Lecture9: *Special Relativity III & General Relativity I*

- A new velocity addition law
- Einstein's formula for energy
- Equivalence of mass and energy
- Mass turning into energy
- Energy turning into mass
- Redshifting of light
- Need for General Relativity

- Physics humor

the joke begins with the barman saying:"I'm sorry,we don't serve neutrinos."then the punchline:a neutrino walks into the bar.

Only if neutrinos travel faster than light

see www.pbs.org/wgbh/nova/physics/theory-behind-equation.html for a detailed history

"I love hearing that lonesome wail of the train whistle as the magnitude of the frequency of the wave changes due to the Doppler effect."

Last time…

- We discussed further aspects of special relativity,including:
  - Simultaneity and causality
  - Invariant intervals and proper time
  - Space-time diagrams

Last time…

- All these ***are consequences*** of Einstein's fundamental postulates
- Postulate 1–the laws of nature are the same in all inertial frames of reference
- Postulate 2–the speed of light in a vacuum is the same in all inertial frames of reference

- These two deceptively simple principles mark the most profound insights into the nature of the universe since Newton's work. From them, Einstein derived an entirely new picture of space and time.
- Postulate 1- The principle of equivalence has historically played an important role in the development of gravitation theory. Newton regarded this principle as such a cornerstone of mechanics that he devoted the opening paragraph of the Principia to it.

Special Relativity can be Important-

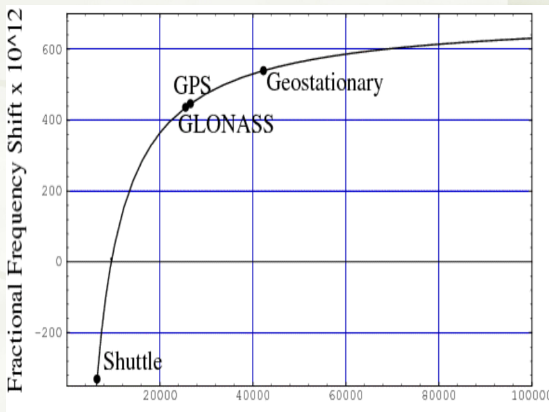
"Relativity in the Global Positioning System" by Neil Ashby *http://relativity.livingreviews.org/Articles/lrr-2003-1/*

- ... after initializing a Cesium clock, and leaving it alone for a day, it should be correct to within about 5 parts in 10^{14}, or 4 nanoseconds. **Relativistic effects are huge compared to this.**

- Although clock velocities are small and gravitational fields are weak near the earth, they give rise to significant relativistic effects. These effects include first- and second-order Doppler frequency shifts of clocks due to their relative motion, gravitational frequency shifts, and the Sagnac effect due to earth's rotation. If such effects are not accounted for properly, unacceptably large errors in GPS navigation and time transfer will result. In the GPS one can find many examples of the application of fundamental relativity principles. ... experimental tests of relativity can be performed with GPS, although generally speaking these are not at a level of precision any better than previously existing tests.

Relativistic Effects on GPS

- At height of GPS satellites effect ~$6x10^{-10}$ sec/sec- ~15km/day
- Five sources of relativistic effects contribute
- For the shuttle the velocity is so great that slowing due to time dilation is the dominant effect, while for a GPS satellite clock, the gravitational blueshift is greater. The effects cancel at a ≈ 9545 km.



I:NEW VELOCITY ADDITION LAW

- Einstein's theory of special relativity was partly motivated by the fact that Galilean velocity transformations (simply adding/subtracting frame velocity) give *incorrect results for electromagnetism*

- Once we've taken into account the way that time and distances change in Einstein's theory, there is a new law for adding velocities

- For a particle measured to have velocity V_p by a observer moving at velocity V_s to a stationary observer ,the particle's velocity as measured by the observer is

$$V = \frac{V_p + V_s}{1 + V_p V_s / c^2}$$

I: NEW VELOCITY ADDITION LAW

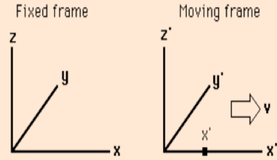
$$V = \frac{V_p + V_s}{1 + V_o V_s / c^2}$$

- Notice that if V_p and V_s are much less than c, the extra term in the denominator <<0 and therefore $V \sim (V_p + V_s)$
- Thus,the Galilean transformation law is *approximately correct* when the speeds involved are small compared with the speed o flight
- This is consistent with everyday experience
- Also notice that if the particle has V_s=c in the moving frame,then it has V_p=c in the stationary frame
- the speed of light is frame-independent!

(algebra e.g $V_p + V_s = 2c$; $V_p * V_s = c^2$; $2c/(1+1) = c$)

Summary

Galilean Transformation



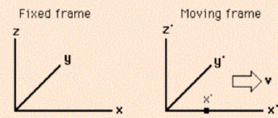
Galilean Transformation

$x' = x - vt$
$y' = y$
$z' = z$
$t' = t$

The primed frame moves with velocity v in the x direction with respect to the fixed reference frame.
The reference frames coincide at t=t'=0.
The point x' is moving with the primed frame.
The Galilean transformation gives the coordinates of the point as measured from the fixed frame in terms of its location in the moving reference frame.

The Galilean transformation is the common sense relationship which agrees with our everyday experience.

Lorentz Transformation



$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The primed frame moves with velocity v in the x direction with respect to the fixed reference frame. The reference frames coincide at t=t'=0. The point x' is moving with the primed frame.

The reverse transformation is:

$$x = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\beta = \frac{v}{c}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Much of the literature of relativity uses the symbols β and γ as defined here to simplify the writing of relativistic relationships.

http://hyperphysics.phy-astr.gsu.edu/hbase/relativ/relcon.html#c1

Now Onto New Stuff

- Einstein's formula for energy
- Equivalence of mass and energy
- Mass turning into energy
- Energy turning into mass
- Redshifting of light
- Need for General Relativity

Red/blueshifting of light- Doppler Effect

- Photons(light particles) are massless,but their energy changes when observer's frame changes
  - Recall (see Chapter4 of text for a review!) light has a wave/particle dual nature
  - Energy of a photon is proportional to the frequency ν of the corresponding wave:E=hν - wavelength λ=c/ν
    - h=6.63×10^{-34}Joule-s (Plancks constant)
  - When changing frames with a velocity ***V***,the frequency of the light waves and energy of the photons changes by a factor

$$\sqrt{\frac{1+\frac{V}{c}}{1-\frac{V}{c}}} = \left(1+\frac{V}{c}\right)\times\gamma$$

  - Moving towards a light source, the frequency and energy increase by this factor=**blueshift** (*bluer*,not necessarily blue)
  - Moving away from a light source, the frequency and energy decrease by this factor=redshift (*redder*,not necessarily red)
  - This is the Doppler effect

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Einstein's Derivation of $E=mc^2$

- Einstein considered a body at rest with mass M. If the body is examined in a frame moving with nonrelativistic velocity v, it is no longer at rest and in the moving frame it has momentum P = Mv.
- Let the body emits two pulses of light to the left and to the right, each carrying an equal amount of energy E/2. In its rest frame, the object remains at rest after the emission since the two beams are equal in strength and carry opposite momentum.
- But if the same process is considered in a frame moving with velocity v to the left, the pulse moving to the left will be redshifted while the pulse moving to the right will be blue shifted (Doppler effect). The blue light carries more momentum* than the red light, so that the momentum of the light in the moving frame is not balanced: the light is carrying some net momentum to the right.
- The object has not changed its velocity before or after the emission. Yet in this frame it has lost some right-momentum to the light. **The only way it could have lost momentum is by losing mass**

* Physics: for a photon momentum, p, $p=h/\lambda$ where h is Plancks constant and λ is the wavelength

Einstein's Derivation cont

- At low velocities the right moving light is blueshifted by the NR Doppler effect factor (1-v/c)
- The momentum of light is its energy divided by c and so the right moving light has 'extra' momentum $\Delta p=(v/c)(E/2c)$
- the left-moving light carries a little less momentum, by the same amount
- So the total momentum 'lost' is $2\Delta p=vE/c^2$
- The momentum of the object in the moving frame after the emission is reduced by this amount: $p'=Mv-2\Delta p= (M-vE/c^2)v$
- So the change in the objects mass is equal to its total energy divided by c^2
- emission of energy is accompanied by a loss of mass. Similarly, by considering absorption, a gain in energy is accompanied by a gain in mass. Einstein concludes that the mass of a body is a measure of its energy content

II:MASS AND ENERGY

see http://www.osti.gov/accomplishments/nuggets/einstein/speedoflight.html for details on how Einstein worked this out

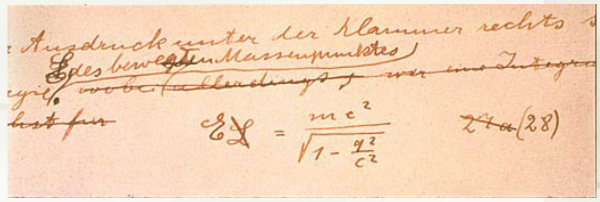
- Einstein reworked Newton's laws of mechanics using his new relativistic formulae.
- He found a formula for the energy of a moving object with mass *m* and speed *V*–

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1 - V^2/c^2}}$$

- Thus energy increases as the speed increases, and energy would become infinite if *V* approaches *c*

Einstein's Insight

- Einstein speculated $E=mc^2$ was not simply an academic exercise; he believed that it might explain how an ounce of radium could emit 4,000 calories of heat per hour indefinitely (Marie Curie), seemingly violating the first law of thermodynamics-

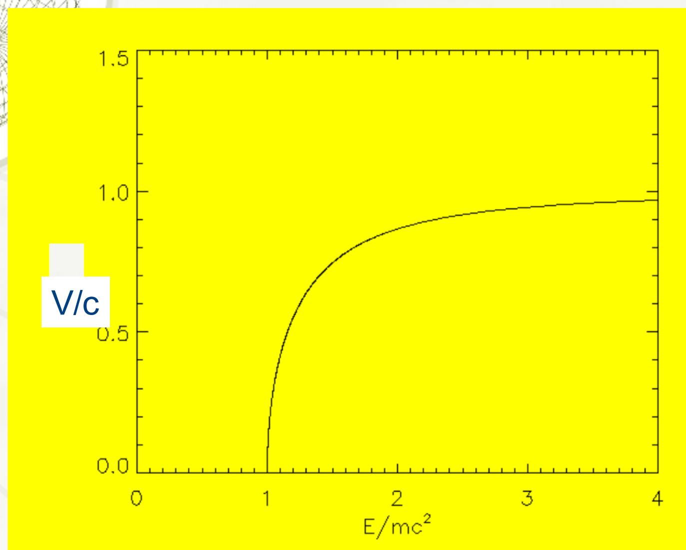


Once again, relativity forced a major revision in classical physics.
Before, the first law of thermodynamics, which states that the total amount of energy can never be created or destroyed.

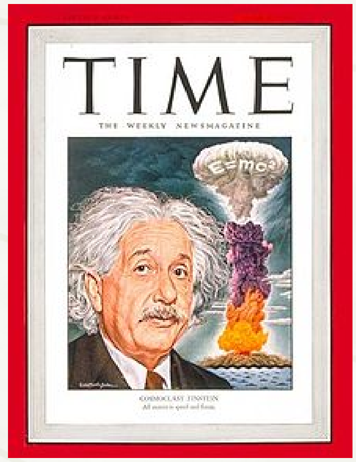
Now the total combined amount of **matter and energy** is the conserved quantity.

adapted from text by Michio Kaku

Energy vs. V/c



Connection of Einstein and the Atomic Bomb



Newton is just slightly wrong

- What about objects moving at "small velocity"? E.g. v <<c
- It can be shown that:(e.g expand 1/sqrt(1-v^2/c^2))

$$E \approx mc^2 + \frac{1}{2}mV^2$$

  - 1/2mv^2 is the Newtonian expression for the **kinetic energy** of a moving object.
- How small is "small velocity"?
  - For car going at 30 mph,approximate formula is wrong by 1 part in10^{30}
  - For a rocket going at 30,000 mph,this approximate formula is wrong by 1 part in 10^{18}
  - So,approximation is fine for all velocities experienced in every day life.

Rest mass energy

- If we put V=0 in Einstein's energy formula,we get…

$$E = mc^2$$

- What does this mean?
  - Maybe it is some fundamental "irreducible" (i.e.inaccessible) energy that every object possesses?
  - Or perhaps this energy can be accessed? In other words, maybe mass can be turned into"usable"energy? **YES!**
  - Also,this can go the other way-energy can be turned into mass.
  - mass and energy are equivalent-particle physicists measure the mass of their particles in energy units-the mass of a electron is 511keV=8.16x10^{-14} watts

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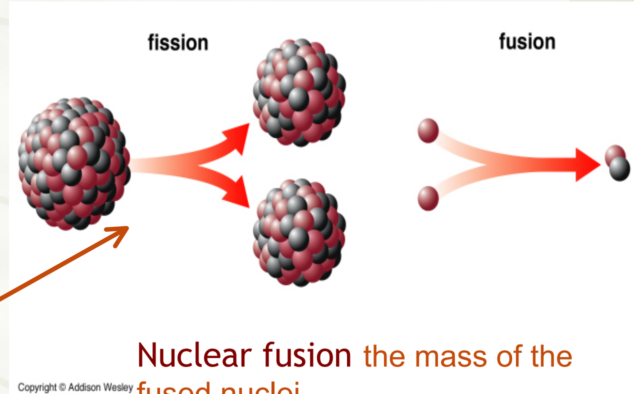
III:EXAMPLES OF CONVERTING MASS TO ENERGY

- Nuclear fission
- Nuclear fusion
- Radioactive decay



Nuclear fission the sum of the mass of the 2 daughters is less-the **difference is energy**

Nuclear fusion the mass of the fused nuclei is less than the sum of the two input nuclei **-the difference is energy**

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Fission

- Nuclear fission (e.g.,of Uranium)
  - Nuclear Fission–the splitting up of atomic nuclei
  - E.g.,Uranium^{235} nuclei are split into fragments when smashed by a (appropriately) moving neutron.One famous nuclear reaction is that used in the atomic bomb (another episode in which Einstein had a hand)

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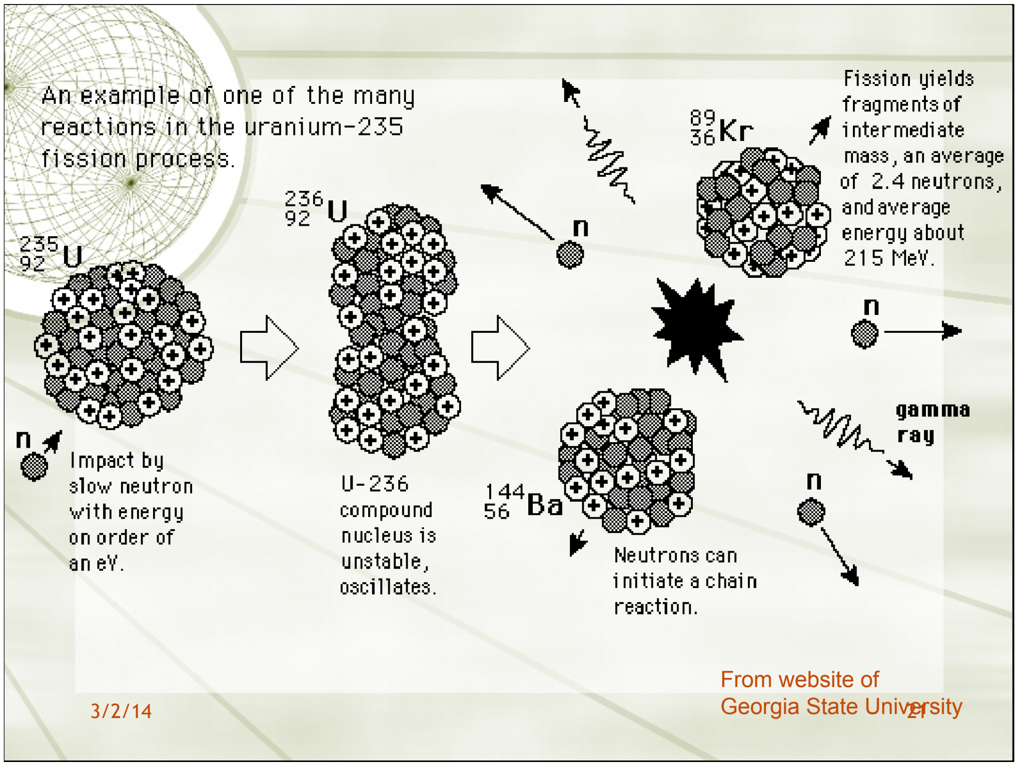

An example of one of the many reactions in the uranium-235 fission process.
235 92 U
n
Impact by slow neutron with energy on order of an eV.
236 92 U
U-236 compound nucleus is unstable, oscillates.
n
89 36 Kr
Fission yields fragments of intermediate mass, an average of 2.4 neutrons, and average energy about 215 MeV.
n
gamma ray
144 56 Ba
n
Neutrons can initiate a chain reaction.
From website of Georgia State University
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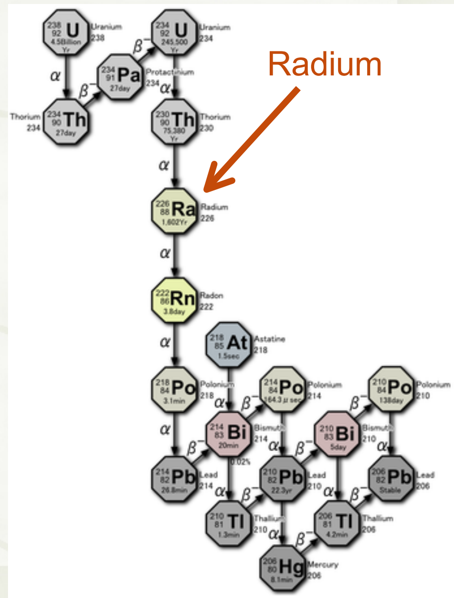
RadioActive Decay

- Decay of radium produces 4.87 MeV/decay and releases an alpha particle decaying into radon
- radium has a 1602 year half life.- remember the importance of radioactive half-lives in measuring the age of the earth
- this is transmutation of the elements - the alchemists dream



Fission

- Nuclear fission(e.g.,of Uranium)
  - Mass of products of reaction (neutrons,Krypton, Barium) is slightly **less** than mass of initial Uranium nucleus+neutron
  - The mass “lost” is converted into energy (gamma-rays and kinetic energy of fragments):
  - $E=mc^2$

Fusion

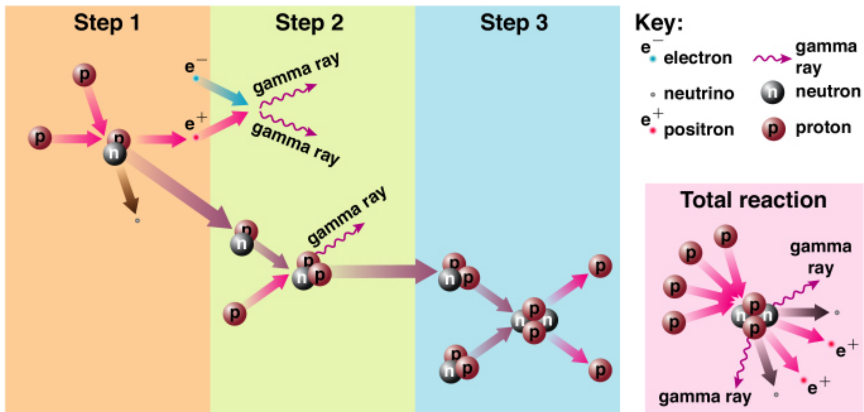
- Nuclear fusion (e.g.Hydrogen into Helium)
  - Fusion-the sticking together of atomic nuclei
  - Much more important for Astronomy (and life on Earth!) than fission- much harder to make controlled fusion on earth
    - Power source for stars,including the Sun
    - Path to making heavy elements (C,N,O,Si,Fe…)
  - Important example-hydrogen fusion.
    - 'Ram together' 4 hydrogen nuclei to form helium nucleus
    - Spits out couple of“positrons” and “neutrinos”in process

$$4\ {}^{1}H \rightarrow {}^{4}He + 2e^{+} + 2\nu$$

Fusion

- Mass of initial helium nucleus plus positrons and neutrinos is less (by about 0.7%)than original 4 hydrogen nuclei $E=mc^2$
- Mass has been converted into energy(gamma-rays and kinetic energy of final particles)
- This nuclear reaction(and similar ones) is the energy source for…
  - Hydrogen Bombs (about1kg of mass converted into energy gives equivalent of 20Megatons of TNT)
  - the Sun converts about4×10^9kg of matter per second into energy,ultimately yielding sunlight)- remember Lord Kelvin's problem

the proton-proton fusion cycle

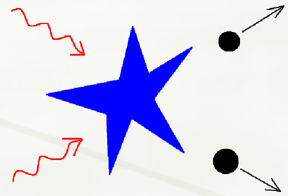


Antimatter

- Anti-matter
  - For every kind of particle,there is an antiparticle…
    - Electron↔anti-electron (also called a positron)
    - Proton↔anti-proton
    - Neutron ↔ anti-neutron
  - Anti-particles have opposite properties from the corresponding particles(e.g.,opposite charge)…but exactly the same mass.
  - When a particle and its antiparticle meet,they completely annihilate each other…**all of their mass is turned into energy (gamma-rays)!**

III:EXAMPLES OF CONVERTING ENERGY TO MASS

- Particle/anti-particle production
  - Opposite process to that just discussed!
  - Energy(e.g.,gamma-rays) can produce particle/anti-particle pairs

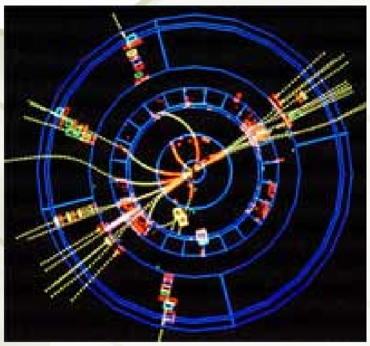


  - **Very fundamental process in Nature…**this process,operating in the early universe, is responsible for all of the mass that exists today!

conservation of energy sets a minimum photon energy required for creation of a pair of fermions: this threshold energy must be greater than the total rest energy of the fermions created.

To create an electron-positron pair the total energy of the photons must be at least $2m_ec^2$=2x0.511KeV=1.022MeV

- Particle production in a particle accelerator
  - Can reproduce conditions similar to early universe in modern particle accelerators…



CERN

General Relativity

see http://forum-network.org/lecture/was-einstein-right

- So far we have discussed only inertial frames
  - Need to take acceleration and gravity into account
- Need to incorporate special relativity ideas of transformation of frames, time and distance into Newton's Laws of force and gravity
- Importance of equivalence principle: gravity is completely indistinguishable to any other acceleration

General Relativity

- Like special relativity, the general theory predicts phenomena which differ significantly from those of classical physics,
  - especially concerning the passage of time,
  - the geometry of space,
  - the motion of bodies in free fall, and the propagation of light.
- Examples of such differences include gravitational time dilation, gravitational lensing, the gravitational redshift of light, and the gravitational time delay

First Newtonian mechanics (special relativity), now his law of gravity (general relativity)

- As we have just learned we have to understand
  - In whose frame do we measure *r*?
  - Does the force depend based on your reference frame?
  - Can gravity information travel (communicate) faster than *c*?

General Relativity

http://forum-network.org/series/nova-einsteins-big-idea-series

- General relativity is crucial for interpreting the 'new' phenomena discovered in astrophysics in the last 40 years
  - Black holes- active galaxies (quasars)
  - Pulsars
  - Accreting Neutron stars (x-ray sources)
  - Microwave background from the big bang
  - Gamma-ray bursts
  - Gravitational lenses
  - Hubble expansion of the universe

General Relativity

- its hard to fully calculate but NOT that hard

- On 6 November 1919, at Carlton House in London presentation on the first test of GR
- Isn't it true, my dear Eddington, that only three persons in the world understand relativity?" Silberstein confidently expects the obvious, polite reply, "But, apart from Einstein, who, my dear Silberstein, who, if not you . . . and I, if you allow me."

Eddington, however, remains aloof, silent, amused. Silberstein insists: "Professor Eddington, you must be one of the three persons in the world who understand general relativity." To which Eddington, unruffled, replies, "On the contrary, I am trying to think who the third person is!"

More than two centuries earlier, a student passed Newton on a Cambridge street and observed in a hushed voice: "There goes the man who has written a book that neither he nor anyone else understands."