dimensions)

  $\Delta s = [(c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2]^{1/2}$

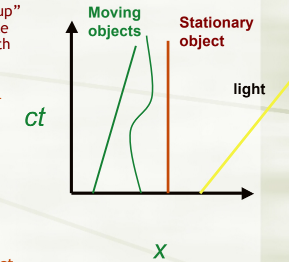
  where $\Delta t = t_A - t_B$, $\Delta x = x_A - x_B$, $\Delta y = y_A - y_B$, $\Delta z = z_A - z_B$,
- The formula is analogous to Pythagorean equation, but modified to account for the difference between space (x) and time (ct)
- The invariant space-time interval is an important quantity because it is **independent** of the frame in which it is measured; ***all*** observers agree on it!
  - This is true even though the Δt, Δx , *etc individually* are different for different observers (due to time dilation, space contraction)
  - The invariant interval is equal in value to (proper time of event) × c
- Space-time interval is zero for any two points on light ray world line
- Proper time between two events connected by a curved world line is computed by adding up results for small straight intervals along curve
  - Even if two curved world lines start and end at the same place, they may result in different proper time intervals

Proper Length and Time

- Proper length is the length of an object as measured in its own frame
  - Proper length is the largest possible length
- Proper time is the time as measured by a clock at rest with respect to the observers inertial frame
  - Proper time is the fastest rate

Space-time diagrams- pg 197 in text

- Because space and time are "mixed up" in relativity, it is often useful to make a diagram of events that includes both their space and time coordinates.
- This is simplest to do for events that take place along a line in space (one-dimensional space)
  - Plot as a 2D graph
  - use two coordinates: *x* and *ct (ct has the units of distance)*
- Can be generalized to events taking place in a plane (two-dimensional space) using a 3D graph (volume rendered image): *x*, *y* and *ct*
- a straight line represents an object moving at constant velocity (slope repsents its velocity)- a wiggly line an object that is accelerating.



World lines of events

world line of an object is the unique path of that object as it travels through 4-dimensional spacetime

World Line and Light Cone

- If we make the vertical axis c*t (speed of light times time) the line at 45 degrees represents objects going at the speed of light